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COMPLETE

ARITHMETIC

ELEMENTS AND COMMERCIAL

NEW SERIES

BY THE

BROTHERS OF THE CHRISTIAN SCHOOLS

46 SECOND STREET, NEW YORK

1888

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PREFACE.

THE leading propositions in the science of Arithmetic are here presented in accordance with the methods in use in the classes of the Brothers of the Christian Schools. Those methods are the result of the united experience flowing from two centuries of teaching. Their chief merit is this: that they dispense with the old way of simply getting the rules by rote and then applying them; in the stead, they speak to the child's intelligence, and lead him up gradually to the understanding of the principles upon which the rules are based. Knowing both the rule and the reason for it, the child no longer labors in the dark, and the teaching of Arithmetic ceases to be mere machine-work. He may afterwards forget the rule; but having learned to reason the problem out independently of a set form of words, he finds no difficulty in reaching the solution of any ordinary case to which he may apply himself.

Indeed, there is no other efficient method of teaching Arithmetic than this of appealing to the child's intelligence. The correct solution of every problem is

an exercise of judgment. It is therefore a good means of mental discipline. Mere memory-work will not produce the same result; it will rather cramp and impede the other faculties. It is not the method of nature. From the moment the child has learned to speak he seeks to know the why and wherefore of things, and ceases to ask questions only when he becomes discouraged by the unsatisfactory answers of inconsiderate parents or incompetent teachers.

All practical life is rooted in number and calculation. Business is transacted upon a basis of counting. Hence the importance of teaching this subject according to approved and practical methods. For this reason, our SCHOOL'S GOVERNMENT lays special stress upon accurate rather than rapid calculation. It says: "*In teaching Arithmetic it is of less importance to teach the pupil to calculate rapidly than to do so accurately.*" It furthermore insists, as a means of acquiring habits of accurate thinking, that precise language be used in all explanations and calculations: "*The pupils should be taught to make use of the correct terms, and never in their work to employ a useless expression.*" And again it says: "*We should insist on the pupils being very exact in the recitation of definitions and principles.*" Finally, it counsels the exercise of the pupil's judgment in words to which nothing can be added: "*After having read the problem to be solved, and written on the blackboard the conditions*

it contains, the pupils should be required to give an account, first, of what is asked of them, or what they have to find out; second, of what they know in reference to it; third, of the operations required by the nature of the problem."

The following pages will be found to embrace these principles in every respect. The language is adapted to the young minds for which it is intended. The explanations aim at being simple. The solutions given are strictly in accordance with the definitions and principles laid down. But the teacher need not confine himself to the forms here given. After the pupil understands these, he may be shown other methods with profit.

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ELEMENTS OF ARITHMETIC.

INTRODUCTORY DEFINITIONS.

1. ***Arithmetic*** is the *science* of *numbers*, and also the *art* of *computation*.

2. ***Number*** is the *result* of the *comparison* of a *quantity* with *unity*.

3. ***Quantity*** is anything that can be increased, diminished, or measured; as, the *length* of a road, the *surface* of a body, the *weight* of an article.

4. A ***Unit*** is a quantity with which we compare others of the same kind.

5. The comparison of quantity with unity produces three kinds of numbers: *Integers*, *Fractions*, and *Mixed Numbers*.

6. An ***Integer*** is a number which contains its unit an exact number of times: as 12, 15; 6 boys, 4 apples.

7. A ***Fraction*** is a number which is less than a unit; as, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$.

8. A ***Mixed Number*** consists of an integer and a fraction. Thus, $1\frac{1}{2}$, $2\frac{1}{3}$, $4\frac{1}{4}$.

9. According to the nature of their unit, numbers are divided into two classes; viz., *Abstract* and *Concrete.*

10. An ***Abstract Number*** is a number the nature of whose unit is not determined; as, 16, 425, 7840.

11. A ***Concrete Number*** is a number the nature of whose unit is determined; as, 16 men, 425 days, 7840 dollars.

TERMS EMPLOYED.

1. Arithmetical Operations are the divers changes to which numbers are subject.

There are four fundamental operations: *Addition, Subtraction, Multiplication,* and *Division.*

2. The ***Proof*** of an operation is a second operation made in order to verify the correctness of the first. The proof does not give an absolute certainty, but only a great probability, that the operation has been correctly performed.

3. A ***Problem*** is a question to be solved.

4. The ***Resolution*** of a problem consists in finding the unknown quantities by means of the known.

The resolution of a problem comprises both the *solution* and the *calculation,* or *operation.*

5. The ***Solution*** is the series of operations necessary to arrive at the required result.

6. The ***Calculation*** is the performance of the operations indicated by the solution.

NOTE.—The calculation refers exclusively to abstract numbers or those considered as such.

7. A ***Principle*** is a fundamental truth from which others are derived.

8. A ***Rule*** is a statement of the method of obtaining a desired result.

NUMERATION AND NOTATION.

12. ***Numeration*** is the method of reading numbers expressed by characters.

13. ***Notation*** is the method of writing numbers.

14. Numbers may be represented as follows:

I. By *words;* as, one, two, three.

II. By *figures,* called the *Arabic Method;* as, 1, 2, 3.

III. By *letters,* called the *Roman Method;* as, I, V, X, C.

15. In the ***Arabic Method,*** numbers are expressed by the following ten

Figures:	1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
Names:	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.

16. The first nine figures are called *significant* because they represent a value. But the tenth, by itself, represents nothing. It is only an auxiliary figure; its office being to hold the place of any order whatever, when there are no units of that order in the number.

17. Each of the first nine numbers expresses simple units, or units of the *first order.*

18. The number which follows the ninth is called *ten.* It is represented by writing the figure 1 with a naught after it; thus, 10.

19. *Ten* is the unit of the *second* order, and is equal to ten units of the *first* order.

20. We count by tens as we count by simple units,

saying: *one ten, two tens, three tens, nine tens.* But custom has replaced these words by the following:

Twenty,	20.	Sixty,	60.
Thirty,	30.	Seventy,	70.
Forty,	40.	Eighty,	80.
Fifty,	50.	Ninety,	90.

NOTE.—The "ty" in these words signifies ten.

21. The names of the numbers included between two consecutive tens, are formed by joining to the name of the first of these tens, the name of each of the first nine numbers, saying:

Twenty-one,	21.	Thirty-one, etc.	31, etc.
Twenty-two,	22.	Forty-one, etc.	41, etc.
Twenty-three,	23.	Fifty-one, etc.	51, etc.
Twenty-four,	24.	Sixty-one, etc.	61, etc.
Twenty-five,	25.	Seventy-one, etc.	71, etc.
Twenty-six,	26.	Eighty-one, etc.	81, etc.
Etc.	Etc.	Ninety-one, etc.	91, etc.

The highest number expressed by two figures, being ninety-nine, 99.

22. But instead of saying ten and one, ten and two, ten and three,.... ten and nine, custom has adopted the expressions:

Eleven,	11.	Fifteen,	15.
Twelve,	12.	Sixteen,	16.
Thirteen,	13.	Seventeen,	17.
Fourteen,	14.	Eighteen,	18.
		Nineteen,	19.

NOTE.—The "teen" in the words thirteen, etc., to nineteen, means *ten.* So that, strictly speaking, thirteen means three and ten; fourteen, four and ten, etc.

23. The number which follows ninety-nine (99) is called *hundred.* It is represented by writing 1 with two naughts after it; thus, 100.

24. *One hundred* is the unit of the *third order*, and is equal to ten units of the *second order*.

25. We count by hundreds as we count by units, saying:

One hundred,	100.	Five hundred,	500.
Two hundred,	200.	Six hundred,	600.
Three hundred,	300.	Seven hundred,	700.
Four hundred,	400.	Eight hundred,	800.
		Nine hundred,	900.

26. The names of the numbers included between two consecutive hundreds, are formed by joining, successively, to the name of the first of these hundreds, the names of all the numbers less than one hundred, thus:

One hundred one,	101.
One hundred two,	102.
One hundred three,	103.
Eight hundred eighty-six,	886.
Nine hundred ninety-eight,	998.

Nine hundred ninety-nine (999) is the highest number that can be expressed by three figures.

27. The group comprising the first three orders of units, viz., *units*, *tens*, and *hundreds*, constitutes the *first period*, that of simple units.

28. The number which follows 999 is called *thousand*, and is represented by writing the figure 1 with three naughts after it; thus, 1000.

29. *Thousand* is the unit of the *second period.* The period of *thousands,* like that of simple units, comprises units, tens, and hundreds. The units of thousands, tens of thousands, and hundreds of thousands, constitute the fourth, fifth, and sixth orders of units.

The units of thousands are :

One thousand, two thousand,.....nine thousand.
1000, 2000, 9000.

The tens of thousands are :

Ten thousand, twenty thousand, ... ninety thousand.
10000, 20000, 90000.

The hundreds of thousands are :

One hundred thousand, Two hundred thousand,
100000, 200000,
.....nine hundred thousand.
900000.

30. The names of the numbers between two consecutive *orders* of thousands, are formed by joining, successively, to the name of the first of these orders, the names of all the numbers less than this order. In this manner we reach the number 999999.

31. Continuing in the same manner, we form the next higher periods, *Millions, Billions, Trillions, etc.*

Million is the unit of the *third* period ; billion, the unit of the *fourth* period ; and trillion, the unit of the *fifth* period.

These three periods, like *units* and *thousands,* comprise, each, three orders ; viz., the order of units, that of tens, and that of hundreds.

32. The names of the various numbers included between their several orders are formed in the same way as those included between the several orders of thousands.

33. Every significant figure has two values. One is called its *simple,* or *absolute* value; and the other, its *local,* or *relative* value.

The *Simple Value* of a figure is that given to it by its form.

The *Local Value* is that which it receives from the place that it occupies in the number.

Thus in the number 4306, the simple value of the first figure to the left is 4; and its local value is 4 units of thousands.

34. For convenience in reading and writing numbers, the figures are divided into *periods,* each of which comprises three places. The *first three* places constitute the *first,* or *units* period; the *second three* places constitute the *second,* or *thousands* period; etc.

35. This division of the periods will be easily understood by a careful examination of the following

NUMERATION TABLE.

Hundreds of Tens of Units of	Hundreds of Tens of Units of	Hundreds of Tens of Units of	Hundreds of Tens of Units of	Hundreds of Tens of Units of
Trillions,	**Billions,**	**Millions,**	**Thousands,**	**Units.**
840	625	074	503	040
89.	031	209	654	061
30	005	800	237	500
201	892	604	083	275
2	000	596	000	082
987	015	003	106	000
400	000	000	500	005

36. If it be required to read or write numbers above trillions, the following is the order of some of the next higher periods: *Quadrillions, Quintillions, Sextillions, Septillions, Octillions, etc.*

EXERCISES IN NUMERATION.

1. What number is expressed by 75346821 ?

SOLUTION.—Separating these figures into periods, we have 75, 346, 821. The third period is 75 *millions;* the second is 346 *thousands;* and the first is 821 *units;* hence, the number is 75 millions, 346 thousands, 821.

NOTE.—If an order, or even an entire period, be wanting, we do not mention it. We also omit to name the last or units period, because it is understood.

Read the following numbers:

1.	*2.*	*3.*	*4.*	*5.*
75	972	1008	60001	704825
63	840	5000	73182	659037
37	569	6300	39502	954308
25	708	2501	18007	100716
12	411	15784	40905	536900
50	4934	29092	56000	213472
225	6527	56311	171360	360005
121	4025	12102	562984	800001
309	7690	20976	630192	780602
406	2416	10083	935871	170112
721	3083	84960	214650	756640
582	6135	37251	836422	151312
391	1237	45162	178245	702514
420	6030	24310	790031	953716
569	1191	37521	110725	803556
230	1688	89111	333470	678520

6.	*7.*	*8.*
5006351	4072634	3563400024
14500021	82791640	7462007302
7040306	7006	82367400210
907001	984270	53024046070
8200000	400200	86920000030
60500005	1807	17629080406
9008600	62876000	294635112211
20036409	900040	909009008007
460072908	900000800	842780062004
111000111	3742680002	1303000170410
305009705	8632073009	3278642197416
99092666	862794846704	14000075001004
200067425	2872819642	167008634216786
180990040	3400641111	3462184390075819

EXERCISES IN NOTATION.

1. Express in figures the number *two thousand four hundred three.*

SOLUTION.—This number consists of two periods, *thousands* and *units.* In the *thousands* period there is but one order, which is 2 units of thousands. We, therefore, write 2 thousands in the fourth place, 4 hundreds in the third place, a naught (0) in the second place, there being no *tens*, and 3 units in the first place. Hence, these figures, 2403, are the proper expression for the given number.

Express the following numbers in figures:

2. Thirteen. Eleven. Seventeen.
3. Nineteen. Thirty-four. Eighty-two.
4. Fifty-five. Ninety-four. Forty-three.
5. Three hundred sixty. Five hundred eighty-two.
6. Nine hundred twelve. Three hundred fourteen.
7. Seven hundred nineteen. Nine hundred three.

8. Nine thousand, seven hundred eight.

9. Eighteen thousand, seven hundred.

10. Fifty thousand, nine hundred one.

11. Seventy thousand ten.

12. Two hundred ninety-one thousand, seven hundred fifty-three.

13. Six hundred seventy-three thousand, twenty-four.

14. One million, four hundred thirty-two thousand, three hundred sixteen.

15. Fourteen million, thirty-five thousand, one hundred ninety-four.

16. One million, three.

17. Seven million, three hundred thousand, ninety-four.

18. Forty million, four thousand, seven hundred.

19. Six hundred three million, fifteen thousand, sixty-one.

20. Fifteen billion, seventy-one million, six thousand, four hundred.

21. Three hundred thousand, five hundred eighty-two.

22. Two hundred million, fourteen thousand, one hundred.

23. Eight hundred thirty billion, twenty thousand, twenty-two.

24. Five million, two hundred six thousand, nineteen.

25. Nine hundred billion, sixteen million, eight thousand.

26. One hundred nine million, four hundred twenty thousand.

27. Five hundred twenty-one million, three thousand, ten.

28. One hundred two billion, two hundred seventy thousand, ten.

29. Twenty-seven billion, fifty million, five hundred ninety-one.

30. Three hundred million, seventy thousand, nine hundred.

31. Three trillion, one hundred twenty billion, two million, five thousand, one.

ROMAN NOTATION.

37. In the *Roman Method* of Notation, numbers are expressed by the following seven letters of the Roman Alphabet:

Letters.	I,	V,	X,	L,	C,	D,	M.
Values.	1,	5,	10,	50,	100,	500,	1000.

PRINCIPLES.—*I. The value of the letter is repeated as often as the letter itself is repeated; as, III expresses the number three; XX, expresses twenty.*

II. A letter placed to the right of one of greater value, adds its own to that of the other; as, XV represents fifteen; VII, seven.

III. The value of a letter placed to the left of one of greater value, must be subtracted from that of the other; as, IV expresses four; IX, nine.

IV. The value of a letter or a combination of letters, is increased a thousand-fold by placing a dash over it. Thus, $\overline{X}$, $\overline{LX}$, denote, respectively, ten thousand, and sixty thousand.

NOTES.—1. If a letter that denotes a less number be placed between two that denote greater numbers, it diminishes the latter, but

does not affect the former. Thus, in the combination LIX, the value of I must be taken from that of X. Hence, the number expressed is fifty-nine (59).

2. It must be observed also that no letter is written four times in succession.

3. There are only six combinations in which a character is placed before one of greater value. Hence, the numbers, 4 (IV), 9 (IX), 40 (XL), 90 (XC), 400 (CD), and 900 (CM), being represented by peculiar double characters, should not be expressed by any other combinations.

38. The application of these principles is shown in the following

TABLE.

I	One	XIX	19
II	Two	XX	20
III	Three	XXX	30
IV	Four	XL	40
V	Five	L	50
VI	Six	LX	60
VII	Seven	LXX	70
VIII	Eight	LXXX	80
IX	Nine	XC	90
X	Ten	C	100
XI	Eleven	CC	200
XII	Twelve	CD	400
XIII	Thirteen	D	500
XIV	Fourteen	DC	600
XV	Fifteen	CM	900
XVI	Sixteen	M	1000
XVII	Seventeen	MM	2000
XVIII	Eighteen	MDCCCLXXXIII	1883

NOTE.—This system of notation is named after the Romans by whom it was invented and used. It is now principally confined to numbering chapters, sections of books, public documents, etc.

EXERCISES.

Read the following numbers and express them in figures:

1.	*2.*	*3.*	*4.*
IV	XXII	LXXXIII	CMLX
XV	XXXII	XIV	DCVIII
XLIV	XVI	LXXXVIII	CXIV
LXXV	LV	LXX	DCLIII
XXVIII	LI	LIX	DCCXLIII
XXXIX	LXII	XCIII	$\overline{\text{V}}$DCCXI
XI	XCI	XXIII	$\overline{\text{IV}}$XC
XLIX	LXXVIII	XCVII	$\overline{\text{D}}$XLIX

Write the following numbers by the *Roman Method:*

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*
17	24	34	30	830	462	262
45	18	46	67	561	689	109
63	57	12	70	326	984	476
71	64	89	82	279	533	324
25	38	77	96	195	372	712
36	27	98	60	914	607	416
52	13	41	29	182	309	967

8.	*9.*	*10.*	*11.*	*12.*
1876	1776	2132	2358	6908
1882	1860	3004	3422	12674
1512	1783	4040	4004	11492
1492	2579	7632	8743	21800
1600	1900	2250	4100	15000
1888	1777	3045	3009	12650
1865	1420	2804	7180	10800

ADDITION.

39. ***Addition*** is the process of finding the sum of two or more numbers of the same kind.

40. The ***Sum*** of several numbers is a number which contains as many units as the numbers added.

41. The ***Sign of Addition,*** $+$, is called *plus.* This sign denotes that the numbers between which it is placed are to be added. Thus, the expression, $6 + 3$, shows that 3 is to be added to 6.

OPERATION OF ADDITION.

1. Let it be required to find the sum of 475, 854, and 696.

OPERATION.

$$\begin{array}{r} 475 \\ 854 \\ \underline{696} \\ 2025 \end{array}$$

SOLUTION.—Having written the numbers so that units of the same order stand in the same column, we begin at the right and add each column separately. The sum of 6 units and 4 units is 10 units; and 10 units and 5 units are 15 units, which are equal to 1 ten and 5 units. We write the 5 units under the column of units, and carry the 1 ten to the column of tens. We next add the column of tens, beginning with the 1 ten which we carried from units' column. The sum of 1 ten and 9 tens is 10 tens, and 5 tens are 15 tens, and 7 tens are 22 tens; that is, 2 hundreds and 2 tens. Writing the 2 tens in the column of tens, we carry the 2 hundreds to the column of hundreds. The sum of the hundreds thus increased, $2 + 6 + 8 + 4$, is 20 hundreds; that is, 2 thousands and 0 hundreds. As this is the last column, we set down the entire sum. The number, 2025, is, therefore, the required sum, because it is the sum of the units, tens, and hundreds of the given numbers.

PROOF.—Perform the operation by commencing at the top of each column and adding downward. If the same result be obtained, it is probable that the work is correct.

ILLUSTRATIONS.

2.	*3.*	*4.*	*5.*	*6.*
412	49	418	4734	4500
343	716	36	8062	982
917	503	672	191	3799
1672	1268	1126	12987	9281

7.	*8.*	*9.*
542 feet.	260 days.	278 pounds.
717 "	428 "	402 "
203 "	600 "	736 "
971 "	26 "	877 "
2433 feet.	1314 days.	2293 pounds.

NOTE.—The operation of adding a column of figures should be abbreviated by simply naming the result of each step. Thus, in example 7, the pupil should say 1, 4, 11, 13 (1st column); 1, 8, 9, 13 (2d column); and 1, 10, 12, 19, 24 (3d column).

WRITTEN EXERCISES.

10.	*11.*	*12.*	*13.*	*14.*
3141	9316	3345	6205	1073
9071	8200	8801	3903	2208
1625	7510	7865	6675	7762
9301	8715	4322	1885	1839
1870	1930	3418	1907	3033
2415	6740	1879	3314	4111
1980	3972	2860	2970	6395
2634	4060	1790	6327	2136
637			18	5
			523	

230

15.	*16.*	*17.*	*18.*
57082	54133	16098	231872
13221	12235	44120	123987
10939	63798	39862	203524
24532	23787	55032	161724
19980	70390	73852	230543
20018	23901	33487	130107

19.	*20.*	*21.*
4 292	874	704 854 927
468 356	68 422	305 476 507
9 746 940	8 792 624	999 767 689
903 806 706	349 508 628	524 783 876
504 784 949	45 302 011	763 525 202
25 376 458	948 306 405	889 731 633

22.	*23.*	*24.*
67 504	6 245 754	42 085
536 789	272 879 182	45 690
547 645 978	94 807 503	649 743
78 456 708	786 546 597	8 437 348
897 604 847	70 964	497 063 547
89 478 600	70 368 483	594 627 795
976 343 269	526 934	99

25. Four hunared fifty thousand, two hundred twenty; two hundred thirty-one thousand, eighty-six; one million, two hundred sixty-two thousand, seven hundred one; four hundred one thousand, six hundred eighteen.

26. Nine thousand; seven hundred thirty-eight; five thousand; seven hundred eighty-nine; six thousand, three hundred four; two hundred thirty-eight thousand.

27. Three thousand, one hundred twenty; two hundred six; five hundred sixteen thousand, three hundred forty-seven; nine hundred thousand, three hundred forty; and five hundred twenty-six.

28. Sixty thousand, seven hundred eighty-five; thirteen thousand, four hundred seventy-eight; ninety-six thousand, five hundred seventy-one; five hundred eight; twelve thousand, six; six hundred eighty-two; and one thousand, seventy-three.

29. 99 million, 127 thousand, 806; 73 million, 156 thousand, 4 hundred 25; 130 million, 17 thousand, 96; 72 million, 45 thousand, 14.

30. 67 million, 849 thousand, 341; 110 thousand, 389; 63 million, 448 thousand, 720; and seven hundred eight.

31. 5 thousand, 163; 12 thousand, 2; 100 thousand, 9; 1 billion, 100 million, 110 thousand, 11; and nine hundred seven.

32. 90 billion, 90; 10 thousand, 6 hundred; and 200 thousand, 780.

33. 87 million, 63 thousand, 11; 14 thousand, 60; 3 million, 3 thousand, 3; 72 thousand, 601; 84 million, 15 thousand, 107; and 162 million, 7 thousand, 23.

UNITED STATES CURRENCY.

42. The ***Sign,*** $, written before a number signifies *dollars.* Thus, the expression $120 is read *one hundred twenty dollars.*

43. *Dollars* and *cents* may be written together, the *cents* being separated from the *dollars* by a point. Thus, the expression $25.35 is read 25 *dollars* and 35 *cents.*

When the number of cents is less than 10, a naught

must be placed between the point and the figure representing the number of cents. Thus, 6 dollars and 7 cents are written $6.07.

Cents, when written alone, may be expressed either by placing $0. before them, or by simply using the word cents. Thus, *twenty-seven cents* are written $0.27, or 27 cents.

WRITTEN EXERCISES.

Express by proper signs and figures the following:

1. Seven dollars and twenty-four cents. Five dollars.
2. Sixteen dollars and forty cents. Thirty-four cents.
3. Eighty-seven cents. Four dollars and nine cents.
4. Seventy-five cents. Seven dollars and six cents.
5. Eleven dollars and eleven cents. Nine dollars.
6. Fifty dollars and twenty-five cents. Forty cents.
7. Nineteen dollars and three cents. Sixty dollars.
8. Fifty cents. Eighty-five cents. Eighteen cents.
9. Three dollars and fifty cents. Ninety-two cents.
10. Sixty-two dollars and nine cents. Thirty cents.

44. In writing dollars and cents for the purpose of adding them, the separating points must stand in the same column.

1. What is the sum of $10.27, $123.06, $206.90, $3.10, and $0.46?

OPERATION.

$10.27
123.06
206.90
3.10
0.46
$343.79

SOLUTION.—Having arranged the numbers according to **Art. 44,** we add them by the principles of simple addition. The separating point is placed in the result immediately under those of the given numbers.

WRITTEN EXERCISES.

In like manner add the following:

2.	*3.*	*4.*	*5.*
$81.05	$217.75	$78.50	$981.34
54.62	83.16	151.63	89.65
125.84	55.32	96.18	326.10
370.62	135.67	12.03	4823.63

6.	*7.*	*8.*	*9.*
$65.14	$75.15	$2245.76	$30000.00
91.73	207.45	791.18	4706.50
182.64	1241.16	33.87	373.33
79.30	79.67	6.75	876.45
20.37	34.81	650.37	1950.55

MENTAL EXERCISES.

1. A grocer sold 67 quarts of strawberries one day and 28 quarts another; how many quarts did he sell in both days?

2. A carpenter earned 54 dollars one month and 76 the next; how much did he earn in the two months?

3. A man bought a horse for $190, and a wagon for $150; what was the cost of both?

4. Eliza is 30 years older than Mary, and Mary is 20 years old; how old is Eliza?

5. Some muslin cost 30 cents, and some silk 400 cents; what was the cost of both?

6. Edward lost 40 cents, and has 30 cents left; how much had he at first?

7. Thomas has studied 60 lessons this month, which is 20 less than the number Francis has studied; how many lessons has Francis studied?

8. A drover purchased 125 sheep at one time and 113 at another; how many did he buy on both occasions?

9. A boy has 14 cents in his vest pocket and 23 in his pants pocket; how many cents has he in both pockets?

10. My cousin gave me 76 marbles, and I had 140 before; how many marbles have I now?

11. William has 36 photographs in one album and 48 in another; how many photographs in both albums?

12. A farmer sold 62 barrels of apples and has 103 barrels remaining; how many barrels had he at first?

13. A boy sold a book for 28 cents less than it cost. If he received 47 cents, what was the buying price?

14. There are 39 roses on one bush and 26 on another; how many roses on both bushes?

15. While repairing a factory 157 men were engaged on the exterior and 33 on the interior · how many men were employed?

16. John's school is 134 rods north of his residence, while his brother Martin's is 63 rods south; how far are the two schools apart?

WRITTEN EXERCISES.

1. A man bought a horse and carriage for $520, a cow for $65, and a wagon for $90. How much did he pay for all?

2. I own a farm worth $56000, and a house worth $30800; how much are both worth?

3. Three men invested in cotton; A invested $7560; B invested $9200; and C invested $12300. How much did they all invest?

4. An army consists of three grand divisions; the first contains 60400 men; the second 47400 men; and the third 85000 men. How many men in the army?

5. A lady paid $85 for a carpet, $60 for some chairs, $25 for a bureau, and $20.50 for a table. How much did she pay for all?

6. If there are 163 sheep in each of 5 pastures, how many sheep in all?

7. Find the sum of all the numbers from 136 to 146.

8. After paying $375.50 to the grocer, $460 to the butcher, and $89.50 to the tailor, Mr. Johnson has $784.90 left. How much had he before he paid his debts?

9. A merchant pays his foreman $1200 a year, three salesmen $640 each, a porter $500, and an errand boy $200. How much does he pay per year in salaries?

10. A grocer sold 10 chickens for $4.98; 6 ducks for $2.45; and 7 turkeys for $9.36. How many fowls did he sell, and what were they worth?

11. A man bought a set of furniture for the parlor, which cost $483.75; one for the sitting-room, costing $265.50; a chamber set for $180.25; and a set for the dining-room, costing $225.90. What was the cost of the four sets? *Ans.* $1155.40.

12. A gentleman donated the following sums for charitable purposes: $4840.65; $7219.80; $975.50; $3265.20; $1783.85. How much did he donate in all?

13. The property of Mr. McLaughlin was valued as follows: land, $15800; house, $3475; barn, $950; horse, $225; carriage, $175; farm implements, $1675; and money in bank, $4900. What was the entire value of his property? *Ans.* $27200.

14. A gentleman owes the following: to Mr. Smith, $67.50; to Mr. Murphy, $122.75; to Mr. Jones, $89.25; to his butcher, $185.62; to his baker, $75.87; and to his grocer, $220.55. What is the amount of his debts?

Add by lines, and by columns, the following:

LEDGER COLUMNS.

	A.	B.	C.	D.	E.	Total.
	8998 6					
1st..	$584.25	$1060.15	$1200.40	$980.00	$103.04	*** **
2d...	79.48	320.80	915.16	90.50	27.01	*** **
3d...	104.28	75.00	812.08	71.55	4005.80	*** **
4th..	2600.00	84.30	2510.24	183.80	981.10	*** **
5th..	403.50	2100.00	630.12	320.42	49.19	*** **
6th..	694.20	64.50	100.25	201.60	800.66	*** **
7th..	1630.75	790.25	4300.95	500.40	313.40	*** **
8th..	15.80	34.00	650.10	63.24	1500.73	*** **
9th..	210.00	815.70	104.50	1000.00	2014.47	*** **
10th..	521.00	431.65	813.60	720.00	613.09	*** **
11th..	69.50	92.80	25.00	50.13	53.84	*** **
12th..	141.12	200.00	161.00	666.78	299.70	*** **
13th..	3260.00	99.00	35.16	1428.00	14.16	*** **
14th..	914.63	71.15	72.00	1126.30	313.91	*** **
15th..	25.00	370.83	912.50	47.21	808.05	*** **
16th..	711.00	1506.90	85.48	11.60	3020.80	*** **
17th..	39.48	400.50	600.12	783.13	706.09	*** **
18th..	1050.65	510.09	566.86	52.51	98.89	*** **
19th..	705.00	1085.00	1022.25	412.17	1206.25	*** **
20th..	874.50	37.48	203.50	97.00	591.75	*** **
21st..	1020.15	4.65	16.73	9000.00	610.00	*** **
22d...	341.00	19.37	9.87	400.00	50.50	*** **
23d...	97.50	114.20	35.48	96.50	100.25	*** **
24th..	2014.25	2917.62	300.16	39.85	312.75	*** **
25th..	120.90	4.96	5000.50	114.76	8.41	*** **
26th..	251.12	12.88	700.35	2.15	800.70	*** **
27th..	305.85	3.75	36.82	83.35	2619.38	*** **
Total.	**** **	**** **	**** **	**** **	**** **	**** **

NOTE.—In adding long columns, it will be found convenient to write in small figures, at the top of each column, the number carried from the preceding column, as shown in No. A.

SUBTRACTION.

45. ***Subtraction*** is the process of finding the difference between two numbers of the same kind.

46. The ***Minuend*** is the greater number.

47. The ***Subtrahend*** is the smaller number.

48. The ***Difference*** is the number obtained by subtracting.

49. The ***Sign of Subtraction,*** —, is called *minus.* It denotes that the number immediately following it is to be subtracted from the number preceding it; as, in the expression, 8 — 5, the sign minus signifies that 5 is to be subtracted from 8.

OPERATION OF SUBTRACTION.

1. Let it be required to find the difference between 837 and 564.

OPERATION.

Minuend,	837
Subtrahend,	564
Difference,	273

SOLUTION.—We write the subtrahend under the minuend so that units of the same order shall stand in the same column. Beginning at the right, we see that 4 units from 7 units leave 3 units, which we write in the line below. Since 6 tens cannot be taken from 3 tens, we borrow one hundred, or ten tens, from the 8 hundreds, and add it to the 3 tens, making 13 tens. Now, 6 tens from 13 tens leave 7 tens, which we write under the tens. To compensate for the 10 tens, or 1 hundred added to the minuend, we diminish the 8 hundreds by one hundred. Then, 5 hundreds taken from 7 hundreds, leave 2 hundreds, which is written under hundreds.

The number 273 is, therefore, the difference between the two given numbers; because it is the sum of the several remainders, obtained by subtracting the parts of the subtrahend from the corresponding parts of the minuend.

PROOF.—Add the difference to the less, and if the work be correct, the result will give the greater number.

ILLUSTRATIONS.

	2.	3.	4.
From	5736	587673 yards.	$5820.54
take	3428	93736 "	2074.18
Difference,	2308	493937 "	3746.36
Proof,	5736	587673 "	$5820.54

WRITTEN EXERCISES.

5.	634—231.	*21.*	79302274—9304506.
6.	635—308.	*22.*	10627304—10416208.
7.	273—156.	*23.*	73462894—62187326.
8.	420—116.	*24.*	598354987—98708905.
9.	481—375.	*25.*	507205304—408609927.
10.	9655—4479.	*26.*	258245071—47847897.
11.	3542—2334.	*27.*	866477955—499393676.
12.	2736—1786.	*28.*	487500654—90579864.
13.	4087—3754.	*29.*	408253695—406211203.
14.	6321—5213.	*30.*	753402500—96677657.
15.	71583—53866.	*31.*	957874009—894377258.
16.	38293—16532.	*32.*	100640001—30786203.
17.	15665861—4329095.	*33.*	320600021—253060522.
18.	87593647—7825636.	*34.*	1290478132—849534119.
19.	59020000—1009284.	*35.*	6784000091—590090099.
20.	32670941—8732804.	*36.*	1100122301—941098877.

UNITED STATES CURRENCY.

50. To subtract *dollars* and *cents* write them as in addition, so that the separating points may fall in the same column.

1. From $26.03 subtract $17.67.

OPERATION.

$26.03, Minuend.
17.67, Subtrahend.
$8.36, Difference.

SOLUTION.—Place the less number under the greater, taking care to have the points in the same column. Then proceed as in ordinary subtraction.

ILLUSTRATIONS.

	2.	*3.*	*4.*	*5.*
From	$39.62	$186.25	$170.00	$2084.62
take	14.37	49.75	37.33	1950.27
	$25.25	$136.50	$132.67	$134.35

6.	*7.*	*8.*	*9.*
$2500.00	$360.01	$8100.75	$10760.00
1750.20	73.09	998.63	8700.75

WRITTEN EXERCISES.

10. Purchased a farm for $16787.99 and sold it for $18000; what was my gain? *Ans.* $1212.01.

11. A tailor bought cloth to the amount of $63.25 and afterward sold it for $59.16; what was his loss? *Ans.* $4.09.

12. A and B began business with a capital of $16000. If A put in $9713.73, what was B's share of the capital? *Ans.* $6286.27.

13. A gentleman having $3800.25 in bank, drew out $468.71; how much has he remaining in bank? *Ans.* $3331.54.

MENTAL EXERCISES.

1. A boy had 75 cents and spent 37 cents; how much had he left?

2. Joseph bought 30 peaches, gave 8 to John, and eat 10; how many had he left?

3. A lady bought some tape for 25 cents, some needles for 15 cents, and some thread for 20 cents; how much less than a dollar did all cost?

4. The sum of two numbers is 32; one of them is 18; what is the other?

5. John had $35, and earned $17 more. If he spend $22, how much will he have left?

6. The difference of two numbers is 19, and the greater is 56; what is the less?

7. A young man completed his college course at the age of 21, having taken 13 years for that purpose; how old was he when he entered college?

8. I sold a cow for $60, which was a loss of $12; what did I pay for it?

9. A boy lost 50 marbles at play, then bought 20, and found he had 45; how many had he at first?

10. A horse was bought for $90 and sold for $122; what was the gain?

11. There are 93 scholars in 3 classes: 23 in the first, and 37 in the second; how many in the third?

12. My watch cost $23. I paid $5 for repairing it, and then sold it for $43; what was my gain?

13. From a cistern containing 100 gallons, 46 gallons were drawn at one time, and 38 at another. How many gallons remained in the cistern?

14. A boy gave his watch and $12 for 2 suits of clothes worth $36. What was the value of the watch?

WRITTEN EXERCISES.

1. A woman went shopping with $140 in her purse; and on her return she had only $65; how much did she spend? *Ans.* $75.

2. A cistern which holds 210 gallons, was filled in the morning, but at night there were but 50 gallons remaining. How many gallons had leaked out?

Ans. 160 gallons.

3. In a certain village there were 587 men drafted for the army; but 296 were rejected by the surgeon. How many were taken? *Ans.* 291 men.

4. The less of two numbers is 6347, and their sum is 15813; what is the greater? *Ans.* 9466.

5. If Mrs. Andrew Johnson was 75 years old in 1880, in what year was she born? *Ans.* 1805.

6. The subtrahend is 919, and the minuend is 4813; what is the remainder? *Ans.* 3894.

7. A jeweller bought a watch for $96, and a chain for $38. He sold both for $235; how much did he gain?

Ans. $101.

8. Two men purchased a horse; one paying 320 dollars, and the other 47 dollars less; how much did both pay? *Ans.* 593 dollars.

9. Arthur had 297 nuts in a basket. He put 85 in one bag and 79 in another; how many were left in the basket? *Ans.* 133 nuts.

10. I sold a farm for $201860, thereby gaining $90990; how much did I pay for it? *Ans.* $110870.

11. An auctioneer received furniture to the value of $7864, which he auctioned off in two lots, one for $4620.75, and the other for $3000; what was the loss on the furniture? *Ans.* $243.25.

MULTIPLICATION.

51. ***Multiplication*** is the process of taking one number as many times as there are units in another.

52. The ***Multiplicand*** is the number taken or multiplied.

53. The ***Multiplier*** is the number by which we multiply. It shows how many times the multiplicand is taken.

54. The ***Product*** is the number obtained by multiplying.

NOTE.—The multiplicand and the multiplier are the *Factors* of the product. Thus, 6 and 3 are the factors of the product 18.

55. The ***Sign of Multiplication*** is the oblique cross, $\times$, and is read *times,* or *multiplied by.* When placed between two numbers it denotes that their product is to be taken. Thus, 8×6 indicates that 8 is to be multiplied by 6, or that 6 is to be multiplied by 8.

CASE I.

56. When the multiplier contains only one figure.

1. Let it be required to multiply 895 by 7.

OPERATION.

Multiplicand,	895
Multiplier,	7
Product,	6265

SOLUTION.—After writing the multiplier under the lowest order of the multiplicand and drawing a line, we begin to multiply at the right. 7 times 5 units are 35 units, which are equal to 3 tens and 5 units. We write the 5 in units' place, and reserve the 3 tens to be added to the

product of the tens. Multiplying 9 tens by 7 we get 63 tens, which increased by the 3 tens reserved, give 66 tens, or 6 hundreds and 6 tens. We set down the 6 tens in tens' place, and reserve the 6 hundreds to add to the next product. 7 times 8 hundreds are 56 hundreds, plus the 6 hundreds reserved, make 62 hundreds, or 6 thousands and 2 hundreds. As we have no more orders to multiply, we put the 6 thousands and the 2 hundreds in their proper places. The resulting number, 6265, is the required product.

PROOF.—If the product be the same when the multiplicand and the multiplier are interchanged, the work is correct.

WRITTEN EXERCISES.

Multiply :

2. 789 by 8.	*10.* 4284 by 8.	*18.* 51847 by 8.
3. 897 by 9.	*11.* 3026 by 9.	*19.* 57432 by 9.
4. 756 by 2.	*12.* 2008 by 7.	*20.* 37476 by 6.
5. 676 by 3.	*13.* 8945 by 6.	*21.* 578045 by 8.
6. 749 by 4.	*14.* 5327 by 5.	*22.* 974834 by 9.
7. 876 by 5.	*15.* 24654 by 5.	*23.* 375406 by 4.
8. 768 by 6.	*16.* 53407 by 6.	*24.* 927454 by 5.
9. 789 by 7.	*17.* 23247 by 7.	*25.* 905453 by 6.

CASE II.

57. When the multiplier contains more than one figure.

1. Find the product of 643 by 58.

OPERATION.

Multiplicand,	643	
Multiplier,	58	
1st partial product.	5144	$= 643 \times 8$
2d partial product.	3215	$= 643 \times 50$
Product,	37294	$= 643 \times 58$

SOLUTION.—Having written the multiplier under the multiplicand, so that units stand under units, tens under tens, etc., we begin at the right hand and multiply all the parts of the multiplicand, successively, by each term of the multiplier. Since 58 is equal to 5 tens and 8 units, 58 times 643 must be equal to 5

tens times 643 plus 8 units times 643. Having found 8 units times 643 by the rule under Case I., to be 5144, we set it down as the first partial product. To find 5 tens times 643, we begin by multiplying 3 units by 5 tens, which gives 15 tens, or 150 units. Omitting the naught for units' place, we set down the 5 tens in tens' place and carry the 1 hundred to the next product. By multiplying the remaining terms of the multiplicand by the 5 tens of the multiplier, and carrying as before, we obtain the second partial product, 32150; which is equal to 5 tens or 50 times 643. Now, by adding the partial products, 5144 and 32150, we find the entire product, 37294. In like manner we may find the product of any two numbers, being careful to place the first figure of each partial product under the corresponding figure of the multiplier.

WRITTEN EXERCISES.

Multiply:

2. 386 by 31.	*13.* 7648 by 325.	*24.* 8759 by 3618.
3. 750 by 52.	*14.* 2185 by 204.	*25.* 3269 by 2943.
4. 674 by 74.	*15.* 9311 by 679.	*26.* 3587 by 7352.
5. 456 by 69.	*16.* 8209 by 123.	*27.* 9145 by 8024.
6. 687 by 95.	*17.* 83672 by 840.	*28.* 7352 by 3806.
7. 984 by 88.	*18.* 21016 by 189.	*29.* 5747 by 1789.
8. 6307 by 44.	*19.* 37804 by 506.	*30.* 4382 by 2849.
9. 9555 by 75.	*20.* 765407 by 257.	*31.* 26937 by 2582.
10. 8616 by 63.	*21.* 709857 by 340.	*32.* 84132 by 4016.
11. 4201 by 99.	*22.* 650074 by 457.	*33.* 73041 by 7364.
12. 1187 by 55.	*23.* 834765 by 518.	*34.* 12345 by 6789.

35. 290008 by 12009.	*42.* 657984854 by 518.
36. 604082 by 351756.	*43.* 696007453 by 673.
37. 615138 by 254/15.	*44.* 973895676 by 1087.
38. 800739 by 700032.	*45.* 475795834 by 2076.
39. 9103256 by 823906.	*46.* 785747827 by 3476.
40. 9400631 by 746201.	*47.* 807954369 by 4637.
41. 2935760 by 342007.	*48.* 584476854 by 5728.

CONTRACTIONS.

58. ***Contractions*** in multiplication are abbreviated methods of multiplying.

1. Multiply 428 by 32.

SOLUTION.—The factors of 32 are 8 and 4, or $4 \times 4 \times 2$, or $2 \times 2 \times 2 \times 2 \times 2$. Hence, $428 \times 32 = 428 \times 8 \times 4$. $428 \times 8 = 3424$; and $3424 \times 4 = 13696$.

WRITTEN EXERCISES.

2. 582×36.	*8.* 1745×63.	*14.* 62816×121.
3. 635×72.	*9.* 2695×45.	*15.* 70432×108.
4. 976×54.	*10.* 2146×96.	*16.* 53006×99.
5. 783×24.	*11.* 1794×144.	*17.* 42315×84.
6. 694×32.	*12.* 2563×132.	*18.* 37640×42.
7. 641×16.	*13.* 3586×120.	*19.* 45607×132.

59. Annexing a cipher to a number moves each of its digits one place to the left, thus converting units into tens, tens into hundreds, etc.; which is the same as multiplying the number by 10. Hence, to multiply a number by 10, we annex one cipher; to multiply it by 100, we annex two ciphers; and so on.

ILLUSTRATIONS.

1. Multiply 35 by 240. *2.* Multiply 3500 by 240.

OPERATION.

```
 35*
 24 | 0
 ----
 140
 70
 ----
 8400
```

OPERATION.

```
 35 | 00
 24 | 0
 ------
 140
 70
 ------
 840000
```

WRITTEN EXERCISES.

Multiply:

3. 8607 by 10.	*10.* 98900 by 24000.
4. 8038 by 100.	*11.* 63000 by 15000.
5. 7415 by 100.	*12.* 70090 by 17500.
6. 1706 by 200.	*13.* 10007 by 20060.
7. 1679 by 300.	*14.* 90061 by 300000.
8. 4164 by 120.	*15.* 60009 by 504000.
9. 1482 by 350.	*16.* 791000 by 11700.

UNITED STATES CURRENCY.

60. When one of the factors contains *cents*, or *dollars* and *cents*, multiply as in simple numbers. Point off two places from the right, in the product, and prefix the sign $.

1. What cost 42 tons of coal at $5.75 a ton?

OPERATION.

```
$5.75
   42
 1150
2300
$241.50
```

SOLUTION.—If one ton cost $5.75, 42 tons will cost 42 times $5.75, which is $241.50. Since the multiplicand contains cents, we must point off two places in the product.

WRITTEN EXERCISES.

2. What will 25 hogsheads of sugar cost at $75.80 a hogshead?

3. Multiply $3502.40 by 162. By 250. By 584.

4. Multiply $4715.45 by 2405. By 810. By 675.

5. Multiply $3010.25 by 16. By 198. By 487.

6. What will be the cost of 144 yards of broadcloth at $3.75 a yard?

7. What is the value of 408 acres of land at $172.75 an acre ?

8. Mr. Spencer bought 40 acres of land at $300 acre, and 75 acres at $252.50 an acre. He sold the whole at $435 an acre ; how much did he gain ?

9. If a yard of cloth cost $2.25, what will 160 pieces, each containing 45 yards, cost ?

10. How much is lost by selling at $4.25 a barrel, 1500 barrels of flour that cost $5.50 a barrel ?

MENTAL EXERCISES.

1. What will 75 pounds of sugar cost at 12 cents a lb. ?

2. At 3 cents apiece what will 80 oranges cost ?

3. What cost 137 pine-apples at 10 cents apiece ?

4. How much will 9 yards of cloth cost at $2.45 a yard ?

5. What will a man spend in 5 days at the rate of $1.50 a day ?

6. From a piece of cloth containing 93 yards, a tailor cut 6 suits of 10 yards each ; how many yards had he left ?

7. How many pounds in 6 loads of wheat, each load containing 20 bushels of 50 pounds each ?

8. In an orchard there are 9 rows of trees, and 65 trees in each row ; how many trees in the orchard ?

9. A gentleman owing $176, gave in payment 6 shares of railroad stock worth $25 a share. How much does he still owe ?

10. If I buy 8 barrels of pork at $15 a barrel, and sell it for $135, how much will I gain ?

11. A boy earns $4.25 a week and pays 50 cents each day for his board, etc.; how much does he save in the week ?

12. I sold 48 cords of wood at $5 a cord, and received in payment $180 cash, and the remainder in cloth. What was the value of the cloth ?

13. How many pounds of cotton in 5 bales, each containing 350 pounds ?

14. If one quart of peanuts cost 7 cents, what will 36 quarts cost ?

15. What will a man pay for 5 pounds of veal at 12 cents a pound, and 9 pounds of beef at 18 cents a pound ?

16. A grocer having 35 melons sold 16 of them, and then bought 4 times as many as he sold ; how many had he then ?

17. At $8 a barrel, what will be the cost of 5 loads, each containing 13 barrels ?

WRITTEN EXERCISES.

1. A fortune of $125000 is to be so divided that each of four sons will receive $5825 ; each of two daughters $3570 ; and the widow the remainder. How much will the widow receive ? *Ans.* $94560.

2. I bought 986 acres of cleared land at $64 an acre, and 60 acres of woodland at $36 an acre. I then sold the whole at $50 an acre. Did I gain or lose, and how much? *Ans.* Lost $12964.

3. If an acre of city land cost $1425, and were sold for $1320, what would be the loss on 37 acres? *Ans.* $3885.

4. A merchant bought at one time 576 pounds of coffee, and at another 425 pounds ; what did it cost at 32 cents a pound ? *Ans.* $320.32.

5. A man sold 8 umbrellas that were worth 85 cents each, for $1.30 each. How much more than their real value did he receive ? *Ans.* $3.60.

6. A grazier bought 92 calves at $14 each, and sold 63 of them at $16 each, and the remainder at $13 each. What did he gain by the bargain? *Ans.* $97.

7. What amount of money will enable me to give $85.75 to each of 354 laborers? *Ans.* $30355.50.

8. A drover bought 140 head of cattle in Texas at $36 a head, and sent them to New York at an expense of $350.65. They were then sold at a profit of $1002.50. What did he receive for the drove? *Ans.* $6393.15.

9. Bought 3 pieces of sheeting containing respectively 25, 30, and 47 yards. The first cost 14 cents, the second 15, and the third 12 cents a yard. What was the entire cost? *Ans.* $13.64.

10. A farmer sold 74 barrels of apples at $5 a barrel, 44 barrels of potatoes at $3.50 a barrel, and 32 tons of hay at $17.75 a ton. What was the amount received? *Ans.* $1092.

11. The President's salary is $50000, and his cabinet, consisting of 7 members, receive each $8000; what is the total salary of the President and his cabinet? *Ans.* $106000.

12. Two men are 860 miles apart. If they travel toward each other, one at the rate of 45 miles a day and the other 24, how far apart will they be in 12 days? *Ans.* 32 miles.

13. In 1861 the national debt of Great Britain was $133.55 for each person, and the population, 29344788. What was the national debt? *Ans.* $3,918,996,437.40.

14. A man worth $15840, received a legacy of $6000. He spent $4900 in traveling, and donated $3875 to a college; how much money has he remaining? *Ans.* $13065.

DIVISION.

61. ***Division*** is the process of finding how many times one number is contained in another of the same kind.

62. The ***Dividend*** is the number to be divided.

63. The ***Divisor*** is the number by which we divide.

64. The ***Quotient*** is the result of the division. It shows how many times the dividend contains the divisor.

65. The ***Sign of Division,*** ÷, read *divided by,* when placed between two numbers, indicates that the number before it is to be divided by the one after it. Thus, the exoression, 63 ÷ 7, is read 63 divided by 7.

SHORT DIVISION.

1. Let it be required to divide 32540 by 5.

OPERATION.

	Dividend
Divisor 5)	32540
Quotient	6508

SOLUTION.—Having written the divisor at the left of the dividend, with a curved line between them, we begin at the left to divide the different parts of the dividend by the divisor. Since 5 is not contained in 3 tens of thousands any tens of thousands times, we divide 32 thousands by 5. This gives 6 thousands for a quotient and 2 thousands for a remainder. We write the 6 thousands under the thousands, and to the remaining 2 thousands we annex the next term, which is 5 hundreds. 2 thousands and 5 hundreds are equal to 25 hundreds. 5 is contained in 25 hundreds 5 hundreds times. Since there is no remainder, and since 4 is less than 5, there are no tens in the quotient. We, therefore, write 0 in the place of tens,

and annex the following term to the 4 tens, making 40 units. Dividing 40 units by 5 we obtain 8 units, which we place in the quotient under units. Hence the number 6508, being the sum of all the partial quotients obtained by dividing the parts of the dividend by the divisor, is the required quotient.

PROOF.—Multiply the quotient by the divisor, and to the product add the remainder. If the result be equal to the dividend, the work is correct.

WRITTEN EXERCISES.

Divide :

2. 605 by 5.	*7.* 432536 by 8.	*12.* 741107808 by 7.
3. 786 by 6.	*8.* 405252 by 9.	*13.* 456904117 by 8.
4. 612 by 7.	*9.* 344688 by 2.	*14.* 741018207 by 9.
5. 872 by 8.	*10.* 478354 by 3.	*15.* 746784320 by 5.
6. 206 by 9.	*11.* 107425 by 4.	*16.* 402084009 by 6.

LONG DIVISION.

1. Let it be required to divide 50289 by 372.

OPERATION.

```
Divisor. Dividend. Quotient.
 372 ) 50289 ( 135
       372
       ----
       1308
       1116
       ----
        1929
        1860
        ----
Remainder,  69
```

SOLUTION. — Since 372 is not contained in 5 tens of thousands, or in 50 thousands any thousands times, there are no thousands in the quotient. Annexing the next term, 2 hundreds, we have 502 hundreds. 372 is contained in 502 hundreds 1 hundred times, with a remainder. Write the 1 hundred in the quotient, and multiply the divisor by it, subtracting the product from the 502 hundreds. This gives for remainder 130 hundreds ; to which we annex the next term, 8 tens, making 1308 tens for the next partial dividend. The quotient of 1308 tens by 372 is greater than 3 and less than 4 ; hence there are 3 tens in the quotient. Multiplying 372 by 3 tens, we have 1116 tens, and this taken from 1308 tens, leaves 192 tens ; to which we annex the next term, 9 units, making 1929 units. 372

is contained in 1929 units 5 times, with a remainder. Writing the 5 units in the quotient, and multiplying and subtracting as before, we obtain the remainder, 69. Hence the quotient is 1 hundred, 3 tens, and 5 units, or 135, with a remainder of 69.

NOTE.—The method of proving long division is the same as that for short division.

WRITTEN EXERCISES.

Divide :

2. 879420 by 64.
3. 796425 by 75.
4. 492630 by 66.
5. 843255 by 87.
6. 943079045 by 535.
7. 633576807 by 493.
8. 476958421 by 207.
9. 897435804 by 356.
10. 116418209 by 627.
11. 896047040 by 770.
12. 431651423 by 921.
13. 427850017 by 999.
14. 1316740006 by 814.
15. 9205639995 by 275.
16. 234827206 by 1047.
17. 764106347 by 5943.
18. 541224807 by 6481.
19. 684124206 by 5398.
20. 541307650 by 4765.
21. 984356401 by 2034.
22. 456087654 by 75979.
23. 864207979 by 79672.
24. 7464804605 by 296489.
25. 1700095084 by 346845.
26. 9467807008 by 374817.

66. ***Contractions*** in division are abbreviated methods of dividing.

1. Divide 1404 by 36, using factors.

OPERATION.

$36 = 9 \times 4$

```
4 ) 1404
    ----
9 ) 351
    ---
     39
```

SOLUTION.—The factors of 36 are 9 and 4. By dividing the dividend by 4, and the quotient thus obtained by 9, we obtain the true quotient. $1404 \div 4 = 351$ *fours.* 351 *fours* $\div 9 = 39$ *thirty-sixes.* Hence 36 is contained in 1404, 39 times.

2. Divide 1162 by 42, using the factors 3, 7, and 2.

OPERATION.

3) 1162
7) 387 1
2) 55 . . . . . 2 × 3 = 6
27 . . . . 1 × 3 × 7 = 21
Rem. 28

SOLUTION.—Since the factors of 42 are 3, 7, and 2, we divide by each in order. 1162 ÷ 3 = 387 *threes* and a remainder of 1 *unit;* 387 *threes* ÷ 7 = 55 *twenty-ones* and a remainder of 2 *threes*, or 6 *units;* 55 twenty-ones ÷ 2 = 27 *forty-twos* and a remainder of 1 *twenty-one*, or 1 × 3 × 7 = 21 *units*. Add the partial remainders, 1 + 6 + 21 = 28. Hence, 42 is contained in 1162, 27 times, with a remainder of 28.

3. Divide 3782 by 400.

OPERATION.

4 | 00) 37 | 82
$9\frac{182}{400}$

SOLUTION.—Cut off the two ciphers in the divisor, and the two terms at the right of the dividend; then four *hundreds* are contained in 37 *hundreds*, 9 times, with a remainder of 1 *hundred;* this 1 *hundred* + 82, the portion cut off = 182. Thus, 3782 ÷ 400 = $9\frac{182}{400}$.

WRITTEN EXERCISES.

Divide:

4. 16848 by 36.
5. 15225 by 125.
6. 26656 by 49.
7. 79645 by 56.
8. 90374 by 124.
9. 45113 by 200.
10. 37341 by 520.
11. 98465 by 2000.
12. 15640 by 860.
13. 72034 by 27000.

UNITED STATES CURRENCY.

67. Reduce the dividend to cents if necessary, and divide as in simple numbers. The quotient will be the answer in cents; which may be reduced to dollars and cents by placing the separating point two places from the right.

When both dividend and divisor are in currency, reduce each to cents if necessary, and divide as in simple numbers. The quotient will be the required number.

ILLUSTRATIONS.

1. Divide $402 equally among 25 men.

2. How many barrels of flour at $8.25 per barrel, can be purchased for $528 ?

1.

```
25 ) $402.00 ( $16.08
     25
     ---
     152
     150
     ----
       200
       200
       ---
```

2.

```
$8.25 ) $528.00 ( 64 barrels.
        4950
        ----
         3300
         3300
         ----
```

Divide:

3. $793.52 by 56.
4. $3924.72 by 237.
5. $30220 by 540.
6. $75612.58 by 784.
7. $299.64 by 33 cents.
8. $45.99 by 21 cents.
9. $45038.75 by $18.75.
10. $1104.00 by $36.80.

MENTAL REVIEW.

1. If 25 sheep cost $200, what will one cost ?

2. A farmer bought a wagon for $99, paying in hay at $11 a ton ; how many tons of hay did he require ?

3. For $216 how many barrels of pork can be bought at $8 a barrel ? At $4 ? At $16 ?

4. If 9 apples are worth 135 chestnuts, how many chestnuts is 1 apple worth ? 15 apples ?

5. If 147 cents be divided among 7 boys, how many cents does each receive?

6. If 5 pounds of coffee cost 45 cents, what will 8 pounds cost? 12? 15? 20?

7. What will be the cost of 16 dozen of eggs, if 9 dozen cost 90 cents? 54 cents? 72 cents?

8. If 24 apples are worth 8 oranges, how many apples are 18 oranges worth? 25 oranges?

9. If 8 barrels of flour cost $72, what will 4 barrels cost? 11 barrels? 6 barrels? 22 barrels?

10. A boy earned 75 cents in 15 days; how much will he earn in 20 days? 25 days? 9 days?

11. A man travels 30 miles in 6 hours; how far will he travel in 14 hours? 16 hours? 32 hours?

12. How many melons can be bought for 75 cents, at the rate of 4 for 60 cents? 3 for 25 cents?

13. What must I pay for 12 pounds of beef, if 5 pounds cost 80 cents? For 8 pounds?

14. If 7 pounds of sugar cost 91 cents, what will 13 pounds cost? 10 pounds? 8 pounds?

15. How many hundredweight of hay can I buy for $65, if 15 hundredweight cost $75?

16. If 9 pigs are worth $63, what will 19 pigs cost?

17. If a man require 100 days to build 25 rods of fence, how long will it take him to build 17 rods?

18. If for 8 meals 8 soldiers have 64 pounds of meat equally divided among them, how many pounds will each receive for one meal?

19. If a steamship make 54 miles in 3 hours, how long will it take her to make 90 miles? 126 miles? 72 miles?

20. If 8 benches are sufficient to accommodate 120 boys, how many benches will be required for 450 boys?

21. If 130 walnuts be equally divided among 13 boys, how many will 5 of them receive? 9? 11?

22. What will 13 sheep cost at the rate of 50 sheep for $150? 16 sheep? 35 sheep?

23. Paid $35 for 5 barrels of flour. At the same price, what will 25 barrels cost?

24. How many barrels of flour at $9 a barrel, are equal in value to 27 yards of cloth at $3 a yard?

25. How many pears at 5 cents apiece, can I get for 85 apples worth 2 cents apiece?

26. If 6 men dig a ditch in 35 days, how many men will do it in 7 days? In 14 days? In 42 days?

27. If it take 15 yards of cloth to make 3 suits of clothes, how many yards will 9 suits require?

28. When peaches are sold for 50 cents a basket, how many baskets must be given in exchange for 5 cords of wood at $3 a cord?

29. If 7 firkins of butter are worth $84, how many barrels of cider at $3 a barrel, will 4 firkins of butter buy?

30. How many quarts of milk at 5 cents a quart, will pay for 15 pounds of sugar at 8 cents a pound?

WRITTEN REVIEW.

1. In how many days will a ship sail 7950 miles, if it sail 150 miles a day? *Ans.* 53 days.

2. A man owing $8637, gives in payment 32 acres of land valued at $26 per acre, and $3500 in cash; how much remains unpaid? *Ans.* $4305.

3. I bought 14 barrels of pork at $12 a barrel, and sold it for $154; how much did I gain or lose per barrel? *Ans.* Lost $1 per barrel.

4. A lady paid $300 for a piano, $56 for a sofa, and $18 for an easy chair; how much less than three times $200 did all cost? *Ans.* $226.

5. How many gallons of molasses at 45 cents a gallon, will pay for 86490 pounds of butter at 25 cents a pound? *Ans.* 48050 gallons.

6. John and Peter started from home together and travelled in opposite directions. John travelled at the rate of 19 miles a day, and Peter at the rate of 27 miles a day; how far apart were they in 37 days? *Ans.* 1702 miles.

7. A farmer sold 356 oxen at the rate of 6 for $192. What did he receive for them? *Ans.* $11392.

8. How much can a person earn in 3 years, if he earn $120 in 24 days? *Ans.* $5475.

9. An estate of $50607 was left to a family of 4 brothers and 9 sisters. The brothers having given up their share to the sisters, how much did each of the latter receive? *Ans.* $5623.

10. A hogshead containing 63 gallons of molasses was bought for 54 cents a gallon. 6 gallons leaked out, and the rest was sold at a gain of 8 cents per gallon. What was the selling price? *Ans.* $35.34.

11. A farmer wishes to exchange 200 bushels of oats at 36 cents a bushel, for flour at $8 a barrel. How many barrels will he receive? *Ans.* 9 barrels.

12. If 34 persons consume 167 pounds of flour every day, how long will 13694 pounds last them? *Ans.* 82 days.

13. What number subtracted 18 times from 97632, will leave 2232 as a remainder? *Ans.* 5300.

14. A gentleman whose salary is $1800, saves $340 every year. What are his daily expenses? *Ans.* $4.

15. The sum of two numbers is 564, and their difference is 126; what is their product? *Ans.* 75555.

16. What number must be multiplied by 102 to produce 110670? *Ans.* 1085.

17. From what number can 251 be subtracted 405 times and leave a remainder of 73? *Ans.* 101728.

18. How long would it take a railway train to go from New York to San Francisco, 3300 miles, at the rate of 20 miles an hour? *Ans.* 165 hours.

19. What will 5 gross of eggs cost at 29 cents a dozen (12 dozen = 1 gross)? *Ans.* $17.40.

20. How many horses worth $124 apiece can be bought for 32 oxen at $74 each, 45 cows at $57 each, 235 sheep at $4 each, and $4543 cash? *Ans.* 84 horses.

21. A's income is 6 times B's; B's income is 4 times C's; and C's income is $2425. What is their entire income? *Ans.* $70325.

22. A man travels due north for 9 days at the rate of 49 miles a day. He then returns on his path at the rate of 36 miles a day; how far is he from the starting point at the end of 12 days? *Ans.* 9 miles.

23. How many yards of muslin at 15 cents a yard must be given in exchange for 75 bushels of oats at 47 cents a bushel? *Ans.* 235 yards.

24. At the rate of 37 miles a day, how long will it take a man to walk from Chicago to San Francisco, a distance of 2442 miles? *Ans.* 66 days.

25. A man having $7603, paid out all but $1735 in 9 weeks. What was the average amount paid out each week? *Ans.* $652.

26. Sold for $6817, a quantity of wood that cost $4812, thereby gaining $5 a cord. What was the cost per cord? *Ans.* $12.

PROPERTIES OF NUMBERS.

68. The ***Properties of Numbers*** are those qualities or elements which necessarily belong to numbers.

69. An ***Even Number*** is a number that is exactly divisible by 2.

All numbers whose unit figure is 0, 2, 4, 6, or 8, are even.

70. An ***Odd Number*** is a number that is not exactly divisible by 2.

All numbers whose unit figure is 1, 3, 5, 7, or 9, are odd.

71. A ***Prime Number*** is a number that has no integral factors except unity and itself; as 2, 3, 7, 13, 23, 31.

72. A ***Composite Number*** is a number that has other integral factors besides unity and itself. Thus, 24 is a composite number; because $24 = 8 \times 3$, 6×4, or 12×2.

73. The ***Factors*** of a composite number are the numbers which, when multiplied together, will produce it; as, 3 and 5 are the factors of 15.

FACTORING.

74. ***Factoring*** is the operation of separating a number into its integral factors.

ORAL EXERCISES.

1. What are the factors of 6? Of 8? Of 15? Of 39? Of 84? Of 63? Of 75?

2. What are the prime numbers from 10 to 43? From 27 to 95? From 37 to 51?

3. What are the composite numbers from 23 to 51? From 47 to 64?

4. Mention the odd numbers between 19 and 31; 73 and 87; 45 and 61.

5. Mention the even numbers between 201 and 237; 506 and 532.

6. What are the prime factors of 21? Of 51? Of 95?

7. What are the composite factors of 24? Of 54?

8. Give both the prime and composite factors of 42.

9. What prime factors are common to 39 and 78?

10. What composite factors are common to 24 and 96?

11. What factors are common to the sum and difference of 60 and 120?

12. What is the greatest composite factor of 120?

OPERATION OF FACTORING.

1. Let it be required to separate 420 into its prime factors.

OPERATION.

$$\begin{array}{r|l} 2 & 420 \\ \hline 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

SOLUTION.—Since the prime number is even, we divide it by 2, the least prime factor; and the quotient we also divide by 2, which gives an odd number for result. This result we divide by 3, its least prime factor. We observe that the last quotient, 35, has for its prime factors 5 and 7. Having divided by 5, we obtain 7 for result. As 7 is not divisible, it must be a prime factor of the given number. Hence, the divisors 2, 2, 3, 5, and the last quotient 7, are all the prime factors or divisors of 420; that is, $420 = 2 \times 2 \times 3 \times 5 \times 7$.

WRITTEN EXERCISES.

Resolve the following numbers into their prime factors:

2. 24.	*8.* 64.	*14.* 112.	*20.* 378.
3. 65.	*9.* 84.	*15.* 128.	*21.* 450.
4. 40.	*10.* 72.	*16.* 154.	*22.* 486.
5. 88.	*11.* 78.	*17.* 165.	*23.* 504.
6. 48.	*12.* 60.	*18.* 360.	*24.* 726.
7. 36.	*13.* 99.	*19.* 160.	*25.* 1155.

GREATEST COMMON DIVISOR.

75. An ***Exact Divisor*** of a number is a number that will divide it without a remainder. Thus, 4 is an exact divisor of 32.

76. ***The Greatest Common Divisor*** of two or more numbers is the greatest number that exactly divides each of them. Thus, 6 is the greatest common divisor of 18 and 42.

NOTE.—A common divisor of the divisor and the dividend, is also a divisor of the remainder; and a common divisor of the remainder and divisor, is also a common divisor of the divisor and dividend. Thus, the common divisor 2, of the divisor 6 and the dividend 32, is also a common divisor of the remainder 2.

ORAL EXERCISES.

1. Name two exact divisors of 6. Of 8. Of 12. Of 32. Of 50.

2. Name three exact divisors of 48. Of 16. Of 96. Of 42.

3. What number is an exact divisor of 52? Of 91? Of 49?

4. Name all the exact divisors of 40. Of 25. Of 63.

5. What divisors are common to 20 and 30? To 35 and 105? To 24 and 60?

6. What are the prime factors of 60? Of 80? Of 50? Of 70? Of 25? Of 45?

7. What prime factors are common to 30 and 50?

8. What is the greatest common divisor of 12 and 18? Of 24 and 48?

9. What two numbers will exactly divide 20 and 40? Their sum and difference?

10. What is the greatest common divisor of 26, 52, and 91?

11. Find the greatest common divisor of the sum and difference of 36 and 90.

12. What is the greatest divisor common to 99 and 121?

Case I.

77. When the numbers are small and can be easily factored.

In the first method of this case, we find the common factors and take their product.

1. Find the greatest common divisor of 48, 72, and 120.

OPERATION.

2	48,	72,	120
2	24,	36,	60
2	12,	18,	30
3	6,	9,	15
	2,	3,	5

Solution.—Having written the numbers as in the margin, we observe that 2 is a factor of each. It is, therefore, a factor of the greatest common divisor.

The quotients may also be divided by 2 as a common factor. The second set of quotients also have 2 for a common factor. We divide the next set of quotients by their common factor 3, and thus obtain for the last quotients, the prime numbers 2, 3, 5. Hence, the numbers 2, 2, 2, and 3, be-

ing all the common factors, their product, 24, will be the greatest common divisor.

WRITTEN EXERCISES.

What is the greatest common divisor of:

2. 10, 30, and 40?
3. 24, 32, and 56?
4. 16, 48, and 64?
5. 25, 65, and 75?
6. 120, 360, and 480?
7. 272, 288, and 396?
8. 96, 144, and 504?
9. 504, 546, and 648?
10. 640, 320, and 480?
11. 392, 504, and 648?
12. 180, 320, and 342?
13. 900, 468, and 342?
14. 126, 210, and 252?
15. 144, 720, and 360?
16. 165, 242, and 726?
17. 330, 420, and 560?

The second method in this case is to resolve the numbers into their prime factors, and take the product of their common factors.

1. Find the greatest common divisor of 48, 72, and 120.

OPERATION.

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$
$$120 = 2 \times 2 \times 2 \times 3 \times 5$$
$$2 \times 2 \times 2 \times 3 = 24$$

SOLUTION.—The factors of 48 are 2, 2, 2, 2, and 3; the factors of 72 are 2, 2, 2, 3, and 3; and the factors of 120 are 2, 2, 2, 3, and 5. We see that 2, 2, 2, and 3 are all the prime factors common to the three given numbers; hence, their product, 24, is the greatest common divisor.

WRITTEN EXERCISES.

Find the greatest common divisor of:

2. 6, 9, and 27.
3. 8, 16, and 24.
4. 10, 40, and 60.
5. 24, 32, and 56.
6. 16, 48, and 64.
7. 25, 65, and 75.
8. 36, 48, and 72.
9. 88, 99, and 154.
10. 24, 40, and 64.
11. 65, 78, and 117.

Case II.

78. When the numbers are large and cannot be readily factored.

1. Find the greatest common divisor of 316 and 664.

OPERATION.

```
316 ) 664 ( 2
      632
       32 ) 316 ( 9
            288
             28 ) 32 ( 1
                  28
                   4 ) 28 ( 7
                       28
```

Solution.—We divide 664 by 316, the divisor 316, by the remainder 32, the divisor 32, by the remainder 28, and the divisor 28, by the remainder 4, and obtain no remainder. Now, the greatest common divisor of 316 and 664, is also the greatest common divisor of 32 and 316 (**76**); and the greatest common divisor of 32 and 316 is also the greatest common divisor of 28 and 32; but the greatest common divisor of 28 and 32 is 4; hence, the greatest common divisor of 316 and 664 is 4.

WRITTEN EXERCISES.

What is the greatest common divisor of:

2. 320 and 496?
3. 972 and 624?
4. 420, 240, and 540?
5. 324, 672, and 720?
6. 1203, 2406, and 4812?
7. 1728 and 1872?
8. 364 and 584?
9. 826, 1372, and 4116?
10. 2040, 8476, and 4238?
11. 2192, 1456, and 1824?

LEAST COMMON MULTIPLE.

79. ***The Least Common Multiple*** of two or more numbers is the least number which is a multiple of each of them. Thus, 24, 48, and 72, are each a common multiple of 2, 3, and 4; but 12 is their least common multiple.

ORAL EXERCISES.

1. What numbers between 3 and 41 are exactly divisible by 5? By 6? By 8? By 9?

2. What numbers less than 60 are exactly divisible by 8? By 5? By 9? By 12?

3. What multiple is common to 5 and 6? To 9 and 11? To 8 and 3? To 7 and 12?

4. What is the least common multiple of 3 and 7? Of 5 and 9? Of 4 and 11? Of 9 and 4?

5. What is the least common multiple of 4 and 6? Of 11 and 12?

6. Name at least four numbers of which 3 and 4 are factors. 2 and 5. 6 and 7.

7. By what prime factors can 63 be divided?

8. Name at least three common multiples of 8 and 9.

9. Mention three numbers of which 2 and 3 are factors.

10. Find at least four common multiples of 3 and 5.

11. What is the least common multiple of 6 and 12? Of 4 and 10? Of 5 and 30?

WRITTEN EXERCISES.

1. Find the least common multiple of 16, 20, and 30.

OPERATION.

$16 = 2 \times 2 \times 2 \times 2$
$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$

SOLUTION.—The least common multiple cannot be less than the largest number, 30, since it must contain 30. Hence, it must contain all the prime factors of 30, which are 2, 3, and 5. But the least common multiple of 30, must also contain all the prime factors of each of the other numbers. Now, since the prime factor 2, of 30, is common also to 20 and 16, we omit it, and annex the factors 2, 2, and 2 to those of 30. The series 2, 3, 5, 2, 2, and 2 are all the prime factors of the given numbers, and their product, 240, is the least common multiple of the given numbers.

SECOND METHOD.

Find the least common multiple of 16, 20, and 30.

OPERATION.

2	16, 20, 30
2	8, 10, 15
5	4, 5, 15
	4, 1, 3

SOLUTION.—Having placed the numbers in a horizontal line, we see that 2 is a factor of each of them. It is, therefore, a factor of the least common multiple. Continuing to divide the quotient by the factors 2 and 5, we obtain the *numbers* 4, 1, and 3, which, being *prime* to one another, are the last quotients. The factors of the given numbers are 2, 2, 5, 4, and 3; and their product, 240, is the least common multiple.

Find the least common multiple of:

2. 60, 40, and 90.
3. 24, 56, and 84.
4. 17, 59, and 85.
5. 48, 72, and 81.
6. 240, 120, and 260.
7. 960, 690, and 810.
8. 30, 42, and 72.
9. 32, 40, and 25.
10. 60, 12, and 63.
11. 56, 49, and 77.
12. 54, 108, and 207.
13. 27, 216, and 414.

CANCELLATION.

80. ***Cancellation*** is the process of abridging operations in division by rejecting equal factors from both dividend and divisor.

1. Divide 32×96 by 12×16.

1ST OPERATION.

$$\frac{32 \times 96}{12 \times 16} = \frac{\cancel{8} \times \cancel{4} \times 16 \times \cancel{3} \times \cancel{2}}{\cancel{3} \times \cancel{4} \times \cancel{8} \times \cancel{2}} = 16$$

SOLUTION.—Having written the numbers that constitute the dividend above a line and those that constitute the divisor below it, resolve each of the numbers into its respective factors. After rejecting equal factors from both dividend and divisor there remains the factor 16 in the dividend. Hence, the quotient is 16.

2D OPERATION.

$$\frac{\overset{2}{\cancel{12}} \times \overset{8}{\cancel{96}}}{\cancel{12} \times \cancel{16}} = 16$$

SOLUTION.—Since 12 will divide both 96 in the dividend and 12 in the divisor, reject 12 as a factor from both, retaining the factor 8 in the dividend. In the same manner reject the factor 16 from both dividend and divisor, leaving the factor 2 in the dividend. There remain the factors 8 and 2 in the dividend, whose product is the required quotient.

WRITTEN EXERCISES.

2. Divide $24 \times 14 \times 16$ by $6 \times 7 \times 32$. *Ans.* 4.

3. Divide $20 \times 48 \times 10$ by $96 \times 8 \times 4$. *Ans.* $3\frac{1}{8}$.

4. Divide $25 \times 120 \times 10$ by $80 \times 40 \times 4$. *Ans.* $2\frac{11}{32}$.

5. Divide $45 \times 63 \times 28$ by $9 \times 126 \times 56$. *Ans.* $1\frac{1}{4}$.

6. Divide $60 \times 39 \times 15$ by $120 \times 78 \times 30$. *Ans.* $\frac{1}{8}$.

WRITTEN REVIEW.

1. If it require 15 men, working 10 hours a day for 24 days, to do a certain work, how many men, working 8 hours a day for 50 days, would be required to do the same work?

2. Find three numbers less than 150, which are multiples of 16 and 24.

3. How many tubs of butter, each containing 45 pounds, worth 30 cents a pound, will cost the same as 25 barrels of sugar, each containing 300 pounds, at 10 cents a pound?

4. The G. C. D. of two numbers is 50, and their L. C. M. is 4200. If one of the numbers is 600, what is the other?

5. I can buy a number of cows at $60 each, sheep at $12 each, and hogs at $18 each; what is the least sum of money that will exactly pay for them?

6. Find the least number which, when divided by 5, 8, 9, 7, or 4, will leave 3 as a remainder in each case.

7. There are three tanks holding, respectively, 1500, 2550, and 3000 gallons. What is the capacity of the largest vessel that will exactly fill them if emptied into them an exact number of times?

8. Divide $39 \times 5 \times 84 \times 36$ by $75 \times 26 \times 96 \times 7$, and multiply the quotient by 20.

9. How many dozen of eggs, at 33 cents per dozen, can be exchanged for 495 pounds of sugar at 12 cents a pound?

10. Find the length of the longest curb-stones that will exactly fit each of three strips of sidewalk, the first of which is 546 feet long, the second 588 feet, and the third 1134 feet.

11. What three numbers between 170 and 850 have the same G. C. D. as these numbers?

12. In a certain house there are four rooms whose widths are, respectively, 51, 45, 30, and 33 feet. What is the width of the widest carpeting that will exactly fit each of them?

13. The L. C. M. of 95, 5, 7, and another number prime to each of them, is 5320; what is the other number?

14. What is the least sum of money with which I can pay five servants whose monthly wages are, respectively, $13, $21, $39, $54, and $63 per month?

15. Find the G. C. D. and also the L. C. M. of 44, 99, 33, and 77.

16. I sold 35 barrels of potatoes, each containing 3 bushels, at 36 cents a bushel, and received in payment 15 boxes of soap each containing 28 pounds. What was the price of the soap per pound?

FRACTIONS.

81. A ***Fraction*** is one or more of the equal parts of a unit.

82. Fractions may be divided into two classes: *common fractions* and *decimal fractions.*

83. A ***Common Fraction*** is one in which the unit is divided into any number of equal parts.

84. A *common fraction* is expressed by two numbers, one written above the other, with a short line between them; as, $\frac{4}{5}$ expresses 4 fifths.

85. The ***Denominator*** of a fraction denotes the number of equal parts into which the unit is divided. It is written below the line.

86. The ***Numerator*** of a fraction denotes the number of equal parts taken. It is written above the line.

87. Common Fractions are divided into three classes: *Simple, Compound,* and *Complex.*

88. A ***Simple Fraction*** is a fraction having a single integral numerator and denominator; as, $\frac{5}{8}$; $\frac{3}{4}$.

89. A ***Proper Fraction*** is a simple fraction whose value is less than a unit; as, $\frac{7}{8}$; $\frac{9}{13}$.

90. An ***Improper Fraction*** is a simple fraction whose value is equal to or greater than a unit; as, $\frac{5}{4}$; $\frac{11}{9}$.

91. A ***Compound Fraction*** is a fraction of a fraction; as, $\frac{3}{4}$ of $\frac{5}{6}$; $\frac{6}{7}$ of $\frac{8}{9}$.

92. A ***Complex Fraction*** is a fraction whose numerator, or denominator, or both, are fractional; as, $\frac{6}{\frac{3}{4}}$; $\frac{\frac{7}{8} \text{ of } \frac{2}{3}}{\frac{9}{10} \text{ of } \frac{11}{12}}$; $\frac{\frac{4}{7} \times 1\frac{3}{4} \times \frac{5}{8}}{2\frac{1}{2} \times \frac{4}{5} \times 4\frac{1}{3}}$.

93. A ***Mixed Number*** is a whole number and a fraction united; as, $9\frac{1}{2}$; $7\frac{3}{4}$.

EXERCISES.

Read the following fractions and mixed numbers:

1. $\frac{1}{2}$.	*9.* $\frac{2}{5}$.	*17.* $\frac{9}{10}$.	*25.* $\frac{4}{4}$.	*33.* $\frac{15}{18}$.	*41.* $\frac{73}{100}$.	*49.* $1\frac{1}{2}$.
2. $\frac{1}{3}$.	*10.* $\frac{3}{7}$.	*18.* $\frac{11}{12}$.	*26.* $\frac{5}{3}$.	*34.* $\frac{21}{25}$.	*42.* $\frac{45}{203}$.	*50.* $3\frac{2}{3}$.
3. $\frac{1}{4}$.	*11.* $\frac{6}{5}$.	*19.* $\frac{5}{13}$.	*27.* $\frac{7}{6}$.	*35.* $\frac{16}{48}$.	*43.* $\frac{127}{320}$.	*51.* $8\frac{1}{4}$.
4. $\frac{1}{5}$.	*12.* $\frac{8}{9}$.	*20.* $\frac{9}{14}$.	*28.* $\frac{3}{3}$.	*36.* $\frac{47}{63}$.	*44.* $\frac{54}{153}$.	*52.* $5\frac{1}{2}$.
5. $\frac{2}{3}$.	*13.* $\frac{4}{5}$.	*21.* $\frac{7}{15}$.	*29.* $\frac{8}{8}$.	*37.* $\frac{19}{52}$.	*45.* $\frac{75}{300}$.	*53.* $7\frac{1}{5}$.
6. $\frac{3}{4}$.	*14.* $\frac{3}{8}$.	*22.* $\frac{8}{11}$.	*30.* $\frac{7}{7}$.	*38.* $\frac{31}{84}$.	*46.* $\frac{67}{150}$.	*54.* $9\frac{2}{7}$.
7. $\frac{1}{6}$.	*15.* $\frac{7}{9}$.	*23.* $\frac{9}{17}$.	*31.* $\frac{2}{8}$.	*39.* $\frac{22}{35}$.	*47.* $\frac{99}{200}$.	*55.* $6\frac{7}{8}$.
8. $\frac{1}{7}$.	*16.* $\frac{5}{6}$.	*24.* $\frac{6}{15}$.	*32.* $\frac{8}{7}$.	*40.* $\frac{25}{45}$.	*48.* $\frac{141}{420}$.	*56.* $3\frac{4}{5}$.

Express by figures:

One third, Two eighths, Five sevenths, Two fifths, Four ninths, Five eighths, Three fourths, One ninth, Four elevenths, Five sixths, Seven eighths, Seven fifteenths, Nine twentieths, Three tenths, Six thirteenths, Nine tenths, Eleven thirty-thirds, Sixteen seventy-fifths, Fifty-one sixty-ninths, Twenty-four sixty-sevenths, Thirteen fourteenths, Forty-nine fiftieths, Sixty-two eighty-fourths, Ninety one-hundredths, Eighty-six one hundred twentieths, Nine sevenths, Six sixths, Thirty-two elevenths, Eighty-four fortieths, Thirty-seven twenty-sixths, One hundred one eighty-fifths.

REDUCTION.

94. ***Reduction of Fractions*** is the operation of changing their form without altering their value.

CASE I.

95. To reduce fractions to higher or lower terms.

ORAL EXERCISES.

1. Reduce $\frac{4}{6}$ to its lowest terms.

SOLUTION.—Since a fraction is expressed in its lowest terms when its numerator and denominator are prime to each other, we divide the terms of the given fraction $\frac{4}{6}$, by their greatest common divisor 2, and obtain $\frac{2}{3}$ for result. This result, therefore, expresses the lowest terms of the given fraction.

2. Change $\frac{6}{8}$ to its lowest terms. $\frac{4}{8}$. $\frac{6}{12}$. $\frac{3}{6}$. $\frac{5}{10}$.

3. Change $\frac{16}{24}$ to its lowest terms. $\frac{4}{12}$. $\frac{5}{15}$. $\frac{6}{24}$. $\frac{10}{18}$.

4. Change to their lowest terms: $\frac{3}{9}$, $\frac{6}{10}$, $\frac{4}{8}$, $\frac{7}{21}$, $\frac{6}{18}$, $\frac{9}{6}$, $\frac{12}{8}$, $\frac{6}{4}$, $\frac{10}{24}$, $\frac{12}{36}$, $\frac{18}{30}$, $\frac{6}{42}$, $\frac{8}{10}$, $\frac{13}{26}$, $\frac{42}{49}$, $\frac{21}{24}$.

5. Change $\frac{1}{3}$ to its higher terms by 2.

SOLUTION.—Since the value of a fraction is not changed by multiplying both numerator and denominator by the same number, we multiply the terms of the given fraction $\frac{1}{3}$ by 2, and obtain $\frac{2}{6}$ for result. This result expresses higher terms of the given fraction. In like manner we may multiply the terms of the given fraction by any number.

6. Change to equivalent higher terms: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{7}$, $\frac{4}{5}$, $\frac{2}{11}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{3}{87}$, $\frac{5}{61}$, $\frac{3}{4}$, $\frac{7}{81}$.

7. Change $\frac{4}{5}$ to an equivalent fraction having 40 for denominator.

8. Reduce $\frac{6}{7}$ to a fraction having 56 for a denominator.

9. Reduce $\frac{15}{16}$ to forty-eighths. $\frac{4}{9}$ to eighty-firsts.

WRITTEN EXERCISES.

I.

Reduce the following to their lowest terms:

1. $\frac{48}{54} = 2)\frac{48}{54} = 3)\frac{24}{27} = \frac{8}{9}$, *Ans.*

2. $\frac{40}{150} = 10)\frac{40}{150} = \frac{4}{15}$, *Ans.*

NOTE.—When the terms of the fraction are large and cannot be readily factored, find the greatest common divisor before reducing. Thus, to reduce $\frac{1261}{3298}$ to its lowest terms, divide both terms by their greatest common divisor, 97.

3. $\frac{21}{63}$.	*9.* $\frac{15}{145}$.	*15.* $\frac{72}{108}$.	*21.* $\frac{609}{696}$.
4. $\frac{36}{40}$.	*10.* $\frac{75}{120}$.	*16.* $\frac{150}{240}$.	*22.* $\frac{250}{1000}$.
5. $\frac{18}{45}$.	*11.* $\frac{17}{153}$.	*17.* $\frac{648}{980}$.	*23.* $\frac{824}{1236}$.
6. $\frac{25}{60}$.	*12.* $\frac{76}{285}$.	*18.* $\frac{252}{404}$.	*24.* $\frac{1555}{2799}$.
7. $\frac{48}{87}$.	*13.* $\frac{91}{406}$.	*19.* $\frac{84}{252}$.	*25.* $\frac{262}{1441}$.
8. $\frac{70}{96}$.	*14.* $\frac{63}{207}$.	*20.* $\frac{840}{980}$.	*26.* $\frac{1080}{1296}$.

II.

Find the value of:

1. $\frac{6 \times 8}{4 \times 18}$. *Ans.* $\frac{2}{3}$.

2. $\frac{9 \times 5}{20 \times 3}$. *Ans.* $\frac{3}{4}$.

3. $\frac{9 \times 8 \times 6}{16 \times 3 \times 5}$. *Ans.* $\frac{9}{5}$.

4. $\frac{3 \times 6 \times 7}{14 \times 3 \times 15}$. *Ans.* $\frac{1}{5}$.

5. $\frac{9 \times 6 \times 8}{12 \times 16 \times 18}$. *Ans.* $\frac{1}{8}$.

6. $\frac{3 \times 11 \times 4}{8 \times 12 \times 22}$. *Ans.* $\frac{1}{16}$.

7. $\frac{7 \times 8 \times 3}{15 \times 14 \times 7}$. *Ans.* $\frac{4}{35}$.

8. $\frac{12 \times 9 \times 1}{13 \times 24 \times 18}$. *Ans.* $\frac{1}{52}$.

9. $\frac{9 \times 7 \times 5 \times 13}{14 \times 1 \times 58 \times 18}$.

10. $\frac{4 \times 2 \times 15 \times 11}{9 \times 33 \times 8 \times 2}$.

11. $\frac{9 \times 5 \times 25 \times 28}{15 \times 3 \times 56 \times 40}$.

12. $\frac{11 \times 27 \times 13 \times 45 \times 16}{15 \times 26 \times 22 \times 54 \times 75}$.

13. $\frac{18 \times 20 \times 125 \times 36 \times 84}{60 \times 12 \times 96 \times 72 \times 120}$.

14. $\frac{35 \times 21 \times 65 \times 44 \times 250}{325 \times 90 \times 40 \times 36 \times 27}$.

CASE II.

96. To reduce whole or mixed numbers to simple fractions.

ORAL EXERCISES.

1. In 2 units how many halves?

SOLUTION.—Since the parts of a unit which has been divided into 2 equal parts, are called halves, every unit must, therefore, contain 2 halves. Now, if one unit contains 2 halves, 2 units will contain twice as many halves as 1 unit, or twice 2 halves, which are 4 halves. Therefore, in 2 units there are 4 halves.

2. In 4 units how many fifths? Halves? Fourths?
3. How many sixths in 3? 8? 7? 10? 2? 12?
4. How many thirds in 4? 5? 9? 11? 7? 8?
5. In \$5 how many half dollars? Quarter dollars?
6. How many \$$\frac{1}{2}$ will buy a barrel of flour worth \$8?
7. Reduce 8 to fourths. 9 to sixths. 3 to halves.
8. Reduce 17 to thirds. 8 bushels to 7ths of bushels.
9. How many 9ths of yards in 3 yards? 5 yards?
10. How do you reduce a number to thirds? Eighths?
11. In $2\frac{1}{2}$ how many halves? $3\frac{1}{2}$? $5\frac{1}{2}$? $8\frac{1}{2}$? $12\frac{1}{2}$?
12. How many thirds in $2\frac{1}{3}$? $3\frac{1}{3}$? $5\frac{2}{3}$? $7\frac{2}{3}$? $10\frac{1}{3}$?
13. How many fifths in $3\frac{1}{5}$? $6\frac{2}{5}$? $2\frac{3}{5}$? $4\frac{4}{5}$? $7\frac{3}{5}$?
14. Reduce to improper fractions $4\frac{2}{3}$. $2\frac{4}{7}$. $4\frac{3}{11}$.
15. Reduce $2\frac{5}{6}$ to sixths. $7\frac{5}{8}$ to eighths. $13\frac{1}{3}$ to thirds.
16. In $8\frac{9}{10}$ how many tenths? $4\frac{3}{10}$? $15\frac{1}{10}$? $6\frac{1}{10}$?
17. How many 20ths in 8? 12? 25? 16?
18. Reduce $9\frac{1}{4}$ to an improper fraction. $15\frac{1}{6}$. $25\frac{1}{8}$.
19. How many 12ths in 30? 70? 80? 14? 21?
20. In 15 how many 8ths? 10ths? 9ths? 7ths?
21. Reduce $8\frac{8}{9}$ to an improper fraction. $2\frac{11}{12}$. $1\frac{15}{16}$.
22. Reduce $22\frac{1}{9}$ to an improper fraction. $33\frac{1}{3}$. $37\frac{1}{2}$.

WRITTEN EXERCISES.

1. Reduce $96\frac{6}{7}$ to sevenths.

OPERATION.

$$1 \text{ unit} = \tfrac{7}{7}$$
$$96 \text{ units} = \tfrac{7}{7} \times 96 = \tfrac{672}{7}$$
$$\tfrac{672}{7} + \tfrac{6}{7} = \tfrac{678}{7}$$

SOLUTION.—As there are 7 sevenths in 1 unit, in 96 units there will be 96 times as many sevenths as in 1 unit, or 96 times $\frac{7}{7}$, which are $\frac{672}{7}$. $\frac{672}{7}$ and $\frac{6}{7}$ make $\frac{678}{7}$. Therefore, $96\frac{6}{7}$ are equal to $\frac{678}{7}$. *Ans.*

2. Express 28 in the form of a fraction having for denominator 3; 7; 11; 9; 16; 19. *Ans.* to first, $\frac{84}{3}$.

3. 7 is equal to what fraction having 18 for denominator? *Ans.* $\frac{126}{18}$.

4. Express 27 as fifths. *Ans.* $\frac{135}{5}$.

5. How many tenths in 38 pounds? *Ans.* $\frac{380}{10}$ pounds.

6. In 42 yards how many $\frac{1}{36}$ yards? *Ans.* $\frac{1512}{36}$ yards.

7. Reduce 3, 27, 63, 18, 42, to 13ths. *Ans.* to first, $\frac{39}{13}$.

8. How many 4ths in 6, 8, 13, 25, 75, 67?

9. How many 3ds in 9, 11, 16, 42, 7, 8?

10. What fraction with 17 for denominator is equal to 9? 16? 20? 12? *Ans.* $\frac{153}{17}$, $\frac{272}{17}$, $\frac{340}{17}$, $\frac{204}{17}$.

11. Reduce $9\frac{7}{12}$ to twelfths. *Ans.* $\frac{115}{12}$.

12. Change $18\frac{17}{23}$ to twenty-thirds. *Ans.* $\frac{431}{23}$.

13. Reduce $37\frac{4}{9}$ to an improper fraction. *Ans.* $\frac{337}{9}$.

14. In $86\frac{3}{4}$ miles how many $\frac{1}{4}$ miles? *Ans.* $\frac{347}{4}$ miles.

15. In \$$50\frac{3}{4}$ how many $\frac{1}{4}$ dollars? *Ans.* $\frac{203}{4}$ dollars.

Reduce the following to improper fractions:

16. $8\frac{3}{7}$.	*21.* $31\frac{8}{19}$.	*26.* $66\frac{2}{5}$.	*31.* $16\frac{1}{2}$.
17. $7\frac{5}{13}$.	*22.* $106\frac{11}{14}$.	*27.* $99\frac{9}{11}$.	*32.* $35\frac{1}{4}$.
18. $9\frac{13}{17}$.	*23.* $273\frac{3}{5}$.	*28.* $30\frac{6}{25}$.	*33.* $150\frac{1}{6}$.
19. $42\frac{8}{11}$.	*24.* $75\frac{1}{2}$.	*29.* $472\frac{23}{75}$.	*34.* $200\frac{5}{8}$.
20. $33\frac{1}{3}$.	*25.* $67\frac{1}{2}$.	*30.* $682\frac{7}{135}$.	*35.* $109\frac{15}{16}$.

CASE III.

97. To reduce improper fractions to whole or mixed numbers.

ORAL EXERCISES.

1. How many bushels in $\frac{19}{4}$ bushels?

SOLUTION.—In 1 bushel, or 1 unit, there are $\frac{4}{4}$; and in $\frac{19}{4}$ bushels there are as many bushels as 4 are contained times in 19, which are $4\frac{3}{4}$ times. Therefore, $\frac{19}{4}$ bushels are equal to $4\frac{3}{4}$ bushels.

2. How many units in $\frac{8}{2}$ units? $\frac{6}{3}$ units? $\frac{10}{5}$ units?

3. Reduce to whole numbers: $\frac{8}{4}$, $\frac{16}{2}$, $\frac{18}{3}$, $\frac{27}{9}$, $\frac{36}{4}$.

4. In $\frac{36}{12}$ feet, how many feet? In $\frac{36}{6}$ feet?

5. Reduce to whole numbers: $\frac{48}{6}$ yards, $\frac{35}{7}$ feet.

6. How many tons in $\frac{23}{5}$ tons? $\frac{13}{4}$ tons? $\frac{19}{7}$ tons?

7. How many cents in $\frac{12}{5}$¢? $\frac{18}{7}$¢? $\frac{32}{6}$¢? $\frac{41}{8}$¢?

8. Reduce to mixed numbers: $\frac{13}{7}$, $\frac{25}{6}$, $\frac{42}{9}$, $\frac{71}{8}$, $\frac{53}{11}$, $\frac{33}{10}$.

WRITTEN EXERCISES.

1. Change $\frac{127}{13}$ to a whole or mixed number.

OPERATION.

$\frac{127}{13} = 127 \div 13 = 9\frac{10}{13}$, *Ans.*

SOLUTION.—Since there are $\frac{13}{13}$ in 1 unit, in $\frac{127}{13}$ there are as many units as 13 are contained times in 127, which are $9\frac{10}{13}$ times. Therefore, $\frac{127}{13} = 9\frac{10}{13}$.

Reduce the following to whole or mixed numbers:

2. $\frac{82}{3}$.	*10.* $\frac{110}{11}$.	*18.* $\frac{118}{9}$.	*26.* $\frac{4812}{307}$.
3. $\frac{59}{9}$.	*11.* $\frac{208}{3}$.	*19.* $\frac{206}{7}$.	*27.* $\frac{4704}{12}$.
4. $\frac{81}{13}$.	*12.* $\frac{244}{121}$.	*20.* $\frac{133}{5}$.	*28.* $\frac{1248}{133}$.
5. $\frac{74}{5}$.	*13.* $\frac{204}{12}$.	*21.* $\frac{277}{12}$.	*29.* $\frac{3721}{162}$.
6. $\frac{97}{8}$.	*14.* $\frac{117}{7}$.	*22.* $\frac{458}{39}$.	*30.* $\frac{8002}{371}$.
7. $\frac{84}{12}$.	*15.* $\frac{234}{18}$.	*23.* $\frac{271}{47}$.	*31.* $\frac{1803}{361}$.
8. $\frac{56}{9}$.	*16.* $\frac{424}{106}$.	*24.* $\frac{632}{78}$.	*32.* $\frac{21762}{806}$.
9. $\frac{35}{16}$.	*17.* $\frac{948}{237}$.	*25.* $\frac{809}{102}$.	*33.* $\frac{87107}{3705}$.

Case IV.

98. To reduce fractions to equivalent fractions having a common denominator.

ORAL EXERCISES.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, to a common denominator.

Solution.—Since a common denominator of two or more fractions is a common multiple of their denominators, take the product of 2 and 4, which is 8, for the required denominator. Next, reduce the given fractions to others having 8 for a common denominator. $\frac{1}{2}$ is equal to $\frac{4}{8}$; and $\frac{3}{4}$ is equal to $\frac{6}{8}$. The given fractions are now said to be reduced to a common denominator.

2. $\frac{1}{3}$, $\frac{1}{6}$.	*7.* $\frac{2}{3}$, $\frac{3}{4}$.	*12.* $\frac{4}{5}$, $\frac{1}{2}$.
3. $\frac{1}{5}$, $\frac{1}{3}$.	*8.* $\frac{1}{8}$, $\frac{1}{6}$.	*13.* $\frac{2}{5}$, $\frac{2}{3}$.
4. $\frac{1}{2}$, $\frac{1}{3}$.	*9.* $\frac{2}{3}$, $\frac{2}{7}$.	*14.* $\frac{3}{8}$, $\frac{2}{5}$.
5. $\frac{5}{6}$, $\frac{1}{8}$.	*10.* $\frac{9}{11}$, $\frac{1}{2}$.	*15.* $\frac{5}{12}$, $\frac{2}{7}$.
6. $\frac{3}{7}$, $\frac{5}{9}$.	*11.* $\frac{9}{10}$, $\frac{6}{7}$.	*16.* $\frac{3}{17}$, $\frac{2}{5}$.

WRITTEN EXERCISES.

1. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to their least common denominator.

OPERATION.

$\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$.

$$\begin{array}{r|l} 3 & 3\,,\ 6\,,\ 8 \\ \hline 2 & 1\,,\ 2\,,\ 8 \\ \hline & 1\,,\ 1\,,\ 4 \end{array}$$

$3 \times 2 \times 4 = 24$, L. C. D.

$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$

$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

$\frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$

Solution.—Having found the least common multiple of the given denominators to be 24, reduce each fraction to equivalent fractions having 24 for denominator.

To reduce $\frac{2}{3}$ to an equivalent fraction, having 24 for denominator, divide 24 by the denominator 3, and multiply both terms by the quotient, 8. $\frac{5}{6}$ and $\frac{7}{8}$ may be reduced in the same way.

Reduce to their least common denominator:

2. $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}$.	*10.* $\frac{3}{4}, \frac{4}{11}, \frac{10}{22}$.	*18.* $\frac{13}{18}, \frac{18}{20}, \frac{22}{36}, \frac{40}{60}$.
3. $\frac{3}{8}, \frac{4}{6}, \frac{2}{5}$.	*11.* $\frac{3}{8}, \frac{7}{25}, \frac{5}{10}$.	*19.* $\frac{3}{14}, \frac{26}{28}, \frac{28}{35}, \frac{17}{63}$.
4. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$.	*12.* $\frac{2}{3}, \frac{4}{15}, \frac{7}{20}$.	*20.* $\frac{5}{12}, \frac{7}{36}, \frac{9}{48}, \frac{11}{72}$.
5. $\frac{5}{6}, \frac{8}{9}, \frac{6}{7}$.	*13.* $\frac{1}{4}, \frac{15}{16}, \frac{9}{20}$.	*21.* $\frac{7}{48}, \frac{17}{96}, \frac{3}{24}, \frac{1}{12}$.
6. $\frac{7}{9}, \frac{5}{8}, \frac{5}{6}$.	*14.* $\frac{5}{9}, \frac{30}{54}, \frac{8}{27}$.	*22.* $\frac{24}{75}, \frac{37}{56}, \frac{14}{15}, \frac{1}{28}$.
7. $\frac{4}{9}, \frac{1}{3}, \frac{3}{4}$.	*15.* $\frac{11}{21}, \frac{16}{39}, \frac{6}{13}$.	*23.* $\frac{13}{42}, \frac{8}{21}, \frac{16}{28}, \frac{9}{10}$.
8. $\frac{3}{5}, \frac{2}{7}, \frac{2}{3}$.	*16.* $\frac{14}{18}, \frac{31}{36}, \frac{61}{72}$.	*24.* $\frac{5}{12}, \frac{7}{15}, \frac{15}{16}, \frac{24}{25}$.
9. $\frac{1}{4}, \frac{2}{9}, \frac{3}{5}$.	*17.* $\frac{5}{12}, \frac{7}{18}, \frac{11}{24}$.	*25.* $\frac{7}{42}, \frac{71}{84}, \frac{101}{168}, \frac{93}{108}$.

ADDITION.

99. ***Addition of Fractions*** is the operation of finding the sum of two or more fractions.

ORAL EXERCISES.

1. What is the sum of $\frac{3}{4}+\frac{2}{4}$? $\frac{2}{3}+\frac{5}{6}$? $5\frac{1}{2}+4\frac{3}{5}$?

SOLUTION.—Addition of fractions implies three operations: 1st, to add fractions having a common denominator; 2d, to add fractions having different denominators; and 3d, to add mixed numbers. The first is performed by simply adding the numerators, for a new numerator, and keeping the common denominator. Thus, $\frac{3}{4}+\frac{2}{4}=\frac{5}{4}=1\frac{1}{4}$, *Ans.* Secondly, before adding fractions of different denominators, it is necessary to reduce them to a common denominator. Thus, $\frac{2}{3}+\frac{5}{6}=\frac{4}{6}+\frac{5}{6}=\frac{9}{6}=1\frac{3}{6}=1\frac{1}{2}$, *Ans.*

In the third operation find the sum of the fractions and that of the integers separately, and then add the results. Thus, to add $5\frac{1}{2}$ and $4\frac{3}{5}$, we first find the sum of $\frac{3}{5}$ and $\frac{1}{2}$, which is $1\frac{1}{10}$. This added to the sum of the integers, 5 and 4, will make $10\frac{1}{10}$, *Ans.*

NOTES.—1. Mixed numbers may also be reduced to improper fractions and then added by the second operation.

2. The sum should always be expressed in its lowest terms; and if an improper fraction, should be reduced to a mixed number.

I.

2. $\frac{1}{2}+\frac{1}{2}$.	*15.* $\frac{1}{6}+\frac{2}{6}$.	*28.* $\frac{3}{7}+\frac{2}{5}$.	*41.* $\frac{8}{10}+\frac{2}{5}$.
3. $\frac{1}{3}+\frac{1}{3}$.	*16.* $\frac{1}{2}+\frac{3}{4}$.	*29.* $\frac{6}{7}+\frac{3}{7}$.	*42.* $2\frac{1}{2}+\frac{1}{2}$.
4. $\frac{1}{4}+\frac{1}{4}$.	*17.* $\frac{2}{5}+\frac{1}{3}$.	*30.* $\frac{4}{9}+\frac{8}{9}$.	*43.* $3\frac{1}{4}+\frac{1}{4}$.
5. $\frac{2}{3}+\frac{1}{3}$.	*18.* $\frac{3}{4}+\frac{1}{3}$.	*31.* $\frac{8}{9}+\frac{6}{9}$.	*44.* $4\frac{2}{3}+1\frac{1}{3}$.
6. $\frac{3}{4}+\frac{1}{4}$.	*19.* $\frac{2}{3}+\frac{1}{9}$.	*32.* $\frac{3}{4}+\frac{3}{4}$.	*45.* $2\frac{1}{7}+3\frac{4}{7}$.
7. $\frac{3}{7}+\frac{2}{7}$.	*20.* $\frac{5}{8}+\frac{3}{4}$.	*33.* $\frac{2}{3}+\frac{4}{3}$.	*46.* $8\frac{4}{9}+4\frac{2}{9}$.
8. $\frac{3}{8}+\frac{2}{8}$.	*21.* $\frac{1}{3}+\frac{1}{6}$.	*34.* $\frac{5}{6}+\frac{1}{12}$.	*47.* $5\frac{2}{5}+6\frac{4}{5}$.
9. $\frac{3}{8}+\frac{1}{8}$.	*22.* $\frac{5}{12}+\frac{3}{12}$.	*35.* $\frac{6}{11}+\frac{1}{3}$.	*48.* $7\frac{7}{8}+3\frac{3}{8}$.
10. $\frac{4}{9}+\frac{2}{9}$.	*23.* $\frac{3}{10}+\frac{5}{10}$.	*36.* $\frac{5}{12}+\frac{5}{6}$.	*49.* $3\frac{3}{4}+2\frac{1}{2}$.
11. $\frac{1}{8}+\frac{5}{8}$.	*24.* $\frac{3}{14}+\frac{9}{14}$.	*37.* $\frac{5}{12}+\frac{9}{4}$.	*50.* $5\frac{2}{3}+4\frac{1}{6}$.
12. $\frac{4}{11}+\frac{3}{11}$.	*25.* $\frac{2}{15}+\frac{4}{15}$.	*38.* $\frac{12}{13}+\frac{10}{13}$.	*51.* $9\frac{3}{8}+8\frac{1}{4}$.
13. $\frac{6}{13}+\frac{5}{13}$.	*26.* $\frac{3}{16}+\frac{9}{16}$.	*39.* $\frac{14}{17}+\frac{10}{17}$.	*52.* $3\frac{1}{7}+4\frac{1}{2}$.
14. $\frac{9}{20}+\frac{8}{20}$.	*27.* $\frac{5}{18}+\frac{7}{18}$.	*40.* $\frac{12}{25}+\frac{14}{5}$.	*53.* $6\frac{3}{5}+1\frac{1}{3}$.

II.

1. James gave $\frac{1}{5}$ of a dollar for a ball and $\frac{2}{5}$ of a dollar for a pair of gloves; how much did he give for both?

2. A farmer sold $\frac{1}{6}$ of his potatoes to A, $\frac{1}{6}$ to B, and $\frac{3}{6}$ to C. What part of the whole did he sell to all?

3. Three boys hired a boat; John gave \$$\frac{1}{2}$, William \$$\frac{1}{4}$, and Henry \$$\frac{1}{4}$; what sum of money did they all pay?

4. A carpenter put up $\frac{3}{4}$ of a rod of fence, his son $\frac{1}{2}$ of a rod, and his apprentice $\frac{1}{2}$ of a rod; how long was the fence?

5. James gave \$$\frac{1}{4}$ for a circus ticket, \$$\frac{4}{5}$ for a straw hat, and had $\frac{1}{10}$ of a dollar remaining; how much had he at first?

6. Paid \$$8\frac{1}{2}$ for a bbl. of flour, \$$5\frac{1}{4}$ for a ton of coal, and \$$3\frac{1}{3}$ for a cord of wood; what was the amount of the bill?

7. How many yards of cloth will be required to make a pair of pants containing $1\frac{1}{2}$ yards, a vest $\frac{2}{3}$ of a yard, and a coat $2\frac{1}{6}$ yards?

8. Bought a mosaic table for $\$87\frac{3}{4}$ and sold it for $\$12\frac{1}{4}$ more than I paid for it; what was the selling price?

9. A man walked $13\frac{3}{4}$ miles on Monday and $14\frac{1}{8}$ miles on Tuesday; how far did he walk in the two days?

10. A boy earned $\$2\frac{1}{4}$ one week, $\$2$ the next week, and $\$3\frac{1}{2}$ the next week; how much did he earn in the 3 weeks?

11. What is the sum of $\$10\frac{1}{2}$, $\$5\frac{1}{2}$, $\$20\frac{1}{2}$, $\$14\frac{1}{2}$?

12. A man sold $5\frac{1}{2}$ bushels of cranberries at one time, $2\frac{1}{4}$ bushels at another, and $10\frac{3}{4}$ bushels at a third time. How many bushels did he sell in all?

13. A farm comprised a peach orchard of $6\frac{1}{8}$ acres, a wheat field of $12\frac{1}{2}$ acres, and a corn field of $9\frac{3}{4}$ acres. How many acres in the farm?

14. If a clerk copy $20\frac{1}{2}$ pages in one day, $30\frac{1}{3}$ pages in another, and $40\frac{5}{6}$ pages in another, how many pages will he copy in the three days?

WRITTEN EXERCISES.

1. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{12}$.

OPERATION.

	24
$\frac{2}{3}$	$8 \times 2 = 16$
$\frac{3}{4}$	$6 \times 3 = 18$
$\frac{5}{8}$	$3 \times 5 = 15$
$\frac{1}{2}$	$12 \times 1 = 12$
$\frac{7}{12}$	$2 \times 7 = 14$
	$\frac{75}{24} = 3\frac{1}{8}$

SOLUTION.—The least common denominator of the given fractions is 24. Having reduced them to 24ths, **Art. 98,** and taken the sum of their numerators, we find $\frac{75}{24}$, or $3\frac{1}{8}$ for result. Hence, the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{12} = 3\frac{1}{8}$.

2. Find the sum of $4\frac{2}{9}$, $\frac{7}{8}$, and $9\frac{5}{6}$.

OPERATION.

		72	
4	$\frac{2}{9}$	$8 \times 2 =$	16
	$\frac{7}{8}$	$9 \times 7 =$	63
9	$\frac{5}{6}$	$12 \times 5 =$	60
13			$\frac{139}{72} = 1\frac{67}{72}$

$13 + 1\frac{67}{72} = 14\frac{67}{72}$.

SOLUTION.—Having separated the integers from the fractions, reduce the latter to equivalent fractions having a common denominator. Find their sum as in the first example, and add it to the sum of the integers. The result will be the answer required.

Find the sum of:

3. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$.
4. $\frac{1}{5}+\frac{1}{4}+\frac{1}{6}$.
5. $\frac{2}{7}+\frac{1}{8}+\frac{1}{2}$.
6. $\frac{4}{7}+\frac{3}{8}+\frac{2}{9}$.
7. $\frac{7}{8}+\frac{3}{4}+\frac{3}{5}$.
8. $\frac{6}{7}+\frac{5}{9}+\frac{1}{3}$.
9. $\frac{5}{6}+\frac{5}{8}+\frac{1}{9}$.
10. $\frac{4}{4}+\frac{1}{7}+\frac{1}{6}$.
11. $\frac{3}{7}+\frac{4}{9}+\frac{5}{21}$.
12. $\frac{4}{5}+\frac{1}{2}+\frac{9}{10}$.
13. $\frac{8}{9}+\frac{1}{3}+\frac{8}{36}$.
14. $\frac{6}{7}+\frac{5}{6}+\frac{3}{14}$.
15. $\frac{5}{8}+\frac{2}{3}+\frac{13}{24}$.
16. $\frac{5}{8}+\frac{3}{4}+\frac{19}{20}$.
17. $\frac{1}{8}+\frac{7}{8}+\frac{27}{64}$.
18. $\frac{5}{9}+\frac{2}{3}+\frac{3}{11}$.
19. $\frac{5}{6}+\frac{8}{11}+\frac{1}{2}+\frac{5}{12}$.
20. $\frac{4}{9}+\frac{7}{36}+\frac{2}{5}+\frac{1}{18}$.
21. $\frac{16}{25}+\frac{7}{8}+\frac{1}{10}+\frac{3}{4}$.
22. $\frac{6}{7}+\frac{6}{35}+\frac{1}{5}+\frac{4}{25}$.
23. $\frac{1}{2}+\frac{4}{13}+\frac{1}{8}+\frac{2}{17}$.
24. $\frac{8}{15}+\frac{3}{7}+\frac{3}{9}+\frac{17}{50}$.
25. $\frac{14}{21}+\frac{19}{35}+\frac{1}{7}+\frac{8}{9}$.
26. $\frac{11}{12}+\frac{5}{24}+\frac{41}{48}+\frac{3}{5}$.

27. $\frac{5}{18}+\frac{1}{4}+\frac{7}{9}+\frac{25}{36}+\frac{3}{12}+\frac{1}{2}$. *Ans.* $2\frac{3}{4}$.
28. $\frac{7}{16}+\frac{2}{5}+\frac{17}{25}+\frac{1}{10}+\frac{2}{3}+\frac{7}{8}$. *Ans.* $3\frac{191}{1200}$.
29. $\frac{19}{45}+\frac{1}{9}+\frac{3}{8}+\frac{47}{55}+\frac{2}{11}+\frac{1}{3}+\frac{7}{10}$. *Ans.* $3\frac{199}{330}$.
30. $\frac{16}{33}+\frac{9}{11}+\frac{21}{22}+\frac{4}{3}+\frac{1}{2}+\frac{1}{4}+\frac{14}{15}$. *Ans.* $5\frac{181}{660}$.
31. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8}+\frac{8}{9}+\frac{9}{10}$. *Ans.* $7\frac{179}{2520}$.

II.

Add the following:

32. $5\frac{1}{8}$, $6\frac{1}{4}$, $1\frac{4}{5}$.
33. $9\frac{4}{5}$, $5\frac{1}{6}$, $3\frac{9}{10}$.
34. $3\frac{9}{11}$, $7\frac{19}{21}$, $10\frac{1}{2}$.
35. $15\frac{3}{8}$, $19\frac{11}{14}$, $5\frac{6}{7}$.
36. $6\frac{1}{3}$, $9\frac{11}{12}$, $4\frac{5}{16}$, $5\frac{7}{48}$.
37. $3\frac{1}{2}$, $4\frac{1}{4}$, $7\frac{1}{5}$, $9\frac{1}{5}$, $2\frac{5}{8}$.
38. $1\frac{1}{6}$, $3\frac{7}{27}$, $15\frac{1}{9}$, $23\frac{8}{54}$.
39. $45\frac{2}{3}$, $63\frac{3}{5}$, $5\frac{7}{8}$, $73\frac{1}{2}$.
40. $\frac{3}{7}$, $\frac{18}{63}$, $\frac{1}{9}$, $17\frac{3}{21}$, $6\frac{1}{3}$.
41. 45, $2\frac{15}{48}$, $9\frac{3}{16}$, $1\frac{3}{4}$.
42. $1\frac{2}{5}$, $9\frac{7}{12}$, $17\frac{8}{15}$, $18\frac{7}{30}$.
43. $2\frac{9}{16}$, $12\frac{1}{8}$, $3\frac{9}{32}$, $21\frac{3}{4}$.

44. $\frac{41}{35}$, $\frac{11}{20}$, $19\frac{3}{4}$, $68\frac{4}{5}$.
45. $\frac{14}{17}$, $5\frac{5}{12}$, $8\frac{16}{51}$, $7\frac{3}{17}$.
46. $9\frac{8}{39}$, $4\frac{1}{13}$, $\frac{18}{78}$, $4\frac{1}{6}$.
47. $\frac{74}{145}$, $6\frac{1}{9}$, $\frac{7}{15}$, $36\frac{1}{3}$.
48. $\frac{16}{35}$, $1\frac{19}{70}$, $2\frac{5}{7}$, $29\frac{9}{10}$.
49. $13\frac{1}{10}$, $1\frac{57}{100}$, $20\frac{131}{1000}$.
50. $6\frac{1}{4}$, $19\frac{17}{24}$, $50\frac{19}{48}$, $8\frac{1}{3}$.
51. $185\frac{1}{63}$, $9\frac{1}{2}$, $16\frac{7}{9}$, $2\frac{13}{81}$.

MENTAL EXERCISES.

1. Joseph bought a pair of skates for $\frac{5}{8}$ of a dollar, a pair of mittens for $\frac{4}{7}$ of a dollar, and a strap for $\frac{3}{4}$ of a dollar; what was the cost of the whole?

SOLUTION.—If the skates cost $\frac{5}{8}$ of a dollar, the mittens $\frac{4}{7}$, and the strap $\frac{3}{4}$, all together must cost the sum of $\frac{5}{8} + \frac{4}{7} + \frac{3}{4}$; which is $\frac{109}{56} = 1\frac{53}{56}$ dollars.

2. A farmer sold $\frac{1}{4}$ of his grain to one man, $\frac{2}{7}$ to another, and $\frac{3}{14}$ to another; what part of his grain did he sell to the three men together?

3. Jane paid $3\frac{1}{2}$ dollars for a dress, $2\frac{2}{3}$ for a pair of shoes, and $\frac{1}{6}$ for some ribbon; what did she pay for all?

4. Thomas saved $\frac{3}{4}$ of a dollar one week, $\frac{2}{5}$ of a dollar the next, $2\frac{1}{3}$ the next, and $5\frac{1}{2}$ the fourth week; how much did he save in the four weeks?

5. Three men bought a cow: Mr. Jones paid $\$15\frac{5}{6}$, Mr. Brown $\$24\frac{7}{24}$, and Mr. Smith $\$12\frac{1}{2}$; what was the cost of the cow?

6. Henry bought a penknife for $76\frac{7}{12}$ cents, and sold it for $23\frac{4}{5}$ more than it cost. What did he sell it for?

7. If it take $2\frac{1}{3}$ yards of cloth for a coat, $2\frac{1}{4}$ yards for a pair of pantaloons, and $\frac{5}{7}$ of a yard for a vest, how many yards will be required for the whole?

8. A man dug $\frac{1}{5}$ of a rod of a ditch the first hour, $\frac{4}{5}$ of a rod the second, $\frac{1}{5}$ the third, and $\frac{11}{12}$ the fourth; how many rods did he dig in the 4 hours?

9. How many pounds of candy in 4 packages containing respectively, $6\frac{1}{2}$, $5\frac{1}{4}$, 17, and $8\frac{9}{10}$ pounds?

10. If coal is worth $\$8\frac{7}{24}$ a ton, and wood is worth $\$6\frac{5}{12}$ a cord, what will be the cost of 2 tons of coal and 1 cord of wood?

11. Having $15\frac{9}{10}$ barrels of flour, I bought $3\frac{3}{20}$ barrels more. How many barrels had I then?

12. If Joseph walk $6\frac{3}{4}$ miles in the forenoon, and $4\frac{1}{5}$ miles in the afternoon, how many miles will he walk in two days at the same rate?

WRITTEN EXERCISES.

1. A gentleman travelled $56\frac{1}{6}$ miles in one day, $63\frac{3}{4}$ the next, $45\frac{1}{3}$ the third, and $20\frac{4}{5}$ the fourth. How far did he travel in all? *Ans.* $185\frac{25}{36}$ miles.

2. A boy earned $\$5\frac{3}{4}$ the first week, $\$6\frac{1}{3}$ the second week, $\$4\frac{7}{16}$ the third, $\$8\frac{1}{2}$ the fourth, $\$6\frac{7}{8}$ the fifth, and $\$2\frac{1}{4}$ the sixth. How much did he earn in the 6 weeks? *Ans.* $\$33\frac{15}{16}$.

3. A farm contains $32\frac{1}{4}$ acres of meadow land, $24\frac{5}{6}$ acres of plough land, $72\frac{3}{5}$ acres of pasture land, and $81\frac{9}{10}$ acres of wood land; how many acres in the farm? *Ans.* $211\frac{7}{12}$ acres.

4. How many pounds of raisins in 4 boxes containing respectively, $14\frac{5}{12}$ pounds, $27\frac{8}{9}$ pounds, $12\frac{14}{15}$ pounds, and $37\frac{9}{12}$ pounds? *Ans.* $92\frac{89}{90}$ pounds.

5. Three cheeses weigh respectively, $18\frac{2}{3}$, $64\frac{11}{12}$, and $14\frac{6}{16}$ pounds. What is their entire weight? *Ans.* $97\frac{23}{24}$ pounds.

6. From a certain quantity of cloth, I cut three pieces, the first containing $124\frac{2}{3}$ yards, the second $234\frac{7}{12}$ yards, and the third $50\frac{15}{36}$ yards. How many yards in the three pieces? *Ans.* $409\frac{2}{3}$ yards.

7. A has $\$150\frac{140}{150}$; B has $\$99\frac{24}{25}$; and C has $\$124\frac{13}{15}$. How much have they all? *Ans.* $\$375\frac{19}{25}$.

8. I sold 2 tubs of butter each containing $254\frac{5}{14}$ pounds, and 1 tub containing $96\frac{6}{7}$ pounds. How many pounds did I sell? *Ans.* $605\frac{4}{7}$ pounds.

9. Bought a ham for $\$1\frac{1}{5}$, a barrel of flour for $\$9\frac{4}{5}$, and some other articles for $\$15\frac{7}{8}$; what was the amount of the bill? *Ans.* $\$27\frac{1}{120}$.

10. What quantity of molasses in 4 casks, containing severally, $56\frac{1}{6}$, $24\frac{1}{3}$, $41\frac{8}{9}$, and $69\frac{2}{3}$ gallons?

Ans. $192\frac{1}{18}$ gallons.

11. One fourth of an army was killed, $\frac{3}{8}$ of them taken prisoners, $\frac{1}{6}$ of them wounded, and the rest remained unhurt. What part of the army was rendered unfit for service? *Ans.* $\frac{19}{24}$.

12. I gave $\$2\frac{1}{5}$ for a knife, $\$1\frac{5}{6}$ for a pocket-book, $\$24\frac{1}{3}$ for a watch, and $\$9\frac{8}{12}$ for a watch-chain, and then found that I had $\$28\frac{9}{12}$ left. How much had I at first?

Ans. $\$66\frac{47}{60}$.

13. A poor person received $\frac{7}{8}$ of a dollar from one person, $\frac{15}{16}$ of a dollar from another, $2\frac{3}{48}$ dollars from a third. How much did he receive from all?

14. After spending $\$\frac{2}{5}$, $\$\frac{3}{10}$, $\$\frac{7}{9}$, and $\$\frac{2}{3}$, I had $\$9\frac{1}{2}$ left. How much had I at first?

SUBTRACTION.

100. ***Subtraction of Fractions*** is the process of finding the difference between two fractions.

ORAL EXERCISES.

What is the value of:

1. $\frac{3}{5}-\frac{2}{5}$? $\frac{3}{4}-\frac{1}{2}$? $3\frac{2}{3}-1\frac{1}{4}$?

SOLUTION.—When the fractions have a common denominator, we simply take the difference between their numerators, and keep the same denominator; thus, $\frac{3}{5}-\frac{2}{5}=\frac{1}{5}$, *Ans.*

Fractions having different denominators, must be reduced to a common denominator before subtracting; as $\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$, *Ans.*

To find the difference between two mixed numbers, we subtract the fractions and the integers separately and then add the results. Thus, the difference between $3\frac{2}{3}$ and $1\frac{1}{4}$ is found as follows: $\frac{2}{3}-\frac{1}{4}=\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$; and $3-1=2$. Adding $\frac{5}{12}$ to the integer 2, we obtain $2\frac{5}{12}$ for the result. Mixed numbers, when small, may also be subtracted by reducing them to improper fractions having a common denominator.

I.

2. $\frac{6}{7}-\frac{3}{7}$.	*11.* $\frac{5}{6}-\frac{1}{3}$.	*20.* $\frac{9}{16}-\frac{3}{8}$.	*29.* $4\frac{6}{8}-1\frac{3}{8}$.
3. $\frac{8}{9}-\frac{1}{9}$.	*12.* $\frac{7}{8}-\frac{3}{4}$.	*21.* $\frac{8}{11}-\frac{2}{3}$.	*30.* $8\frac{6}{7}-2\frac{1}{7}$.
4. $\frac{1}{2}-\frac{1}{2}$.	*13.* $\frac{4}{9}-\frac{1}{3}$.	*22.* $\frac{7}{15}-\frac{1}{5}$.	*31.* $9\frac{3}{4}-2\frac{1}{2}$.
5. $\frac{2}{3}-\frac{1}{3}$.	*14.* $\frac{4}{7}-\frac{1}{2}$.	*23.* $\frac{8}{13}-\frac{1}{4}$.	*32.* $3\frac{7}{8}-2\frac{3}{4}$.
6. $\frac{7}{8}-\frac{3}{8}$.	*15.* $\frac{3}{5}-\frac{1}{2}$.	*24.* $\frac{13}{2}-\frac{7}{4}$.	*33.* $5\frac{1}{2}-4\frac{1}{3}$.
7. $\frac{3}{4}-\frac{1}{4}$.	*16.* $\frac{8}{3}-\frac{7}{4}$.	*25.* $\frac{11}{7}-\frac{2}{3}$.	*34.* $8\frac{3}{5}-2\frac{1}{4}$.
8. $\frac{5}{11}-\frac{3}{11}$.	*17.* $\frac{11}{12}-\frac{5}{12}$.	*26.* $\frac{6}{5}-\frac{5}{15}$.	*35.* $4\frac{1}{7}-2\frac{1}{8}$.
9. $\frac{6}{13}-\frac{1}{13}$.	*18.* $\frac{4}{18}-\frac{1}{18}$.	*27.* $\frac{8}{2}-\frac{20}{6}$.	*36.* $6\frac{1}{4}-5\frac{2}{5}$.
10. $\frac{20}{21}-\frac{8}{21}$.	*19.* $\frac{18}{25}-\frac{3}{25}$.	*28.* $\frac{27}{18}-\frac{1}{9}$.	*37.* $8\frac{1}{3}-6\frac{1}{5}$.

II.

1. Joseph had $\$\frac{3}{4}$ and gave away $\$\frac{1}{2}$; how much had he left?

2. Lucy paid $\$\frac{4}{5}$ for a Geography and $\$\frac{3}{5}$ for a Grammar; how much more did the Geography cost than the Grammar?

3. A man owning a mill sold $\frac{5}{18}$ of it; what part does he still own?

4. Philip bought $2\frac{1}{2}$ pounds of candy and eat $1\frac{1}{4}$ pounds; how much had he left?

5. A girl having $\$7\frac{3}{5}$ loses $\$3\frac{1}{5}$; how much has she remaining?

6. A farmer owning $6\frac{1}{4}$ acres of land sells $2\frac{1}{4}$ acres; how many acres has he remaining?

7. A man having \$$10\frac{2}{3}$, paid \$$5\frac{1}{4}$ for a ton of coal, and \$$2\frac{3}{4}$ for a cord of wood; how much had he left?

8. From $\frac{7}{10}$ of a dollar William spent $\frac{3}{10}$ of a dollar. What part of a dollar had he left?

WRITTEN EXERCISES.

1. Find the difference between $\frac{5}{9}$ and $\frac{3}{8}$.

OPERATION.

$$\frac{5}{9}-\frac{3}{8}=\frac{13}{72}$$

$$\begin{array}{c|l} & 72 \\ \hline \frac{5}{9} & 8\times 5=40 \\ \frac{3}{8} & 9\times 3=27 \\ \hline & \qquad\qquad \frac{13}{72} \end{array}$$

SOLUTION.—Reducing the given fractions to equivalent fractions having the least common denominator, we get $\frac{40}{72}$ and $\frac{27}{72}$. Then, $\frac{27}{72}$ from $\frac{40}{72}$ leave $\frac{13}{72}$.

2. From $157\frac{3}{5}$ take $85\frac{5}{6}$.

OPERATION.

$$\begin{array}{r|l} & 30 \\ \hline 157\frac{3}{5} & 6\times 3=18 \\ 85\frac{5}{6} & 5\times 5=25 \\ \hline 71\frac{23}{30} & \qquad\qquad \frac{23}{30} \end{array}$$

SOLUTION.—Having reduced the fractions to equivalent fractions having the least common denominator, we find that $\frac{25}{30}$ cannot be taken from $\frac{18}{30}$. We, therefore, take $1=\frac{30}{30}$, from 157 and add it to $\frac{18}{30}$, which gives $\frac{48}{30}$. Now, $\frac{25}{30}$ from $\frac{48}{30}$ leave $\frac{23}{30}$, and 85 from 156 leave 71. Hence, the required difference is $71\frac{23}{30}$.

Find the value of:

3. $\frac{5}{9}-\frac{1}{3}$.	*11.* $\frac{14}{15}-\frac{1}{3}$.	*19.* $\frac{8}{17}-\frac{12}{51}$.	*27.* $25-18\frac{4}{5}$.
4. $\frac{6}{7}-\frac{4}{5}$.	*12.* $\frac{9}{16}-\frac{3}{8}$.	*20.* $\frac{17}{25}-\frac{27}{50}$.	*28.* $37-15\frac{5}{6}$.
5. $\frac{2}{3}-\frac{5}{9}$.	*13.* $\frac{11}{12}-\frac{6}{7}$.	*21.* $\frac{31}{63}-\frac{8}{21}$.	*29.* $42-12\frac{1}{9}$.
6. $\frac{5}{6}-\frac{2}{3}$.	*14.* $\frac{12}{17}-\frac{2}{3}$.	*22.* $\frac{11}{30}-\frac{3}{10}$.	*30.* $56-10\frac{3}{5}$.
7. $\frac{8}{9}-\frac{3}{4}$.	*15.* $\frac{11}{25}-\frac{1}{6}$.	*23.* $\frac{19}{40}-\frac{7}{15}$.	*31.* $71-20\frac{2}{3}$.
8. $\frac{7}{8}-\frac{6}{7}$.	*16.* $\frac{5}{21}-\frac{1}{7}$.	*24.* $\frac{21}{35}-\frac{9}{28}$.	*32.* $90-31\frac{1}{2}$.
9. $\frac{4}{5}-\frac{1}{3}$.	*17.* $\frac{7}{20}-\frac{2}{7}$.	*25.* $\frac{27}{28}-\frac{9}{40}$.	*33.* $64-29\frac{3}{7}$.
10. $\frac{3}{4}-\frac{1}{6}$.	*18.* $\frac{4}{13}-\frac{2}{9}$.	*26.* $\frac{14}{41}-\frac{3}{13}$.	*34.* $47-40\frac{1}{8}$.

35. $39\frac{4}{15}-12$.
36. $28\frac{9}{16}-20$.
37. $71\frac{3}{17}-31$.
38. $48\frac{6}{19}-15$.
39. $82\frac{7}{18}-27$.
40. $56\frac{4}{13}-49$.
41. $42\frac{3}{4}-36\frac{4}{9}$.
42. $92\frac{4}{7}-25\frac{1}{2}$.
43. $75\frac{3}{8}-63\frac{1}{6}$.
44. $87\frac{1}{2}-39\frac{4}{5}$.
45. $51\frac{2}{3}-37\frac{1}{2}$.
46. $67\frac{3}{8}-24\frac{7}{8}$.
47. $200\frac{4}{57}-191\frac{2}{19}$.
48. $324\frac{1}{10}-180\frac{7}{40}$.
49. $502\frac{7}{15}-121\frac{16}{25}$.
50. $2\frac{161}{450}-1\frac{145}{876}$.
51. $600\frac{201}{300}-437\frac{47}{600}$.
52. $1001\frac{3}{77}-906\frac{4}{11}$.

MENTAL EXERCISES.

1. Joseph paid $\frac{9}{10}$ of a dollar for an Arithmetic, and $\frac{5}{6}$ of a dollar for a History. For which did he pay the more, and how much?

2. A man owning a factory, sold $\frac{9}{37}$ of it; how much did he still own?

3. Thomas gathered $9\frac{3}{4}$ quarts of strawberries, and sold $7\frac{2}{9}$ quarts; how many quarts had he left?

4. Henry will be 27 years old $12\frac{11}{15}$ years hence; how old is he now?

5. Having $\$39\frac{11}{20}$ in the bank, what amount must I add to it to make the sum $60?

6. A grocer sold $\frac{1}{3}$ of a box of tea from a box $\frac{4}{5}$ full: what part of the whole box was left?

7. William earns $\$15\frac{9}{20}$ a week, and Louis $\$30\frac{13}{40}$: how much does one earn more than the other?

8. If I buy a slate for $6\frac{3}{4}$ cents, a Reader for $65\frac{2}{3}$ cents, and a Grammar for $27\frac{7}{12}$ cents, how much change will I receive from a $5 bill?

9. Mr. Brown sold $\frac{1}{5}$ of his estate to A, $\frac{1}{9}$ to B, and $\frac{4}{15}$ to C; what part remained unsold?

10. If I had $\$8\frac{2}{3}$ more than I have, I should have $\$23\frac{1}{2}$. How much have I?

11. One man travels $37\frac{1}{4}$ miles a day, and another $31\frac{3}{8}$ miles a day. How many miles does one travel more than the other?

WRITTEN EXERCISES.

1. A man bought a farm, $236\frac{5}{8}$ acres, and sold $187\frac{9}{16}$ acres of it; how many acres had he remaining?

Ans. $49\frac{1}{16}$ acres.

2. Having $\$184\frac{9}{10}$, I bought a certain amount of cloth and had $\$63\frac{13}{15}$ left; what did the cloth cost me?

Ans. $\$121\frac{1}{30}$.

3. What number must be added to $164\frac{5}{6}$ to make $237\frac{7}{18}$? *Ans.* $72\frac{5}{9}$.

4. Sold a barrel of flour for $\$9\frac{3}{4}$, and a barrel of potatoes for $\$4\frac{5}{7}$. Which cost the more, and how much?

Ans. Flour, $\$4\frac{25}{28}$.

5. From a cask containing $60\frac{3}{4}$ gallons of cider I drew off $17\frac{11}{12}$ gallons; how many gallons were left?

Ans. $42\frac{5}{6}$ gallons.

6. A has $\$16\frac{1}{2}$, B has $\$17\frac{2}{3}$, and C has $\$8\frac{4}{5}$; how much less than $\$78\frac{19}{30}$ have they all? *Ans.* $\$35\frac{2}{3}$.

7. A lady having $150, made the following purchases: a bonnet for $\$10\frac{1}{2}$, a cloak for $\$47\frac{1}{6}$, and a shawl for $\$65\frac{1}{3}$. How much had she left? *Ans.* $27.

8. A merchant has 2 casks of wine each containing $56\frac{5}{8}$ gallons. If he sell $33\frac{4}{7}$ gallons from each of them, how many gallons will he have left?

Ans. $46\frac{3}{28}$ gallons.

9. A father bequeathed $5000 to his three sons. The first received $\$816\frac{5}{9}$; the second, $\$1684\frac{3}{5}$; and the third, the remainder. What did the third receive?

Ans. $\$2498\frac{28}{45}$.

10. Sold a quantity of grain, that cost me $\$6782\frac{3}{4}$, for $\$4896\frac{1}{2}$. I afterwards bought a second quantity for $\$6115\frac{11}{12}$, and sold it for $8975. What did I gain or lose on both transactions? *Ans.* Gained $\$972\frac{5}{6}$.

11. A grocer earned 65\frac{2}{3}$ and received a present of 30\frac{3}{4}$. He then bought a barrel of sugar for 12\frac{1}{2}$, and groceries to the amount of 25\frac{4}{15}$; how much had he remaining? *Ans.* 58\frac{39}{54}$.

12. A drover bought 5 cows for 240\frac{1}{8}$, and after paying 45\frac{11}{12}$ for pasturage, he sold them for 375\frac{3}{8}$. Did he gain or lose, and how much? *Ans.* 89\frac{1}{3}$ gained.

13. To what must I add 199$\frac{3}{5}$ to make 574$\frac{61}{75}$?

14. A man having to make a journey of 89$\frac{3}{16}$ miles, walked 16$\frac{4}{5}$ miles the first day, 27$\frac{3}{4}$ miles the second day, 29$\frac{9}{16}$ miles the third day, and finished the journey on the fourth day. How far did he walk the fourth day?

Ans. 15$\frac{3}{40}$ miles.

15. A merchant has on hand 495 pounds of tea, of which 242$\frac{3}{4}$ pounds are Hyson, and the remainder Oolong. How many pounds of Oolong tea has he?

Ans. 252$\frac{1}{4}$ pounds.

16. What number must be added to the difference between 175$\frac{6}{7}$ and 204$\frac{1}{2}$ to make 301$\frac{19}{28}$? *Ans.* 273$\frac{1}{28}$.

17. What fraction added to the sum of $\frac{1}{5}$, $\frac{3}{50}$, $\frac{9}{100}$, and $\frac{7}{20}$ will make $\frac{193}{240}$? *Ans.* $\frac{5}{48}$.

18. How much will be left of a piece of cloth containing 73$\frac{7}{36}$ yards, after cutting from it 5 coats and 7 vests, allowing $\frac{5}{8}$ of a yard for a vest and 3$\frac{7}{9}$ yards for a coat?

Ans. 49$\frac{47}{72}$ yards.

19. Two men being 8496 feet apart, travel towards each other, one at the rate of 190$\frac{3}{7}$ feet in an hour, and the other at the rate of 101$\frac{19}{28}$ feet in an hour. How far apart are they at the end of two hours?

Ans. 7912$\frac{1}{14}$ feet.

20. A man owing $85, paid at one time 19\frac{7}{12}$, and at another 32\frac{4}{5}$. How much does he still owe?

Ans. 32\frac{37}{60}$.

MULTIPLICATION.

101. ***Multiplication of Fractions*** is the process of finding the product of two or more fractions.

CASE I.

102. To multiply a fraction by a whole number.

ORAL EXERCISES.

1. Multiply $\frac{5}{8}$ by 4. $3\frac{1}{2} \times 7$.

SOLUTION.—To multiply a fraction by an integer, we may either multiply the numerator, or divide the denominator by the integer. Thus, $\frac{5}{8} \times 4 = \frac{20}{8} = 2\frac{1}{2}$, *Ans.* Or, $\frac{5}{8} \times 4 = \frac{5}{2} = 2\frac{1}{2}$, *Ans.*

In multiplying mixed numbers, we may reduce them to improper fractions, or multiply their parts separately, and add the results. Thus, $3\frac{1}{2} \times 7 = \frac{7}{2} \times 7 = \frac{49}{2} = 24\frac{1}{2}$, *Ans.* Or, $3\frac{1}{2} \times 7 = (3 \times 7) + (\frac{1}{2} \times 7) = 21 + 3\frac{1}{2} = 24\frac{1}{2}$, *Ans.*

I.

2. $\frac{1}{5} \times 4$.	*10.* $\frac{7}{9} \times 3$.	*18.* $\frac{7}{15} \times 2$.	*26.* $2\frac{1}{2} \times 4$.
3. $\frac{3}{5} \times 2$.	*11.* $\frac{2}{5} \times 5$.	*19.* $\frac{7}{18} \times 3$.	*27.* $3\frac{3}{4} \times 5$.
4. $\frac{5}{6} \times 3$.	*12.* $\frac{7}{8} \times 6$.	*20.* $\frac{2}{17} \times 7$.	*28.* $7\frac{3}{5} \times 4$.
5. $\frac{4}{7} \times 5$.	*13.* $\frac{8}{9} \times 7$.	*21.* $\frac{5}{21} \times 7$.	*29.* $8\frac{1}{6} \times 8$.
6. $\frac{4}{5} \times 6$.	*14.* $\frac{4}{3} \times 4$.	*22.* $\frac{8}{11} \times 9$.	*30.* $6\frac{3}{4} \times 5$.
7. $\frac{3}{7} \times 4$.	*15.* $\frac{9}{6} \times 5$.	*23.* $\frac{3}{24} \times 4$.	*31.* $9\frac{2}{7} \times 6$.
8. $\frac{2}{3} \times 3$.	*16.* $\frac{6}{7} \times 3$.	*24.* $\frac{3}{22} \times 5$.	*32.* $3\frac{5}{8} \times 4$.
9. $\frac{3}{4} \times 2$.	*17.* $\frac{9}{8} \times 8$.	*25.* $\frac{3}{13} \times 6$.	*33.* $8\frac{3}{4} \times 7$.

II.

1. At $\$\frac{3}{4}$ apiece, what will five penknives cost?

2. At $\$\frac{3}{8}$ a yard, what will 7 yards of alpaca cost?

3. What will 6 base balls cost at $\$\frac{2}{3}$ each?

4. What will 8 pounds of cheese cost at $\$\frac{1}{7}$ a pound?

5. Find the value of 16 barrels of flour at \$$8\frac{1}{2}$ per barrel.

6. If I gain \$$\frac{5}{8}$ on one yard of cloth, how much will I gain on 15 yards?

7. What will eleven bushels of apples cost at \$$\frac{5}{6}$ a bushel? 18 bushels? 30 bushels?

8. If a pound of tea cost \$$\frac{3}{8}$, what will 16 pounds cost?

9. What is the cost of 3 barrels of flour at \$$8\frac{1}{3}$ a barrel? \$$7\frac{1}{2}$? \$$9\frac{1}{4}$? \$$6\frac{3}{4}$?

10. What is the cost of 6 tons of coal at \$$6\frac{1}{4}$ a ton?

11. What amount of money will pay for 5 weeks' board at \$$7\frac{2}{3}$ a week?

12. What is the product of $2\frac{1}{2} + 3\frac{3}{4}$, multiplied by 8?

WRITTEN EXERCISES.

1. Multiply $\frac{4}{9}$ by 3.

OPERATION.

$\frac{4}{9} \times 3 = \frac{12}{9} = 1\frac{1}{3}$, *Ans.*

Or,

$\frac{4}{9} \times 3 = \frac{4}{3} = 1\frac{1}{3}$, *Ans.*

SOLUTION.—To multiply the fraction $\frac{4}{9}$ by 3, we multiply the numerator 4 by 3, and keep the same denominator. This operation gives $\frac{12}{9} = 1\frac{1}{3}$, for the required result.

Or, since 3, the multiplier, is a factor of 9, the denominator of the multiplicand, we may also perform the multiplication by dividing the denominator by the multiplier. Thus we obtain $\frac{4}{3} = 1\frac{1}{3}$, *Ans.*

What is the value of:

2. $\frac{2}{3} \times 12$.	*10.* $\frac{19}{21} \times 7$.	*18.* $131\frac{4}{5} \times 9$.	*26.* $\frac{57}{132} \times 66$.
3. $\frac{3}{4} \times 20$.	*11.* $\frac{16}{51} \times 3$.	*19.* $215\frac{5}{9} \times 6$.	*27.* $\frac{102}{325} \times 25$.
4. $\frac{8}{9} \times 36$.	*12.* $\frac{9}{37} \times 6$.	*20.* $182\frac{5}{6} \times 4$.	*28.* $19\frac{99}{100} \times 20$.
5. $\frac{7}{8} \times 40$.	*13.* $\frac{11}{15} \times 5$.	*21.* $360\frac{2}{3} \times 5$.	*29.* $121\frac{11}{24} \times 36$.
6. $\frac{6}{7} \times 32$.	*14.* $\frac{8}{65} \times 7$.	*22.* $542\frac{3}{4} \times 8$.	*30.* $518\frac{9}{10} \times 45$.
7. $\frac{5}{8} \times 17$.	*15.* $\frac{7}{17} \times 9$.	*23.* $418\frac{1}{2} \times 7$.	*31.* $363\frac{8}{15} \times 56$.
8. $\frac{4}{9} \times 54$.	*16.* $\frac{13}{85} \times 5$.	*24.* $204\frac{5}{8} \times 9$.	*32.* $284\frac{7}{12} \times 48$.
9. $\frac{2}{3} \times 45$.	*17.* $\frac{22}{25} \times 4$.	*25.* $605\frac{1}{5} \times 6$.	*33.* $110\frac{19}{20} \times 50$.

CASE II.

103. To multiply a whole number by a fraction.

ORAL EXERCISES.

1. Multiply 6 by $\frac{2}{3}$.

SOLUTION.—Multiply the whole number by the numerator and divide the product by the denominator. Thus, $6 \times \frac{2}{3} = \frac{12}{3} = 4$, *Ans.* Or, $6 \times \frac{2}{3} = 2 \times \frac{1}{3}$ of $6 = 4$, *Ans.* When mixed numbers occur, perform the operation as in Case I.

I.

2. $3 \times \frac{1}{4}$.	*10.* $7 \times \frac{3}{7}$.	*18.* $5 \times \frac{8}{15}$.	*26.* $9 \times 3\frac{1}{9}$.
3. $6 \times \frac{2}{3}$.	*11.* $6 \times \frac{7}{8}$.	*19.* $6 \times \frac{7}{54}$.	*27.* $8 \times 5\frac{1}{8}$.
4. $9 \times \frac{3}{4}$.	*12.* $3 \times \frac{5}{9}$.	*20.* $4 \times \frac{17}{24}$.	*28.* $7 \times 8\frac{3}{7}$.
5. $7 \times \frac{1}{2}$.	*13.* $4 \times \frac{8}{9}$.	*21.* $8 \times \frac{11}{16}$.	*29.* $5 \times 3\frac{4}{5}$.
6. $9 \times \frac{3}{5}$.	*14.* $9 \times \frac{4}{5}$.	*22.* $9 \times \frac{14}{15}$.	*30.* $3 \times 7\frac{5}{6}$.
7. $3 \times \frac{4}{7}$.	*15.* $7 \times \frac{3}{2}$.	*23.* $7 \times \frac{8}{21}$.	*31.* $4 \times 8\frac{8}{9}$.
8. $8 \times \frac{5}{6}$.	*16.* $8 \times \frac{9}{4}$.	*24.* $6 \times \frac{11}{36}$.	*32.* $9 \times 5\frac{2}{3}$.
9. $5 \times \frac{2}{9}$.	*17.* $6 \times \frac{8}{3}$.	*25.* $3 \times \frac{14}{35}$.	*33.* $8 \times 7\frac{3}{4}$.

II.

1. Joseph having 3 cents worth of candy gave John $\frac{1}{3}$ of it. How much did John receive?

2. What will $\frac{6}{15}$ of a spool of thread cost at 5 cents a spool? $\frac{4}{5}$ of a spool? $\frac{9}{10}$ of a spool?

3. What is the $\frac{6}{63}$ of \$9? Of \$36? Of \$27? Of \$54? Of \$72? Of \$21? Of \$48?

4. At \$8 a week, what will a man earn in $\frac{1}{2}$ of a week?

5. When sugar is worth 13 cents a pound, what will $\frac{7}{26}$ of a pound cost? $\frac{8}{19}$ of a pound? $\frac{2}{3}$? $\frac{5}{6}$?

6. At 30 days to a month, how many days in $\frac{4}{15}$ of a month? In $\frac{5}{6}$? In $\frac{9}{10}$? In $\frac{2}{3}$?

7. At \$6 a yard, what will $\frac{7}{10}$ of a yard of cloth cost?

WRITTEN EXERCISES.

1. Multiply 8 by $\frac{3}{4}$.

OPERATION.

$8 \times \frac{3}{4} = (8 \div 4) \times 3 = 6$, *Ans.*

Or,

$8 \times \frac{3}{4} = \frac{\overset{2}{\cancel{8}} \times 3}{\cancel{4}} = 6$, *Ans.*

SOLUTION.—To multiply 8 by $\frac{3}{4}$ is the same as to find $\frac{3}{4}$ of 8. But $\frac{3}{4}$ of 8 is equal to 3 times $\frac{1}{4}$ of 8, which is 6.

Or, multiplying the whole number 8, by the numerator 3, and dividing by the denominator 4, we obtain 6 for result.

2. Multiply 25 by $4\frac{3}{5}$.

OPERATION.

$$\begin{array}{r} 25 \\ 4\frac{3}{5} \\ \hline 15 = \frac{3}{5} \text{ of } 25 \\ 100 = 4 \text{ times } 25 \\ \hline 115 = 4\frac{3}{5} \text{ times } 25 \end{array}$$

Or,

$25 \times 4\frac{3}{5} = 25 \times \frac{23}{5} = 115$, *Ans.*

SOLUTION.—To multiply 25 by $4\frac{3}{5}$ is the same as to add the $\frac{3}{5}$ of 25 to 4 times 25. Now, $\frac{3}{5}$ of 25 is equal to 15; and 4 times 25 is equal to 100. Adding 15 and 100 we get 115. Or, before multiplying we may reduce the mixed number, $4\frac{3}{5}$, to an improper fraction, and then perform the operation as in the preceding example.

Find the product of:

3. $35 \times \frac{5}{7}$.	*11.* $120 \times \frac{7}{10}$.	*19.* $396 \times 4\frac{1}{4}$
4. $75 \times \frac{3}{5}$.	*12.* $352 \times \frac{11}{12}$.	*20.* $463 \times 6\frac{2}{3}$
5. $81 \times \frac{8}{9}$.	*13.* $460 \times \frac{7}{20}$.	*21.* $850 \times 3\frac{4}{5}$.
6. $78 \times \frac{2}{3}$.	*14.* $500 \times \frac{8}{15}$.	*22.* $580 \times 7\frac{3}{4}$.
7. $67 \times \frac{3}{4}$.	*15.* $148 \times \frac{3}{16}$.	*23.* $325 \times 8\frac{5}{8}$.
8. $44 \times \frac{5}{6}$.	*16.* $256 \times \frac{8}{13}$.	*24.* $630 \times 7\frac{2}{9}$.
9. $58 \times \frac{4}{7}$.	*17.* $692 \times \frac{4}{11}$.	*25.* $245 \times 9\frac{2}{7}$.
10. $95 \times \frac{3}{8}$.	*18.* $428 \times \frac{5}{14}$.	*26.* $199 \times 3\frac{1}{2}$.

27. $66 \times 19\frac{5}{6}$.	*33.* $625 \times 9\frac{1}{25}$.	*39.* $316 \times 22\frac{1}{4}$.
28. $18 \times 50\frac{5}{9}$.	*34.* $420 \times 2\frac{13}{21}$.	*40.* $659 \times 14\frac{1}{7}$.
29. $61 \times 12\frac{1}{2}$.	*35.* $760 \times 7\frac{9}{10}$.	*41.* $285 \times 50\frac{5}{9}$.
30. $47 \times 41\frac{1}{3}$.	*36.* $840 \times 6\frac{9}{14}$.	*42.* $245 \times 25\frac{8}{35}$.
31. $89 \times 12\frac{2}{3}$.	*37.* $101 \times 3\frac{2}{11}$.	*43.* $461 \times 32\frac{9}{10}$.
32. $91 \times 16\frac{4}{7}$.	*38.* $238 \times 5\frac{9}{13}$.	*44.* $576 \times 40\frac{17}{36}$.

Case III.

104. To find the product of two or more fractions or mixed numbers.

ORAL EXERCISES.

1. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

Solution.—Multiply the numerators together for a new numerator, and the denominators for a new denominator. Thus, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$, *Ans.* Reduce mixed numbers to improper fractions.

I.

2. $\frac{1}{2} \times \frac{1}{2}$.	*10.* $\frac{3}{4} \times \frac{9}{10}$.	*18.* $\frac{8}{15} \times \frac{10}{40}$.	*26.* $\frac{3}{4} \times 1\frac{1}{2}$.
3. $\frac{1}{2} \times \frac{1}{4}$.	*11.* $\frac{5}{7} \times \frac{14}{15}$.	*19.* $\frac{9}{14} \times \frac{11}{4}$.	*27.* $\frac{5}{8} \times 2\frac{2}{5}$.
4. $\frac{1}{3} \times \frac{2}{5}$.	*12.* $\frac{3}{5} \times \frac{10}{21}$.	*20.* $\frac{17}{18} \times \frac{9}{20}$.	*28.* $\frac{1}{2} \times 9\frac{1}{2}$.
5. $\frac{3}{4} \times \frac{2}{3}$.	*13.* $\frac{4}{7} \times \frac{21}{32}$.	*21.* $\frac{24}{25} \times \frac{15}{16}$.	*29.* $\frac{6}{7} \times 3\frac{1}{6}$.
6. $\frac{2}{7} \times \frac{3}{5}$.	*14.* $\frac{2}{5} \times \frac{11}{16}$.	*22.* $\frac{7}{30} \times \frac{10}{13}$.	*30.* $\frac{4}{5} \times 5\frac{1}{4}$.
7. $\frac{1}{3} \times \frac{3}{4}$.	*15.* $\frac{5}{6} \times \frac{12}{5}$.	*23.* $\frac{8}{45} \times \frac{15}{9}$.	*31.* $\frac{2}{3} \times 1\frac{5}{7}$.
8. $\frac{1}{4} \times \frac{1}{8}$.	*16.* $\frac{8}{5} \times \frac{7}{11}$.	*24.* $\frac{8}{13} \times \frac{39}{40}$.	*32.* $\frac{3}{5} \times 6\frac{2}{3}$.
9. $\frac{5}{6} \times \frac{1}{7}$.	*17.* $\frac{7}{9} \times \frac{9}{10}$.	*25.* $\frac{7}{10} \times \frac{12}{35}$.	*33.* $\frac{8}{9} \times 2\frac{3}{4}$.

II.

1. If a ton of coal cost \$$5\frac{1}{2}$, what will $\frac{2}{3}$ of a ton cost?

2. What cost $\frac{3}{4}$ of a yard of muslin at $12\frac{1}{2}$ cents a yard?

3. What will be the cost of $\frac{3}{5}$ of a quart of chestnuts at $7\frac{1}{2}$ cents a quart?

4. What is the $\frac{1}{7}$ of $14\frac{7}{10}$? $\frac{3}{8}$ of $21\frac{1}{2}$? $\frac{6}{7}$ of $9\frac{1}{3}$?

5. James having \$$\frac{2}{3}$ gave his brother $\frac{1}{8}$ of it; what part of a dollar did his brother receive?

6. When rye is worth \$$\frac{5}{6}$ a bushel, what will $\frac{4}{5}$ of a bushel cost?

7. At \$$21\frac{1}{2}$ a yard, what will $\frac{3}{4}$ of a yard of cloth cost?

8. What will $\frac{1}{2}$ a pound of tea cost at \$$\frac{2}{3}$ a pound?

WRITTEN EXERCISES.

1. Multiply $\frac{5}{9}$ by $\frac{3}{7}$.

OPERATION.

$$\frac{5}{9} \times \frac{3}{7} = \frac{5}{9} \times 3 = \frac{15}{9}$$

$$\frac{15}{9} \div 7 = \frac{15}{9 \times 7} = \frac{15}{63} = \frac{5}{21}, \textit{Ans.}$$

Or,

$$\frac{5}{9} \times \frac{3}{7} = \frac{5 \times \cancel{3}}{\underset{3}{\cancel{9}} \times 7} = \frac{5}{21}, \textit{Ans.}$$

SOLUTION.—To multiply $\frac{5}{9}$ by $\frac{3}{7}$ we first multiply $\frac{5}{9}$ by 3, which gives $\frac{15}{9}$. But this result is 7 times the required product, because 3 is 7 times the given multiplier. Hence, to find the true product we must divide the product of $\frac{5}{9} \times 3$ by 7. This gives $\frac{5 \times \cancel{3}}{\underset{3}{\cancel{9}} \times 7} = \frac{5}{21}$, *Ans.*

NOTE.—Cancel all factors common to the numerators and denominators.

2. Multiply $64\frac{1}{2}$ by $48\frac{2}{3}$.

OPERATION.

$$\begin{array}{rrl} & 64\frac{1}{2} & \\ & 48\frac{2}{3} & \\ & \frac{1}{3} & = \frac{1}{2} \times \frac{2}{3} \\ 3\,)\,128 = & 42\frac{2}{3} & = 64 \times \frac{2}{3} \\ 2\,)\,\ 48 = & 24 & = \frac{1}{2} \times 48 \\ & \left.\begin{array}{r}512\\256\end{array}\right\} & = 64 \times 48 \\ & 3139 & = 64\frac{1}{2} \times 48\frac{2}{3} \end{array}$$

SOLUTION.—Instead of reducing to improper fractions, as when the numbers are small, multiply the whole number and the fraction of the multiplicand separately, first by the fraction and then by the whole of the multiplier. Take the sum of the partial products for the required product.

Find the product of:

3. $\frac{5}{13} \times \frac{2}{3}$.
4. $\frac{13}{31} \times \frac{1}{2}$.
5. $\frac{22}{35} \times \frac{7}{9}$.
6. $\frac{14}{15} \times \frac{5}{8}$.
7. $\frac{12}{25} \times \frac{5}{6}$.
8. $\frac{10}{31} \times \frac{1}{5}$.
9. $\frac{7}{23} \times \frac{3}{7}$.
10. $\frac{7}{36} \times \frac{4}{5}$.

11. $\frac{3}{5} \times \frac{5}{8} \times \frac{6}{7}$.
12. $\frac{1}{2} \times \frac{7}{9} \times \frac{3}{4}$.
13. $\frac{6}{7} \times \frac{7}{8} \times \frac{2}{3}$.
14. $\frac{1}{4} \times \frac{2}{3} \times \frac{3}{5}$.
15. $\frac{3}{8} \times \frac{4}{7} \times \frac{1}{3}$.
16. $\frac{4}{5} \times \frac{1}{8} \times \frac{5}{7}$.
17. $\frac{3}{9} \times \frac{1}{2} \times \frac{1}{7}$.
18. $\frac{2}{7} \times \frac{3}{2} \times \frac{4}{7}$.

19. $\frac{5}{8} \times 12\frac{1}{2}$.
20. $\frac{3}{5} \times 22\frac{1}{2}$.
21. $\frac{2}{3} \times 35\frac{1}{4}$.
22. $\frac{4}{7} \times 76\frac{3}{5}$.
23. $\frac{5}{8} \times 29\frac{1}{3}$.
24. $\frac{1}{7} \times 84\frac{2}{5}$.
25. $\frac{4}{5} \times 12\frac{4}{5}$.
26. $\frac{3}{4} \times 62\frac{1}{2}$.

27. $41\frac{2}{3} \times 12\frac{1}{2}$.
28. $15\frac{1}{6} \times 10\frac{1}{3}$.
29. $62\frac{1}{2} \times 15\frac{1}{4}$.
30. $27\frac{1}{3} \times 42\frac{3}{4}$.
31. $78\frac{3}{4} \times 26\frac{4}{5}$.
32. $34\frac{2}{7} \times 15\frac{7}{9}$.
33. $48\frac{4}{5} \times 30\frac{1}{4}$.
34. $52\frac{5}{7} \times 65\frac{1}{2}$.

35. $230\frac{3}{4} \times \frac{1}{2}$.
36. $189\frac{2}{3} \times \frac{2}{5}$.
37. $350\frac{1}{4} \times \frac{3}{5}$.
38. $436\frac{2}{5} \times \frac{5}{8}$.
39. $170\frac{2}{5} \times \frac{3}{4}$.
40. $520\frac{3}{7} \times \frac{1}{3}$.
41. $280\frac{1}{4} \times \frac{8}{9}$.
42. $600\frac{1}{2} \times \frac{4}{5}$.

43. $360\frac{1}{4} \times 50\frac{1}{3}$.
44. $135\frac{1}{2} \times 42\frac{1}{4}$.
45. $270\frac{2}{3} \times 32\frac{1}{5}$.
46. $307\frac{1}{4} \times 18\frac{1}{2}$.
47. $405\frac{2}{5} \times 32\frac{4}{7}$.
48. $154\frac{1}{6} \times 21\frac{1}{3}$.
49. $193\frac{2}{3} \times 24\frac{1}{8}$.
50. $200\frac{3}{4} \times 27\frac{2}{5}$.

51. $221\frac{1}{3} \times 20\frac{3}{4}$.
52. $430\frac{1}{2} \times 16\frac{4}{7}$.
53. $387\frac{2}{5} \times 47\frac{1}{2}$.
54. $564\frac{2}{9} \times 32\frac{1}{7}$.
55. $102\frac{1}{4} \times 56\frac{1}{3}$.
56. $670\frac{2}{5} \times 10\frac{5}{12}$.

57. $\frac{6}{17} \times \frac{34}{120} \times 5\frac{5}{8}$.
58. $16\frac{2}{3} \times 17\frac{8}{9} \times \frac{27}{50}$.
59. $48 \times \frac{2}{7} \times 3\frac{2}{11} \times \frac{11}{15}$.
60. $(\frac{2}{3} + \frac{4}{7}) \times (\frac{3}{7} - \frac{3}{14})$.
61. $(2\frac{2}{5} - 1\frac{1}{2}) \times (16\frac{1}{4} + 8\frac{2}{5}) \times 1\frac{3}{17}$.
62. $[(140 - 19\frac{2}{3}) \times (52\frac{1}{2} + 3\frac{1}{4})] \times 21\frac{3}{5}$.

MENTAL EXERCISES.

1. What will 4 barrels of flour cost at $\$7\frac{5}{8}$ a barrel?

2. How many bushels of wheat will 27 bags hold, if each hold $3\frac{1}{3}$ bushels?

3. If it take $4\frac{1}{2}$ yards of cloth for a coat, and $3\frac{1}{3}$ yards for a pair of pantaloons, how many yards will be required to make 4 of each?

4. How long will 1 man take to do a certain piece of work, if 8 men require $12\frac{1}{2}$ days to do it?

5. If a person's expenses be \$15 a week, what will he spend in $7\frac{3}{5}$ weeks? $8\frac{2}{5}$ weeks?

6. Joseph having $\frac{3}{4}$ of a pound of candy, gave $\frac{1}{3}$ of it to James, $\frac{1}{6}$ to William, and $\frac{1}{4}$ to Henry. What part of a pound had he left?

7. Mr. Smith owning $\frac{3}{8}$ of an iron foundry, sold $\frac{1}{6}$ of his share. What part had he left?

8. A man lost $\frac{1}{3}$ of his money at play, and then won $\frac{1}{4}$ as much as he had lost. What part of his money had he then?

9. At \$$3\frac{1}{6}$ a yard what will $4\frac{4}{5}$ yards of cloth cost?

10. Bought a wagon for \$$35\frac{7}{8}$, and sold it for $\frac{4}{7}$ of what it cost. What was the loss?

11. What will $\frac{1}{8}$ of $\frac{1}{9}$ of 216 gallons of oil cost, at \$$\frac{7}{8}$ a gallon? At \$$\frac{3}{4}$? At \$$\frac{5}{6}$?

12. A man paid \$$15\frac{2}{3}$ for an overcoat, and $\frac{1}{2}$ as much for a vest. How much did he pay for both?

13. Thomas gave each of 6 poor persons $\frac{3}{7}$ of a dollar, and had $5\frac{1}{4}$ dollars left; how much had he at first?

14. If 6 pears cost $\frac{7}{8}$ of a dime apiece, what will 24 cost? 16? 32? 12? 14?

15. James having spent $\frac{5}{9}$ of his money, found as much as $\frac{1}{4}$ of the remainder. What part of his money did he then have?

16. An orchard contains 56 trees, $\frac{3}{8}$ of which are pear trees; $\frac{2}{7}$ of the remainder are peach trees; and the rest bear apples. How many apple trees in the orchard?

17. If one dozen of eggs cost \$$\frac{7}{20}$, what will 12 dozen cost? 25 dozen? 44 dozen?

18. Find the cost of 20 pounds of cheese at $11\frac{3}{4}$ cents a pound. 30 pounds. 16 pounds.

WRITTEN EXERCISES.

1. A farmer bought 27 acres of land at 96\frac{3}{4}$ an acre, and 75 head of cattle at 23\frac{1}{2}$ per head. What did he pay for all? *Ans.* 4374\frac{3}{4}$.

2. A flour merchant bought 565 barrels of flour at 5\frac{2}{3}$ per barrel. He sold 420 barrels at 3\frac{1}{4}$ a barrel, and the remainder at 8\frac{2}{5}$. How much did he lose? *Ans.* 618\frac{2}{5}$.

3. What will be the cost of $\frac{4}{9}$ of a piece of cloth containing 337$\frac{1}{2}$ yards, at 5\frac{1}{4}$ a yard? *Ans.* 787\frac{1}{2}$.

4. A has 371\frac{7}{8}$; B has 16$\frac{2}{3}$ times as much; and C has 270\frac{1}{4}$ more than A and B together. How much have they all? *Ans.* 13409\frac{5}{6}$.

5. A daughter is 19$\frac{5}{12}$ years old, and her mother lacks 9$\frac{2}{7}$ years of being 3 times as old. What are their united ages? *Ans.* 67$\frac{17}{21}$ years.

6. A man travelled 218$\frac{1}{2}$ miles in three days. The first day he travelled $\frac{1}{5}$ of the whole distance, and the second, $\frac{1}{3}$ as far as he travelled on the first day. How far did he travel on the third day? *Ans.* 160$\frac{7}{30}$ miles.

7. If a train of cars run 25$\frac{1}{4}$ miles in an hour, how far will it run in 6$\frac{3}{7}$ hours? *Ans.* 162$\frac{9}{28}$ miles.

8. Paid 249\frac{5}{8}$ for a horse and sold him for $\frac{3}{8}$ of what he cost. What was the loss? *Ans.* 83\frac{5}{24}$.

9. When raisins are worth $\frac{9}{10}$ a box, what will 135$\frac{4}{5}$ boxes cost? *Ans.* 122\frac{11}{50}$.

10. Two sevenths of a pole is in the mud, $\frac{1}{6}$ of it is in the water, and the rest of it is in the air. How much less than $\frac{3}{4}$ of it is in the air? *Ans.* $\frac{17}{84}$ feet.

11. One person spent $2 for coal worth $5 a ton; and another $4 for a quantity worth $6 per ton. How much does one purchase more than the other? *Ans.* $\frac{4}{15}$ ton.

12. From a flock of sheep containing 700, I lost $\frac{5}{14}$, and afterward added $\frac{2}{5}$ as many as remained. By what part of itself was the original flock increased or diminished? *Ans.* Diminished $\frac{3}{14}$.

13. A gentleman having $\frac{7}{8}$ of $213\frac{5}{6}$ acres of land, sold $\frac{3}{7}$ of his share. How many acres did he sell?

Ans. $80\frac{3}{16}$ acres.

14. What will be the cost of $\frac{3}{5}$ of $\frac{9}{10}$ of a ton of hay at $\$4\frac{1}{3}$ per ton? *Ans.* $\$2\frac{74}{125}$.

15. A boy sold $9\frac{3}{4}$ dozen of eggs at 3 cents apiece. He received in payment $6\frac{5}{8}$ pounds of butter at 20 cents a pound, and $2\frac{1}{3}$ yards of ribbon at 4 cents a yard; how much was still due him? *Ans.* $\$2.09\frac{1}{6}$.

16. Mr. Carson rented a farm at $\$54\frac{1}{4}$ a month, taking a lease for six years, but disposed of the lease at the end of $4\frac{2}{3}$ years. How much rent did he pay?

Ans. $3038.

17. A man at his death left his wife $18400, which was $\frac{1}{3}$ of his estate. She at her death left $\frac{3}{8}$ of her share to her daughter. What part of the father's estate did the daughter receive? *Ans.* $\frac{1}{8}$.

18. If a horse eat $\frac{2}{3}$ of a bushel of oats in a day, how many bushels will 20 horses eat in $8\frac{1}{2}$ weeks?

Ans. $793\frac{1}{3}$ bushels.

19. At the rate of 3640 words in an hour, how many words will a man utter in $\frac{17}{20}$ of an hour?

Ans. 3094 words.

20. If by working 13 hours a day, a piece of work can be done in $53\frac{2}{3}$ days, in what time can it be done by working one hour a day? *Ans.* $697\frac{2}{3}$ days.

21. Find the cost of the following: 27 barrels of apples at $\$3\frac{2}{3}$ a barrel, 39 bushels of strawberries at $\$4\frac{1}{2}$ a bushel, and 140 quarts of nuts at $14\frac{1}{2}$ cents a quart.

DIVISION.

105. ***Division of Fractions*** is the process of dividing when one or both of the terms are fractional.

CASE I.

106. To divide a fraction by an integer.

ORAL EXERCISES.

1. Divide $\frac{4}{5}$ by 2.

SOLUTION.—Divide the numerator by the whole number, and keep the same denominator; or multiply the denominator by the whole number, and keep the same numerator. Thus, $\frac{4}{5} \div 2 = \frac{2}{5}$, *Ans.* Or, $\frac{4}{5} \div 2 = \frac{4}{10} = \frac{2}{5}$, *Ans.*

I.

2. $\frac{4}{5} \div 2$.	*10.* $\frac{3}{7} \div 6$.	*18.* $\frac{10}{11} \div 5$.	*26.* $3\frac{3}{4} \div 3$.
3. $\frac{2}{3} \div 2$.	*11.* $\frac{5}{7} \div 4$.	*19.* $\frac{9}{10} \div 4$.	*27.* $5\frac{1}{2} \div 6$.
4. $\frac{3}{4} \div 4$.	*12.* $\frac{8}{9} \div 9$.	*20.* $\frac{14}{15} \div 3$.	*28.* $8\frac{3}{4} \div 7$.
5. $\frac{1}{2} \div 3$.	*13.* $\frac{5}{8} \div 3$.	*21.* $\frac{6}{13} \div 2$.	*29.* $6\frac{7}{8} \div 2$.
6. $\frac{2}{5} \div 6$.	*14.* $\frac{4}{5} \div 8$.	*22.* $\frac{12}{17} \div 6$.	*30.* $4\frac{4}{5} \div 8$.
7. $\frac{8}{9} \div 4$.	*15.* $\frac{7}{9} \div 6$.	*23.* $\frac{8}{25} \div 4$.	*31.* $9\frac{1}{2} \div 5$.
8. $\frac{7}{8} \div 3$.	*16.* $\frac{7}{8} \div 5$.	*24.* $\frac{15}{2} \div 7$.	*32.* $7\frac{3}{5} \div 2$.
9. $\frac{5}{6} \div 5$.	*17.* $\frac{5}{4} \div 8$.	*25.* $\frac{49}{9} \div 5$.	*33.* $17\frac{7}{8} \div 5$.

II.

1. James shared $\frac{3}{4}$ of a dollar equally between two companions; how much did he give each?

2. Henry paid $\frac{8}{9}$ of a dollar for 3 bats; what was each worth?

3. Mary did $\frac{3}{8}$ of her work in 4 days; what part of the work did she do in a day?

4. If 10 rubber balls cost $\frac{3}{8}$ of a dollar, what will 1 rubber ball cost?

5. Divide $\frac{6}{7}$ of a ton of coal among 3 poor families; what part of a ton will each receive?

6. Divide $\frac{5}{6}$ of a dollar into 5 equal parts.

7. How many times are 3 contained in $\frac{7}{8}$?

WRITTEN EXERCISES.

1. Divide $\frac{6}{13}$ by 3.

OPERATION.

$$\frac{6}{13} \div 3 = \frac{6 \div 3}{13} = \frac{2}{13}$$

Or,

$$\frac{6}{13} \div 3 = \frac{6}{13 \times 3} = \frac{2}{13}$$

SOLUTION.—To divide $\frac{6}{13}$ by 3 we may either divide the numerator or multiply the denominator by 3; which in each case gives $\frac{2}{13}$ for result. By the first operation we diminish the *number* of parts; and by the second, we diminish their *value*. In both cases the result is the same.

NOTES.—1. Whenever the numerator is a multiple of the divisor, the operation should be performed by dividing the numerator.

2. Mixed numbers should be reduced to improper fractions before dividing.

Find the value of:

2. $\frac{10}{17} \div 4$.	*10.* $\frac{36}{47} \div 12$.	*18.* $10\frac{3}{5} \div 8$.
3. $\frac{20}{33} \div 5$.	*11.* $\frac{28}{30} \div 14$.	*19.* $25\frac{1}{2} \div 6$.
4. $\frac{14}{27} \div 4$.	*12.* $\frac{45}{82} \div 15$.	*20.* $37\frac{1}{3} \div 5$.
5. $\frac{16}{21} \div 8$.	*13.* $\frac{11}{12} \div 16$.	*21.* $16\frac{1}{3} \div 7$.
6. $\frac{3}{11} \div 7$.	*14.* $\frac{81}{93} \div 27$.	*22.* $15\frac{1}{8} \div 9$.
7. $\frac{15}{28} \div 9$.	*15.* $\frac{135}{171} \div 15$.	*23.* $10\frac{1}{4} \div 6$.
8. $\frac{9}{17} \div 6$.	*16.* $\frac{128}{204} \div 16$.	*24.* $12\frac{1}{3} \div 5$.
9. $\frac{18}{25} \div 8$.	*17.* $\frac{225}{316} \div 25$.	*25.* $75\frac{3}{5} \div 4$.

26. $125\frac{1}{4} \div 12$.	*30.* $156\frac{1}{2} \div 33$.	*34.* $135\frac{7}{8} \div 25$.
27. $250\frac{2}{3} \div 20$.	*31.* $194\frac{1}{8} \div 16$.	*35.* $440\frac{2}{7} \div 30$.
28. $327\frac{1}{5} \div 15$.	*32.* $260\frac{1}{9} \div 12$.	*36.* $137\frac{1}{2} \div 50$.
29. $400\frac{3}{4} \div 24$.	*33.* $345\frac{1}{3} \div 16$.	*37.* $210\frac{4}{5} \div 60$.

CASE II.

107. To divide an integer by a fraction.

ORAL EXERCISES.

1. Divide 5 by $\frac{3}{4}$.

SOLUTION.—Multiply the whole number by the denominator of the fraction, and divide the product by the numerator. Thus, $5 \div \frac{3}{4} = \frac{20}{3} = 6\frac{2}{3}$, *Ans.*

I.

2. $6 \div \frac{2}{3}$.	*10.* $12 \div \frac{4}{5}$.	*18.* $16 \div \frac{4}{5}$.	*26.* $39 \div 3\frac{1}{4}$.
3. $9 \div \frac{3}{4}$.	*11.* $15 \div \frac{5}{6}$.	*19.* $10 \div \frac{5}{8}$.	*27.* $40 \div 1\frac{3}{5}$.
4. $6 \div \frac{4}{5}$.	*12.* $16 \div \frac{7}{8}$.	*20.* $20 \div \frac{2}{3}$.	*28.* $36 \div 1\frac{1}{2}$.
5. $8 \div \frac{1}{5}$.	*13.* $27 \div \frac{9}{2}$.	*21.* $14 \div \frac{7}{8}$.	*29.* $49 \div 6\frac{1}{8}$.
6. $5 \div \frac{3}{7}$.	*14.* $12 \div \frac{8}{7}$.	*22.* $25 \div \frac{5}{3}$.	*30.* $25 \div 8\frac{1}{3}$.
7. $8 \div \frac{6}{7}$.	*15.* $18 \div \frac{3}{5}$.	*23.* $30 \div \frac{3}{4}$.	*31.* $42 \div 8\frac{2}{5}$.
8. $7 \div \frac{2}{3}$.	*16.* $32 \div \frac{3}{4}$.	*24.* $18 \div \frac{6}{7}$.	*32.* $18 \div 4\frac{1}{2}$.
9. $9 \div \frac{1}{4}$.	*17.* $15 \div \frac{6}{7}$.	*25.* $24 \div \frac{8}{9}$.	*33.* $32 \div 1\frac{7}{9}$.

II.

1. How many packages of tea, containing $\frac{2}{3}$ of a pound each, can be made from 8 pounds?

2. 18 pounds are $\frac{3}{4}$ of how many pounds?

3. If $\frac{3}{7}$ of a piece of work can be done in 6 days, what time will it take to do $\frac{1}{7}$ of it?

4. How many fourths in 6 pounds? In 8 pounds?

5. If a boy earn $\frac{5}{8}$ of a dollar a day, in what time will he earn \$8? \$10? \$5?

6. At $\frac{2}{5}$ of a dollar a yard, how many yards can be bought for \$10?

7. How many penknives at $\$\frac{4}{7}$ each, can be bought for \$12? For \$6? For \$9? For \$11?

WRITTEN EXERCISES.

1. Divide 16 by $\frac{3}{4}$.

OPERATION.

$$16 \div \tfrac{3}{4} = \tfrac{64}{4} \div \tfrac{3}{4} = 21\tfrac{1}{3}.$$

Or,

$$16 \div \tfrac{3}{4} = \frac{16 \times 4}{3} = 21\tfrac{1}{3}.$$

SOLUTION.—Before dividing we reduce 16 to fourths. 16 is equal to $\frac{64}{4}$; which, divided by $\frac{3}{4}$, gives $\frac{64}{3} = 21\frac{1}{3}$.

Or, multiply 16 by the denominator 4, and divide the product by the numerator 3. This operation gives $\frac{64}{3} = 21\frac{1}{3}$, the required result.

Find the value of:

2. $21 \div \frac{3}{4}$.	*10.* $18 \div \frac{8}{15}$.	*18.* $605 \div 7\frac{1}{3}$.
3. $15 \div \frac{3}{7}$.	*11.* $54 \div \frac{10}{18}$.	*19.* $322 \div 1\frac{3}{8}$.
4. $11 \div \frac{5}{6}$.	*12.* $49 \div \frac{14}{15}$.	*20.* $124 \div 3\frac{3}{7}$.
5. $37 \div \frac{2}{3}$.	*13.* $36 \div \frac{8}{17}$.	*21.* $428 \div 3\frac{2}{3}$.
6. $29 \div \frac{2}{5}$.	*14.* $90 \div \frac{15}{16}$.	*22.* $350 \div 3\frac{3}{4}$.
7. $65 \div \frac{5}{8}$.	*15.* $72 \div \frac{16}{21}$.	*23.* $272 \div 6\frac{1}{2}$.
8. $78 \div \frac{3}{7}$.	*16.* $63 \div \frac{21}{25}$.	*24.* $837 \div 5\frac{2}{5}$.
9. $56 \div \frac{4}{5}$.	*17.* $18 \div \frac{54}{35}$.	*25.* $982 \div 7\frac{1}{7}$.

CASE III.

108. To divide a fraction by a fraction.

ORAL EXERCISES.

1. Divide $\frac{2}{7}$ by $\frac{3}{5}$.

SOLUTION.—Invert the terms of the divisor, and proceed as in multiplication. Thus, $\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times \frac{5}{3} = \frac{10}{21}$, *Ans.*

I.

2. $\frac{1}{2} \div \frac{3}{4}$.	*6.* $\frac{7}{8} \div \frac{1}{4}$.	*10.* $\frac{8}{9} \div \frac{5}{6}$.	*14.* $\frac{5}{7} \div \frac{5}{8}$.
3. $\frac{5}{6} \div \frac{2}{3}$.	*7.* $\frac{1}{4} \div \frac{3}{8}$.	*11.* $\frac{3}{9} \div \frac{6}{9}$.	*15.* $\frac{7}{8} \div \frac{7}{6}$.
4. $\frac{3}{4} \div \frac{1}{2}$.	*8.* $\frac{2}{3} \div \frac{5}{6}$.	*12.* $\frac{9}{5} \div \frac{1}{4}$.	*16.* $\frac{1}{2} \div \frac{1}{9}$.
5. $\frac{4}{5} \div \frac{2}{5}$.	*9.* $\frac{2}{5} \div \frac{7}{9}$.	*13.* $\frac{8}{3} \div \frac{4}{5}$.	*17.* $\frac{6}{7} \div \frac{1}{5}$.

18. $\frac{5}{12} \div \frac{1}{2}$.	*22.* $\frac{2}{3} \div \frac{8}{15}$.	*26.* $3\frac{1}{2} \div 2\frac{1}{3}$.	*30.* $8\frac{1}{2} \div 2\frac{1}{4}$.
19. $\frac{9}{10} \div \frac{3}{5}$.	*23.* $\frac{6}{7} \div \frac{2}{14}$.	*27.* $5\frac{1}{4} \div 1\frac{3}{4}$.	*31.* $4\frac{2}{5} \div 1\frac{5}{6}$.
20. $\frac{14}{15} \div \frac{7}{8}$.	*24.* $\frac{1}{4} \div \frac{18}{25}$.	*28.* $2\frac{1}{6} \div 3\frac{1}{3}$.	*32.* $7\frac{1}{2} \div 8\frac{3}{4}$.
21. $\frac{8}{25} \div \frac{4}{5}$.	*25.* $\frac{5}{6} \div \frac{7}{24}$.	*29.* $5\frac{1}{3} \div 1\frac{7}{9}$.	*33.* $6\frac{3}{7} \div 1\frac{4}{5}$.

II.

1. How many pounds of butter can be bought for \$$\frac{3}{4}$, at \$$\frac{1}{4}$ a pound?

2. How many 6ths in a third? In a $\frac{1}{2}$?

3. How many pounds of cheese at \$$\frac{1}{8}$ a pound, can be bought for \$$\frac{1}{4}$? \$$\frac{1}{2}$? \$$\frac{5}{6}$? \$$\frac{3}{4}$? \$$\frac{8}{5}$?

4. How many $\frac{1}{4}$ pints can be put in a bottle containing $\frac{3}{2}$ pints?

5. How many pieces of ribbon $\frac{3}{7}$ of a yard long, can be cut from a piece $\frac{11}{12}$ of a yard long?

6. Sarah having $\frac{3}{4}$ of a pie, gave each of her sisters $\frac{3}{16}$ of a pie; how many sisters had she?

7. Divide the sum of $\frac{3}{8}$ and $\frac{1}{8}$ by their difference.

WRITTEN EXERCISES.

1. Divide $\frac{4}{7}$ by $\frac{3}{8}$.

OPERATION.

$$\frac{4}{7} \div \frac{3}{8} = \frac{32}{56} \div \frac{21}{56} = \frac{32}{21} = 1\frac{11}{21}.$$

Or,

$$\frac{3}{8} \text{ of quotient} = \frac{4}{7}$$

$$\frac{1}{8} \quad \text{" "} \quad = \frac{4}{7 \times 3}$$

$$\frac{8}{8}, \text{ or} \quad \text{"} \quad = \frac{4 \times 8}{7 \times 3} = 1\frac{11}{21}.$$

SOLUTION. — Having reduced the two given fractions to a common denominator, divide their numerators, which will give $\frac{32}{21} = 1\frac{11}{21}$.

2. In this problem we have to find a quotient, which, when multiplied by the divisor $\frac{3}{8}$, will give the dividend $\frac{4}{7}$, for product. Hence, $\frac{3}{8}$ of the quotient is equal to $\frac{4}{7}$; $\frac{1}{8}$ of the quotient is equal to $\frac{1}{3}$ of $\frac{4}{7}$, or $4 \div (7 \times 3)$; and $\frac{8}{8}$, the quotient, is equal to 8 times $4 \div (7 \times 3)$, or $(4 \times 8) \div (7 \times 3)$, which is equal to $1\frac{11}{21}$.

2. If $\frac{3}{5}$ of a gallon of molasses cost $\frac{7}{8}$ of a dollar, how much will a gallon cost?

OPERATION.

$\frac{3}{5}$ gal. $= \$\frac{7}{8}$

$\frac{1}{5}$ " $= \$\frac{7}{8 \times 3} = \$\frac{7}{24}$

$\frac{5}{5}$ " $= \$\frac{7 \times 5}{8 \times 3} = \$1\frac{11}{24}$.

SOLUTION.—If $\frac{3}{5}$ of a gallon cost $\$\frac{7}{8}$, $\frac{1}{5}$ of a gallon will cost $\frac{1}{3}$ of $\$\frac{7}{8}$, which is $\$\frac{7}{24}$; and $\frac{5}{5}$, or 1 gallon, will cost 5 times $\$\frac{7}{24}$, or $\$\frac{35}{24}$, which is equal to $\$1\frac{11}{24}$.

NOTE.—The solution of this problem affords a practical illustration of the following concise and valuable rule: "*Invert the terms of the divisor and proceed as in multiplication.*" But care must be taken that the pupils understand how to distinguish between the divisor and the dividend. To attain this end they should, at least in the beginning, be required to give a thorough analysis of every problem.

Find the value of:

3. $\frac{9}{11} \div \frac{3}{7}$.
4. $\frac{16}{27} \div \frac{8}{9}$.
5. $\frac{13}{31} \div \frac{7}{9}$.
6. $\frac{18}{19} \div \frac{2}{3}$.
7. $\frac{21}{25} \div \frac{4}{5}$.
8. $\frac{23}{18} \div \frac{5}{6}$.
9. $\frac{9}{20} \div \frac{1}{4}$.
10. $\frac{1}{15} \div \frac{3}{4}$.
11. $\frac{15}{16} \div \frac{9}{10}$.
12. $\frac{24}{37} \div \frac{8}{15}$.
13. $\frac{9}{11} \div \frac{15}{8}$.
14. $\frac{21}{55} \div \frac{2}{11}$.
15. $\frac{11}{31} \div \frac{12}{13}$.
16. $\frac{28}{65} \div \frac{7}{16}$.
17. $\frac{26}{27} \div \frac{13}{18}$.
18. $\frac{34}{25} \div \frac{7}{30}$.
19. $15\frac{3}{4} \div 2\frac{1}{3}$.
20. $37\frac{1}{2} \div 3\frac{1}{6}$.
21. $33\frac{1}{3} \div 5\frac{1}{2}$.
22. $84\frac{1}{4} \div 6\frac{1}{3}$.
23. $52\frac{4}{5} \div 1\frac{6}{7}$.
24. $20\frac{5}{6} \div 4\frac{8}{9}$.
25. $40\frac{3}{7} \div 3\frac{3}{8}$.
26. $66\frac{2}{3} \div 2\frac{7}{9}$.
27. $125\frac{1}{2} \div 10\frac{1}{3}$.
28. $214\frac{2}{3} \div 20\frac{1}{2}$.
29. $321\frac{1}{4} \div 15\frac{2}{3}$.
30. $140\frac{5}{8} \div 19\frac{3}{8}$.
31. $260\frac{6}{7} \div 24\frac{1}{2}$.
32. $300\frac{5}{8} \div 16\frac{1}{4}$.
33. $459\frac{2}{3} \div 30\frac{3}{7}$.
34. $283\frac{8}{9} \div 60\frac{5}{6}$.
35. $4163\frac{2}{5} \div 33\frac{1}{3}$.
36. $6075\frac{1}{8} \div 62\frac{1}{2}$.
37. $3982\frac{1}{2} \div 25\frac{1}{4}$.
38. $5100\frac{1}{4} \div 72\frac{3}{4}$.
39. $2095\frac{7}{8} \div 30\frac{1}{8}$.
40. $1776\frac{2}{5} \div 70\frac{1}{5}$.
41. $2612\frac{2}{7} \div 72\frac{1}{7}$.
42. $1087\frac{1}{5} \div 25\frac{1}{5}$.

COMPOUND AND COMPLEX FRACTIONS.

109. ***Reduction of Compound and Complex Fractions*** is the process of changing them to simple fractions.

CASE I.

110. To reduce compound fractions to simple ones.

1. What is the value of $\frac{5}{7}$ of $\frac{3}{4}$?

OPERATION.

$\frac{5}{7}$ of $\frac{3}{4} = \frac{5}{7} \times \frac{3}{4} = \frac{15}{28}$

SOLUTION.—$\frac{1}{7}$ of $\frac{3}{4}$ equals $\frac{3}{28}$; and since $\frac{1}{7}$ of $\frac{3}{4}$ equals $\frac{3}{28}$, $\frac{5}{7}$ of $\frac{3}{4}$ equals 5 times $\frac{3}{28}$, which gives $\frac{15}{28}$ for the required value of the given fractions.

WRITTEN EXERCISES.

What is the value of:

2. $\frac{3}{5}$ of $\frac{1}{5}$? *Ans.* $\frac{3}{25}$.

3. $\frac{4}{7}$ of $\frac{5}{6}$? *Ans.* $\frac{10}{21}$.

4. $\frac{6}{7}$ of $\frac{9}{10}$? *Ans.* $\frac{27}{35}$.

5. $\frac{11}{12}$ of $\frac{15}{16}$? *Ans.* $\frac{55}{64}$.

6. $\frac{18}{19}$ of $\frac{57}{72}$? *Ans.* $\frac{3}{4}$.

7. $\frac{7}{15}$ of $\frac{45}{91}$? *Ans.* $\frac{3}{13}$.

8. $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{7}{8}$?

9. $\frac{9}{10}$ of $\frac{5}{6}$ of $\frac{24}{35}$?

10. $\frac{7}{9}$ of $\frac{67}{77}$ of $\frac{112}{134}$?

11. $\frac{6}{27}$ of $\frac{64}{81}$ of $\frac{9}{27}$?

12. $\frac{117}{931}$ of $\frac{33}{89}$ of $\frac{7}{351}$?

13. $\frac{4}{13}$ of $\frac{26}{24}$ of $\frac{96}{3}$ of $\frac{39}{16}$.

CASE II.

111. To reduce complex fractions to simple ones.

1. Reduce $\frac{\frac{7}{9}}{\frac{3}{5}}$ to a simple fraction.

OPERATION.

$\frac{\frac{7}{9}}{\frac{3}{5}} = \frac{7}{9} \div \frac{3}{5} = \frac{7}{9} \times \frac{5}{3}$

$\frac{7}{9} \times \frac{5}{3} = \frac{35}{27} = 1\frac{8}{27}$

SOLUTION.—This expression signifies that $\frac{7}{9}$ is to be divided by $\frac{3}{5}$. Hence, by applying the rule for division we have $\frac{7}{9} \times \frac{5}{3} = 1\frac{8}{27}$.

WRITTEN EXERCISES.

Reduce the following to simple fractions:

2. $\frac{\frac{3}{4}}{\frac{4}{7}}$. *3.* $\frac{\frac{11}{12}}{\frac{3}{4}}$. *4.* $\frac{\frac{15}{16}}{\frac{9}{10}}$. *5.* $\frac{\frac{25}{37}}{\frac{5}{6}}$.

6. $\frac{3\frac{1}{2}}{6\frac{1}{3}}$. *7.* $\frac{4\frac{1}{5}}{5\frac{1}{6}}$. *8.* $\frac{12\frac{3}{4}}{16\frac{5}{9}}$. *9.* $\frac{22\frac{2}{3}}{30\frac{3}{4}}$.

10. $\frac{\frac{4}{7}\times\frac{14}{15}}{\frac{32}{45}\times\frac{7}{8}}$. *11.* $\frac{\frac{10}{17}\times\frac{17}{35}}{\frac{10}{28}\times\frac{3}{15}}$. *12.* $\frac{\frac{89}{105}\times\frac{35}{89}}{\frac{6}{13}\times\frac{39}{48}}$.

13. $\frac{\frac{2}{3}\times\frac{5}{6}}{3\frac{1}{4}\times 1\frac{1}{2}}$. *14.* $\frac{14\frac{1}{2}}{2}\times\frac{6\frac{1}{3}}{5\frac{1}{7}}$. *15.* $\frac{\frac{8\frac{1}{4}}{5}\times\frac{9}{10}}{\frac{20}{9\frac{1}{7}}\times\frac{12\frac{1}{3}}{2\frac{1}{3}}}$.

MENTAL EXERCISES.

1. If 2 pounds of tea cost $\$\frac{5}{8}$, what will 1 pound cost?

2. How much butter can be bought for \$8 at $\$\frac{2}{5}$ a pound?

3. Joseph divided $\frac{8}{9}$ of a melon equally between his two sisters; what part did each receive?

4. At $\$\frac{6}{7}$ a yard, how many yards of silk will \$32 buy?

5. If a train run $\frac{9}{14}$ of a mile a minute, how long will it take for a trip of 50 miles?

6. How many bushels of oats worth $\$1\frac{1}{3}$ a bushel, will pay for some hay worth \$63?

7. If you divide $16\frac{2}{3}$ barrels of flour among 7 poor families, how much does each family receive?

8. When potatoes are worth $\$\frac{5}{8}$ per bushel, how many bushels can be bought for $\$27\frac{1}{2}$?

9. If $23\frac{1}{3}$ barrels of sugar cost \$120, what will one barrel cost?

10. If a turkey cost $9\frac{1}{2}$ shillings, how many turkeys can be bought for $47\frac{1}{2}$ shillings?

11. How long will it take a horse to eat $\frac{1}{12}$ of a bushel of oats, if he eat $\frac{1}{4}$ of a bushel in a day?

12. What will 1 dozen of eggs cost, if $26\frac{1}{4}$ dozen cost $\$10\frac{1}{2}$?

13. A gentleman having $\frac{9}{10}$ of a factory, divided his share equally among his 5 children; what did each child receive?

14. If $\frac{8}{9}$ of a yard of broadcloth cost $\$2\frac{1}{2}$, what will 1 yard cost?

15. How many men will be required to do in 1 day, a piece of work which 9 men can do in $6\frac{1}{3}$ days?

WRITTEN EXERCISES.

1. If $32\frac{2}{3}$ ounces of silver cost $\$27\frac{1}{2}$, what is the cost of one ounce? *Ans.* $0.85.

2. If $\$48\frac{1}{6}$ will buy $56\frac{2}{3}$ bushels of wheat, how many bushels can be bought for $\$95\frac{5}{8}$? *Ans.* $112\frac{1}{2}$ bushels.

3. What will 1 barrel of sugar cost, if $52\frac{3}{4}$ barrels cost $\$461\frac{9}{16}$? *Ans.* $\$8\frac{3}{4}$.

4. Bought $342\frac{1}{4}$ acres of land for $\$8605\frac{1}{4}$; what was that an acre? *Ans.* $\$25\frac{1}{4}$.

5. If $39\frac{1}{4}$ tons of Lehigh coal cost $\$215\frac{7}{8}$, what is the price per ton? *Ans.* $\$5\frac{1}{2}$.

6. If $\frac{16}{25}$ of a house be valued at $21800, what is the value of the whole house? *Ans.* $\$34062\frac{1}{2}$.

7. At the rate of $\$35\frac{1}{2}$ for $9\frac{7}{8}$ yards of cloth, how many yards can be bought for $\$88\frac{3}{4}$? *Ans.* $24\frac{11}{16}$.

8. The greater of two numbers is 2739, and their product is 27; what is the less? *Ans.* $\frac{9}{913}$.

9. The product of two numbers is 442 and one of them is $52\frac{1}{4}$; what is the other? *Ans.* $8\frac{75}{184}$.

10. A grocer sold 36 bushels of potatoes at $62\frac{1}{2}$ cents a bushel, and received in payment, eggs worth 45 cents a dozen. How many eggs did he get? *Ans.* 50 dozen.

11. If 25 men consume $\frac{5}{7}$ of 1400 pounds of meat in a month, how much does 1 man consume in the same time? *Ans.* 40 pounds.

12. A boy divided 306 apples among his companions, giving $19\frac{1}{8}$ apples to each. How many companions had he? *Ans.* 16.

13. Henry Perkins bought land for $85340, and sold it so as to gain $\frac{5}{17}$ of the cost, or $25 an acre; how many acres did he buy? *Ans.* 1004 acres.

14. A person having $6500, distributed $\frac{8}{15}$ of it among some poor families, giving each family $181\frac{9}{11}$. How many poor families were there? *Ans.* 22.

MENTAL REVIEW.

I.

1. 56 is $\frac{7}{8}$ of what number?

OPERATION.

$\frac{7}{8} = 56$
$\frac{1}{8} = 8$
$\frac{8}{8} = 64$

SOLUTION.—Since $\frac{7}{8}$ of a certain number is 56, $\frac{1}{8}$ of the number must be $\frac{1}{7}$ of 56, which is 8. And if $\frac{1}{8}$ of the number be 8, $\frac{8}{8}$ of the number, or the number itself, must be 8 times as much as $\frac{1}{8}$ or 8 times 8, which are 64. Therefore, 56 is $\frac{7}{8}$ of 64.

2. 126 is $\frac{14}{15}$ of what number?
3. $16\frac{1}{3}$ is $\frac{1}{4}$ of what number?
4. 36 is $\frac{4}{7}$ of what number?
5. 60 is $\frac{9}{10}$ of what number?
6. $15\frac{1}{5}$ is $\frac{19}{20}$ of what number?
7. 96 is $\frac{12}{31}$ of what number?
8. $17\frac{1}{2}$ is $\frac{5}{9}$?
9. $19\frac{1}{3}$ is $\frac{2}{5}$?
10. 13 is $\frac{3}{8}$?
11. 25 is $\frac{3}{5}$?
12. 48 is $\frac{3}{4}$?
13. 56 is $\frac{7}{13}$?

II.

1. 63 is $\frac{9}{10}$ of how many times 5?

OPERATION.

$\frac{1}{10} = 63$
$\frac{1}{10} = 7$
$\frac{10}{10} = 70$
$0 \div 5 = 14.$

SOLUTION.—If $\frac{9}{10}$ of a certain number of times 5 be 63, $\frac{1}{10}$ of that number of times 5, must be $\frac{1}{9}$ of 63, or 7; and $\frac{10}{10}$ of that number of times 5 is 10 times 7, or 70. Hence, 70 is as many times 5 as 5 is contained times in 70, which is 14 times. Therefore, 63 is $\frac{9}{10}$ of 14 times 5.

2. 40 is $\frac{5}{6}$ of how many times 16? 32? 64? 48?
3. 75 is $\frac{3}{11}$ of how many times 25? 75? 50? 100?
4. 27 is $\frac{3}{16}$ of how many times 72? 24? 48? 16?
5. 42 is $\frac{14}{17}$ of how many times 3? 17? 51? 34?
6. $12\frac{1}{2}$ is $\frac{5}{4}$ of how many times 60? 20? 30? 40?
7. 60 is $\frac{1}{2}$ of how many times $\frac{2}{3}$ of 90? 60? 120?
8. $3\frac{3}{4}$ is $\frac{1}{8}$ of how many times $\frac{4}{5}$ of 25? 40? 50?
9. $1\frac{1}{2}$ is $\frac{1}{10}$ of how many times $\frac{1}{2}$ of 100? 50? 40?
10. 84 is $\frac{18}{13}$ of how many times $\frac{2}{3}$ of $10\frac{1}{2}$? 42? 30?
11. 18 is $\frac{1}{4}$ of how many times $\frac{2}{3}$ of 15? 25? 45?
12. $\frac{6}{7}$ is $\frac{3}{8}$ of how many times $\frac{1}{4}$ of $9\frac{1}{7}$? $18\frac{2}{7}$? $4\frac{4}{7}$?

III.

1. $\frac{4}{7}$ of 84 is $\frac{3}{8}$ of what number? $\frac{6}{7}$ of what number?
2. $\frac{4}{5}$ of 30 is how many twelfths of 72? 48? 96?
3. $\frac{8}{9}$ of 27 is $\frac{5}{7}$ of how many times 6? 7? 8? 9?
4. $\frac{9}{4}$ of 32 are $\frac{5}{7}$ of 21 times what number? $\frac{2}{3}$ of 30?
5. $\frac{4}{9}$ of $\frac{6}{8}$ of 42 is 7 times what part of 120?
6. At the rate of 8 oranges for 17 cents, how many oranges could you buy for 85 cents?
7. If 9 bushels cost $2, what will 14 bushels cost?
8. In what time can 12 men do a piece of work for which 4 men require 13 days?

9. Sold a horse for $80, which was only $\frac{5}{8}$ of its cost; what did I lose?

10. If $\frac{6}{7}$ of a barrel of flour is worth $12, what will $4\frac{1}{2}$ barrels be worth?

11. If a boy earn 17\frac{2}{3}$ in 4 weeks, what will he earn in $13\frac{1}{2}$ weeks?

12. How much less than $15 will 10 yards of satin cost, if 5 yards cost 7\frac{1}{3}$?

13. If a horse travel $27\frac{3}{4}$ miles in 7 hours, how far will he travel in 5 hours?

14. A man having $24\frac{3}{4}$ bushels of potatoes, sold $\frac{2}{3}$ of them; how many bushels had he left?

15. If $\frac{1}{3}$ of a box of raisins cost 1\frac{1}{6}$, what will $\frac{6}{7}$ of a box cost?

16. How many pounds of sugar worth $13\frac{4}{7}$ cents can be had for $4\frac{4}{5}$ pounds of butter at 20 cents a pound?

17. Henry and William together picked 8 times $4\frac{7}{8}$ quarts of blackberries, and shared them equally with their companions. How many companions had they, if each received $3\frac{1}{4}$ quarts?

18. Bought 32 yards of cloth for $96; but, being damaged, I sold $\frac{5}{8}$ of it at 2\frac{1}{3}$ a yard, and the remainder for what it cost. How much did I lose?

19. How many chestnuts will pay for 25 walnuts, if 9 walnuts are worth $16\frac{1}{3}$ chestnuts?

20. A man bought 35 sheep, at the rate of 10 for 14\frac{2}{7}$. How much did they cost?

21. A boy gave $13\frac{1}{2}$ cents for oranges at the rate of 15 oranges for $22\frac{1}{2}$ cents. How many did he buy? How much would 20 oranges cost at the same rate?

22. Grafton is 162 miles from Albany, and $\frac{5}{9}$ of this distance is 2 miles less than $\frac{1}{3}$ of 3 times the distance to Westfield. How far is it to Westfield?

23. The insurance of a ship amounted to $1200, and $\frac{1}{3}$ of that is $\frac{1}{20}$ of 8 times the value of the cargo; what is the value of the cargo?

24. If a sleigh cost $150, what would be the cost of a wagon, if $\frac{3}{5}$ of the cost of the sleigh were $\frac{5}{11}$ of twice the cost of the wagon?

25. A coat cost $30; the $\frac{4}{5}$ of which is $\frac{2}{3}$ of 12 times the price of a hat. What is the price of a hat?

26. A boy being asked his age, said that 27 years equalled $\frac{3}{4}$ of 3 times his age. How old was he?

27. Mr. Brighton gave 45\frac{1}{3}$ for a cow, which was $\frac{1}{2}$ as much as Mr. Anthony paid for his. How much did both cows cost?

28. A man after spending $\frac{2}{3}$ of his fortune, found that $500 was $\frac{5}{16}$ of what he had remaining. What was his fortune?

29. If a 4-pound loaf cost 13 cents when flour is 6\frac{1}{2}$ a barrel, what will it cost when flour is $9 a barrel?

30. To $\frac{3}{4}$ of a gross, add $\frac{4}{5}$ of a hundred, and subtract the sum from $\frac{7}{25}$ of a thousand.

31. If it require $45 worth of provisions to serve 12 men 3 days, how many dollars' worth will serve 3 men 6 days?

32. Philip spent $\frac{5}{8}$ of all his money, and gave the remainder for 15 yards of cloth at 2\frac{2}{5}$ a yard. How much had he at first?

33. When A and B engaged in play, A's money was $\frac{4}{5}$ of B's. B gained $25, which was $\frac{1}{5}$ of $2\frac{1}{2}$ times as much as he commenced with. How much had A left?

34. A merchant, after selling from a cask of wine 12 gallons more than $\frac{1}{4}$ of the whole, found that the number of gallons left was just twice the number of gallons sold. How many gallons did the cask contain at first?

35. I sold 2 barrels of flour for 15\frac{3}{8}$, which is $\frac{1}{5}$ of what I received for all I had left, at 6\frac{1}{2}$ a barrel; how many barrels did I sell in all?

36. Thomas has 75 cents and James 63. Thomas spent $\frac{2}{5}$ of his, and James lost $\frac{4}{7}$ of his; what part of Thomas's money then equalled James's?

WRITTEN REVIEW.

1. A gentleman divides $5000 into 5 equal shares, and gives $3\frac{1}{2}$ of these shares to a benevolent society. What amount has he left? *Ans.* $1500.

2. How many pounds of coffee at $19\frac{1}{2}$ cents a pound, must be given for $23\frac{2}{5}$ pounds of butter at $24\frac{1}{2}$ cents a pound? *Ans.* $29\frac{2}{5}$ pounds.

3. A farmer gave $\frac{3}{7}$ of all his money for a horse, $\frac{1}{8}$ of it for a cow, and had $175 left. How much had he at first? *Ans.* $392.

4. I have $563\frac{3}{5}$ acres of land. How much will be left after selling $\frac{5}{7}$ of $\frac{4}{5}$ of it? *Ans.* $177\frac{2}{5}$ acres.

5. How many pairs of sheets requiring $4\frac{2}{3}$ yards can be made from a piece of sheeting containing 54 yards, leaving a remainder of $11\frac{7}{9}$ yards? *Ans.* 10 pairs.

6. If a boy earn $32\frac{1}{4}$ in $\frac{1}{5}$ of a year, how much will he earn in 8 months? *Ans.* $107.50.

7. The difference of two numbers is $47\frac{3}{21}$, and their sum is $183\frac{5}{7}$; what are the numbers?

Ans. $68\frac{2}{7}$ and $115\frac{3}{7}$.

8. Mr. Bultitude's stock increased by $\frac{7}{8}$ of its value plus $484\frac{3}{10}$ = $6785; what is the value of his stock?

Ans. $3360\frac{8}{15}$.

9. A mechanic received $1.25½ a day for a certain time. After paying his board and other expenses with the ⅜ of his earnings, he had $15.06 left. How many days did he work? *Ans.* 30 days.

10. Two pipes will fill a cistern in 62 minutes, and one of them in 93 minutes. In what time will the other fill it? *Ans.* 186 minutes.

11. A and B can do a certain piece of work in 7 days; A and C, in 9 days; and B and C, in 6 days. In what time will all working together do it?

Ans. 4 49/53 days.

12. Matthew agreed to work 32 days for $21; but was obliged to leave off after 13¼ days. What part of the $21 did he earn? *Ans.* 53/128 of it.

13. Sold 1640 bushels of wheat which cost 62½ cents a bushel, for 49 cents a bushel. What was the loss?

Ans. $221.40.

14. If a carpenter earn $3.50 a day, how many days will he have to work to pay for a suit of clothes, of which the coat costs $28⅓, the pants $9.75, and the vest $4½? *Ans.* 12⅙ days.

15. Exchanged 63 tubs of butter containing each 37⅘ pounds, at 22½ cents a pound, for 50 barrels of flour at $8¼ per barrel, and received the balance in cash. What was the balance? *Ans.* $118.27½.

16. Sold my house and farm of 85¾ acres for $8132. Allowing $2750 for the house, what did I receive per acre for the land? *Ans.* $63.

17. If a man take ⅜ of a day to do a certain piece of work, how long will it take him to do ¾ of ⅝ of it?

Ans. 3/16 of a day.

18. If 2 be added to both terms of the fraction ⅘, will the value of the fraction be increased or diminished?

19. A merchant sold 43 barrels of flour for 506\frac{2}{3}$, which was $\frac{5}{6}$ as much as he received for all he had left at $8 a barrel. How many barrels did he sell in all?

Ans. 119 barrels.

20. A and B bought 15900 acres of land, of which B was to have 7 acres as often as A 8. How many acres did each buy? *Ans.* A, 8480; B, 7420.

21. If 2 be subtracted from both terms of the fraction $\frac{4}{5}$, will the value of the fraction be increased or diminished? *Ans.* Diminished $\frac{2}{15}$.

22. Will the value of the fraction $\frac{5}{4}$, be increased or diminished if 2 be added to both of its terms? If 2 be subtracted from both terms?

23. Two carpenters can do a piece of work in 37 days; the first can do $\frac{7}{8}$ as much as the second. In what time can each do it? *Ans.* $69\frac{3}{8}$ and $79\frac{2}{7}$ days.

24. The sum of two numbers is 75840, and their difference is equal to $\frac{2}{3}$ of the less. What are the numbers?

25. How many yards of carpet, at 1.87\frac{1}{2}$ a yard, can I get for 200 quarts of strawberries at 27 cents a quart?

Ans. $28\frac{4}{5}$ yards.

26. The difference between $\frac{3}{4}$ and $\frac{2}{3}$ of a number is 20 more than $\frac{5}{84}$ of the number. What is the number?

Ans. 840.

27. A capitalist invested $\frac{3}{8}$ of his capital in flour, $\frac{2}{5}$ of the remainder in real estate, and the remainder, $22560, in stocks. What was the amount of his capital?

Ans. $60160.

28. The sum of three numbers is $60\frac{1}{12}$; the greatest is $30\frac{1}{4}$, and the least is $12\frac{1}{3}$. Find their product.

29. A woman bought eggs at the rate of 3 for 5 cents, and sold them at $2\frac{1}{2}$ cents apiece, thereby gaining $1.00. How many did she buy? *Ans.* 120.

DECIMAL FRACTIONS.

112. A ***Decimal Fraction*** is a fraction whose denominator is 10, or some higher power of 10. Thus, $\frac{7}{10}$, $\frac{19}{100}$, $\frac{23}{1000}$, etc., are decimal fractions.

113. ***Decimals,*** or *decimal fractions,* are like common fractions, except that their denominators increase and decrease by the uniform ratio of 10. Thus, the common fraction $\frac{18}{100}$, is the same as the decimal .18.

114. A ***Mixed Decimal*** is a mixed number whose fractional part is a decimal; as, 9.31.

NOTATION.

115. To express a decimal fraction, we place a point before the numerator and omit the denominator.

116. When integers and decimals are written together, the former are placed on the left and the latter on the right of the decimal point, as shown in the following

NUMERATION TABLE.

Ten-millions.	Millions.	Hund.-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	**Decimal Point.**	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund.-thousandths.	Millionths.	Ten-millionths.	Etc.
9	6	8	1	5	7	2	4	.	3	4	5	2	7	9	6	Etc.
Integers.									**Decimals.**							

WRITTEN EXERCISES.

1. Write six tenths, as a decimal.

2. Write forty-seven hundredths.

3. Write twenty-five thousandths.

SOLUTION.—In number 1, the numerator, 6, is written as in simple numbers, and the denominator, tenths, is indicated by placing the decimal immediately before it; thus, .6.

In number 2, since 47 hundredths is equal to 4 tenths and 7 hundredths, we write 4 in tenths' place and 7 in hundredths' place, putting the point before the 4; thus, .47.

In number 3, because 25 thousandths is the same as 0 tenths, 2 hundredths, 5 thousandths, we write 0 in tenths' place, 2 in hundredths', and 5 in thousandths', placing the point before the naught; thus, .025.

4. Eighty-nine thousandths. *Ans.* .089.

5. Three hundred four ten-thousandths.

6. One hundred sixty-three millionths.

7. Nine hundred nineteen thousandths.

8. Seventy-six ten-millionths. *Ans.* .0000076.

9. Five hundred two hundred-thousandths.

10. Forty-two hundred-millionths.

11. Fourteen thousandths. *Ans.* .014.

12. Eight hundredths. *Ans.* .08.

NOTE.—When an integer and a decimal occur in the same written number, a comma should always be placed after the integer, and the decimal part should be introduced by the word and. Thus, 200.004 should be written two hundred, and four thousandths; but .204, two hundred four thousandths.

13. Fifty-eight, and ten hundredths.

14. One thousand three, and sixteen thousandths.

15. Seventeen thousand nine hundred, and ten hundred-thousandths.

16. Thirty, and five hundred seven millionths.

17. Three hundred three million, and two thousand seventy ten-millionths.

18. Sixty-one thousand, and eleven hundredths.

19. Seventy-five thousand, and ninety-six hundred-thousandths.

20. Twelve million, and twelve hundred-millionths.

21. Six million sixty thousand, and two hundred sixty-four millionths.

22. Thirty billion one hundred thousand thirty, and three thousand two hundred six hundred-millionths.

23. 8 million 40 thousand 9 hundred 1, and 901 hundred-thousandths.

NUMERATION.

WRITTEN EXERCISES.

Read and write the following: *1.* .83. *2.* .057.

SOLUTION.—In number 1, .83 expresses 8 tenths and 3 hundredths; or, since 8 tenths is equal to 80 hundredths, and 80 hundredths plus 3 hundredths equal 83 hundredths, this decimal may also be read 83 hundredths.

In number 2, .057 expresses 5 hundredths and 7 thousandths; or, as 5 hundredths is equal to 50 thousandths, and 50 thousandths plus 7 thousandths equal 57 thousandths, it is also read fifty-seven thousandths.

I.

3. .1298.	*9.* .8644.	*15.* 1408.00016.
4. .0204.	*10.* .420369.	*16.* 871914.00809.
5. .0039.	*11.* .0047812.	*17.* 500001.0000051.
6. .00204.	*12.* 240.1800.	*18.* 9000000.0000029.
7. .50608.	*13.* 542.0871.	*19.* 50047.080060037.
8. .01829.	*14.* 300.0094.	*20.* 130900.00041396.

II.

21. .7854.	*25.* 3.1416.	*29.* .433013.
22. .0034.	*26.* .07958.	*30.* 277.274.
23. .0169.	*27.* 4.1888.	*31.* .4971499.
24. .5236.	*28.* 1.4142.	*32.* 15.198684.

REDUCTION.

Case I.

117. To reduce a common fraction to a decimal.

WRITTEN EXERCISES.

1. Reduce $\frac{7}{8}$ to a decimal.

OPERATION.

$\frac{7}{8} = 7 \div 8 =$

8) 7000

875 = .875.

Solution.—Since the value of a fraction is equal to the quotient obtained by dividing the numerator by the denominator, the value of $\frac{7}{8}$ is equal to $7 \div 8$. To find this quotient we annex three ciphers to 7, which is equivalent to multiplying it by 1000. But the result obtained by dividing 7000 by 8 is 1000 times the true value of the fraction. We, therefore, divide this result, or quotient, by 1000, which is done by pointing off three decimal places. Hence, the common fraction, $\frac{7}{8}$, is equal to the decimal, .875.

Reduce the following to equivalent decimals:

2. $\frac{4}{5}$.	*Ans.* .8.	*10.* $23\frac{19}{25}$.	*Ans.* 23.76.
3. $\frac{14}{15}$.	*Ans.* .933 +.	*11.* $\frac{11}{12}$.	*Ans.* .9166+.
4. $\frac{15}{16}$.	*Ans.* .9375.	*12.* $15\frac{35}{75}$.	*Ans.* 15.466+.
5. $\frac{3}{4}$.	*Ans.* .75.	*13.* $203\frac{5}{12}$.	*Ans.* 203.4166 +.
6. $\frac{18}{20}$.	*Ans.* .9.	*14.* $\frac{16}{17}$.	*Ans.* .9411+.
7. $\frac{114}{120}$.	*Ans.* .95	*15.* $\frac{129}{1500}$.	*Ans.* .086.
8. $\frac{5}{1250}$.	*Ans.* .004.	*16.* $501\frac{32}{33}$.	*Ans.* 501.9696+.
9. $\frac{9}{625}$.	*Ans.* .0144.	*17.* $96\frac{13}{19}$.	*Ans.* 96.6842+.

CASE II.

118. To reduce a decimal to a common fraction.

WRITTEN EXERCISES.

1. Reduce .025 to a common fraction.

OPERATION.

$.025 = \frac{25}{1000} = \frac{1}{40}$.

SOLUTION.—In this decimal, the numerator is 25, and the denominator is 1000; therefore, the fractional expression is $\frac{25}{1000}$; which, reduced to its lowest terms, gives $\frac{1}{40}$. Hence, $.025 = \frac{1}{40}$.

2. Reduce the complex decimal, $.15\frac{1}{4}$, to an equivalent common fraction.

OPERATION.

$.15\frac{1}{4} = \frac{15\frac{1}{4}}{100} = \frac{\frac{61}{4}}{100}$

$\frac{\frac{61}{4}}{100} = \frac{61}{400}$, *Ans.*

SOLUTION.—The mixed decimal, $.15\frac{1}{4}$, is $15\frac{1}{4}$ hundredths, which expressed as a common fraction, becomes $\frac{61}{400}$.

Change the following to equivalent common fractions:

3.	$.62\frac{1}{2}$.	*Ans.* $\frac{5}{8}$.	*12.*	24.075.	*Ans.* $24\frac{3}{40}$.
4.	.125.	*Ans.* $\frac{1}{8}$.	*13.*	$184.037\frac{1}{2}$.	*Ans.* $184\frac{3}{80}$.
5.	.05.	*Ans.* $\frac{1}{20}$.	*14.*	$67.0093\frac{1}{3}$.	*Ans.* $67\frac{7}{750}$.
6.	.1260.	*Ans.* $\frac{63}{500}$.	*15.*	$.008\frac{5}{6}$.	*Ans.* $\frac{53}{6000}$.
7.	.0255.	*Ans.* $\frac{51}{2000}$.	*16.*	.93750.	*Ans.* $\frac{15}{16}$.
8.	$.008\frac{1}{3}$.	*Ans.* $\frac{1}{120}$.	*17.*	.3125.	*Ans.* $\frac{5}{16}$.
9.	$.0033\frac{1}{3}$.	*Ans.* $\frac{1}{300}$.	*18.*	$12.0018\frac{3}{4}$.	*Ans.* $12\frac{3}{1600}$.
10.	.1872.	*Ans.* $\frac{117}{625}$.	*19.*	17.00125.	*Ans.* $17\frac{1}{800}$.
11.	$.0142\frac{1}{2}$.	*Ans.* $\frac{57}{4000}$.	*20.*	2.09125.	*Ans.* $2\frac{73}{800}$.

21. $.0087\frac{1}{2}$.	*25.* .88048.	*29.* 25.1850.
22. $.1265\frac{1}{3}$.	*26.* .20625.	*30.* 70.9040.
23. $.2537\frac{1}{2}$.	*27.* .00058.	*31.* 48.0080.
24. $.7001\frac{1}{3}$.	*28.* .15250.	*32.* 40.6009.

ADDITION.

119. ***Addition of Decimals*** is the process of finding the sum of two or more decimals.

WRITTEN EXERCISES.

1. Find the sum of 15.09, 435.6082, .549, and 3.6.

OPERATION.

```
  15.09
 435.6082
    .549
    3.6
 -------
 454.8472
```

SOLUTION.—Write the numbers so that units of the same order shall stand in the same column. This will bring all the decimal points in the same column. Then beginning at the right, add each column separately, setting down and carrying as in simple numbers. The decimal point in the sum must be in the same column with those of the given numbers.

Find the sum ·

2. Of .524, .16, .0623, .800039, .0000403, .508, 3.002, $60\frac{5}{7}$, .12900, and 912.00972. *Ans.* 977.909385.

3. Of 708.53, 6.007, 904.0031, .000082, and 60.00005.

4. Of 80.016, 391.25, .0100096, and 8427.63050.

5. Add 90, and seven tenths; six hundred, and 36 thousandths; forty-nine thousand, and 2 ten-thousandths; and 50 millions. *Ans.* 50049690.7362.

6. Find the sum of $\frac{4}{80}$, $\frac{5}{7}$, $\frac{6}{50}$, $\frac{25}{425}$, and $\frac{94}{125}$ in decimals, correct to the fourth place. *Ans.* 1.6951.

7. Add $18\frac{3}{4}$, $\frac{19}{25}$, $.65\frac{2}{5}$, 44.05, $83\frac{4}{25}$, $\frac{9}{35}$, and $210.06\frac{1}{2}$.

8. How many rods of fence will enclose a field the which are, respectively, 97.185, $75\frac{11}{16}$, $134.002\frac{1}{2}$, ·ods? *Ans.* 451.7958 rods.

$3\frac{1}{5}$, $.6\frac{2}{3}$, $4.2\frac{1}{2}$, $.44\frac{4}{5}$, and $3.875\frac{5}{6}$.

d the sum of $12\frac{3}{16}$, 480.625, $37\frac{2}{9}$, 62.5, $222.22\frac{2}{9}$, d $3864.5\frac{5}{9}$.

ow many bushels of corn in four bins, contain- ›ctively, 16.24, 24.5, $130\frac{5}{8}$, and $64\frac{1}{10}$ bushels?

SUBTRACTION.

120. ***Subtraction of Decimals*** is the process of finding the difference between two decimals.

WRITTEN EXERCISES.

1. Subtract 6.0451 from 129.852.

OPERATION.

Minuend,	129.852
Subtrahend,	6.0451
Remainder,	123.8069

SOLUTION.—We write the subtrahend under the minuend so that units of the same order shall stand in the same column. If both decimals have not the same number of places, we annex ciphers or suppose them to be annexed, and then proceed as in subtraction of simple numbers. Place the decimal point in the remainder in the same column as that of the minuend and subtrahend.

2. From 9040.187 take 867.348. *Ans.* 8172.839.

3. From 884.13 take 197.487. *Ans.* 686.643.

4. From 8431.00194 take 769.100086.

5. From 4.009 take 3.641963. *Ans.* .367037.

6. From .5$\frac{1}{2}$ take five hundred-thousandths.

7. From 1 million take 9 millionths.

8. From 42$\frac{5}{9}$ take .93$\frac{5}{16}$. *Ans.* 41.622430.

9. From 11.00011 take 1111 hundred-thousandths.

10. What is the difference between fourteen million one, and nine million sixteen ten-millionths?

11. From the sum of 5.16$\frac{1}{4}$ and 38.5 take the difference between 33.3$\frac{1}{3}$ and 20.05.

12. Find the difference between nine thousand twelve hundred-thousandths and 2.

13. From eighty-four, and twenty-five thousandths yards of cloth, 39, and eight hundredths yards were cut. How many yards remained?

MULTIPLICATION.

121. ***Multiplication of Decimals*** is the process of finding the product when one or both factors are decimals.

WRITTEN EXERCISES.

1. Multiply 8.29 by .43.

OPERATION.

```
 8.29
  .43
 ----
 2487
3316
 ----
3.5647
```

SOLUTION.—The decimals may be reduced to equivalent common fractions and then multiplied; thus, $\frac{829}{100} \times \frac{43}{100} = \frac{35647}{10000} = 3.5647$. Or, multiply as integers; and since hundredths multiplied by hundredths produce ten thousandths, the product must contain four decimal places, that is, as many as the sum of those in the multiplicand and the multiplier.

2. Multiply 62.18 by .24. *Ans.* 14.9232.

3. Multiply 127.05 by 1.8. *Ans.* 228.690.

4. Multiply 6.0009 by 123.12. *Ans.* 738.830808.

Multiply:

5. 748.136 by 7.004.

6. 2.014 by .735.

7. 596.10 by 33.

8. 6.35 by $87\frac{2}{5}$.

9. 83.416 by 1000.

10. 91.26 by 10000.

11. .0019 by 100.

12. 40.83 by 10.

13. What is the value of $16\frac{2}{3} \times .012\frac{1}{2} \times 100.02$?

14. What is the value of $\frac{7}{8} \times .01 \times 140.08\frac{4}{5}$?

15. What is the product of one billion by one billionth? *Ans.* 1.

16. Find the value of $3.4 \times 3.4 \times .7854$.

17. What is the area of a field 123.875 rods by 74.6 rods?

18. At 9.75 dollars per barrel, what is the cost of 27.5 barrels of flour?

19. What must I pay for 32.75 yards of cloth at $4.25 per yard, and 29.125 yards at $3.50 per yard?

DIVISION.

122. ***Division of Decimals*** is the process of finding the quotient when one or both terms are decimals.

WRITTEN EXERCISES.

1. Divide 9.5 by .625.

OPERATION.

```
.625 ) 9.5000 ( 15.2
       6 25
       -----
       3 250
       3 125
       -----
         1250
         1250
         ----
```

SOLUTION.—As the dividend must contain, at least, as many decimal places as the divisor, we must, in this problem, annex two ciphers to the dividend. Then dividing as in simple numbers, we find a remainder of 125; to which we annex another cipher and continue the division. This gives a final quotient of 152. Now, since the dividend is the product of the quotient by the divisor, we subtract the number of decimal places in the divisor from the number in the dividend, and the remainder will be the number required for the quotient. Counting the three ciphers annexed, we have four decimal places in the dividend and three in the divisor. Hence, there will be one in the quotient.

Divide:

2. 70.48316 by 5.802.
3. 3414.52 by 100.
4. 101.6688 by 43.08.
5. 71.1022 by 18.86.
6. 4171.15 by 1000.
7. 1.5108 by .019.
8. 17.1031 by .00053.
9. 125 by 2480.
10. .004 by .0016.
11. .000625 by 250.
12. .051 by .0015.
13. .001 by 100.
14. .03 by $18\frac{3}{4}$.
15. $.56\frac{2}{5}$ by 4.076.

16. $(81.96 \times \frac{3}{4}) \div (18\frac{1}{2} - 12.25) = ?$
17. $(47.008 + .125) \times (810 \div .018) = ?$
18. $300.163\frac{5}{8} \times (80.005 \div 16.001) - 900.0000099 = ?$

WRITTEN REVIEW.

1. What will 47 horses cost at $125¼ apiece?

2. A lady bought $96 worth of muslin at 32 cents a yard; how many yards did she buy? *Ans.* 300.

3. Mr. Simpson bought 19 hogsheads of molasses, each containing 58 gallons, at the rate of $0.37½ a gallon. What was the cost? *Ans.* $413.25.

4. A student paid $3¼ for a book, 50 cents for some paper, and $3.85 for a fancy inkstand. How much change did he have left from a ten-dollar bill?

5. H. L. Murphy & Co. bought a cargo of coal for $472.75, and by selling it at $8.50 a ton, they gained $190.25. How many tons did they buy? *Ans.* 78.

6. What will be the cost of the following: 32 chests of tea at $41.25 each; 24 bags of coffee, 75 pounds each, at $0.23½ per pound; 58 casks of rice, at $29 each; and 84 barrels of oil, 45 gallons each, at $2.05 per gallon?

7. What will 39 pounds of sugar cost, if 160 pounds cost $11.20? *Ans.* $2.73.

8. How many pounds of rice, at $.06 a pound, are equivalent to 27 barrels of flour at $7.50 a barrel?

9. Mr. Kelly gave $539.63½ for the 4 thirteenths of a vessel; what is ½ of the vessel worth?

10. At 40 cents a yard, how many yards of cloth can be bought for $8.23⅓? *Ans.* 20.583 yards.

11. In 1863 I sold cotton at $1.05½ a pound, for which, in 1860, I had paid $.12⅓ a pound. What was the gain on 2500 pounds? *Ans.* $2329.16⅔.

12. A merchant tailor bought 32 pieces of broadcloth, each piece containing 23⅓ yards, at $5.62¼ a yard; and sold it so as to gain $256.37½, after deducting $9.33⅓ for freight. What did he receive per yard? *Ans.* $5.97.

BUSINESS OPERATIONS.

123. In ***Business Operations*** there are generally three things considered; viz., the *quantity*, the *price*, and the *cost*.

124. The ***Quantity*** is the amount of anything bought or sold.

125. The ***Price*** is the value of a unit of the quantity.

126. The ***Cost*** is the value of the entire quantity.

127. An ***Aliquot Part*** of a number is one of the equal parts into which the number can be divided.

128. The principal aliquot parts of a dollar are as follows:

TABLE.

5 cents	=	$\frac{1}{20}$ of \$1.00.	$6\frac{1}{4}$ cents	=	$\frac{1}{16}$ of \$1.00.
10 cents	=	$\frac{1}{10}$ of \$1.00.	$8\frac{1}{3}$ cents	=	$\frac{1}{12}$ of \$1.00.
20 cents	=	$\frac{1}{5}$ of \$1.00.	$12\frac{1}{2}$ cents	=	$\frac{1}{8}$ of \$1.00.
25 cents	=	$\frac{1}{4}$ of \$1.00.	$16\frac{2}{3}$ cents	=	$\frac{1}{6}$ of \$1.00.
50 cents	=	$\frac{1}{2}$ of \$1.00.	$33\frac{1}{3}$ cents	=	$\frac{1}{3}$ of \$1.00.
75 cents	=	$\frac{3}{4}$ of \$1.00.	$66\frac{2}{3}$ cents	=	$\frac{2}{3}$ of \$1.00.

CASE I.

129. To find the cost of any quantity when the price of a unit is an aliquot part of a dollar.

1. What will 75 bushels of wheat cost at 50 cents a bushel?

OPERATION.

50 cts. = $\frac{1}{2}$.
2) $75
37\frac{1}{2}$

SOLUTION.—If it cost $1 per bushel, the cost would be as many dollars as there are bushels. But since the price is $\frac{1}{2}$ of a dollar a bushel, the whole cost will be $\frac{1}{2}$ as many dollars as there are bushels; or $\frac{1}{2}$ of $75 = $75 ÷ 2 = 37\frac{1}{2}$.

WRITTEN EXERCISES.

2. What cost 1297 dozen of eggs at 16$\frac{2}{3}$ cents a dozen?

3. At 6$\frac{1}{4}$ cents a spool, what cost 9245 spools of thread?

4. What cost 7842 yards of muslin at 33$\frac{1}{3}$ cents a yard?

5. What is the cost of 525 pounds of sugar at 12$\frac{1}{2}$ cents a pound? *Ans.* $65.625.

6. Find the cost of 2500 melons at 25 cents each.

7. What must be paid for 6 bales of cotton, containing 420 pounds each, at 16$\frac{2}{3}$ cents a pound?

8. What will 18 pieces of calico, each containing 45 yards, cost at 25 cents a yard? *Ans.* $202.50.

CASE II.

130. To find the cost when the quantity and the price of 100 or 1000 are given.

1. What is the cost of 8409 feet of boards, at $28.25 a thousand?

OPERATION.

$28.25
8.409
$237.55425

SOLUTION.—If 1000 feet cost $28.25, 1 foot will cost $\frac{1}{1000}$ of $28.25, and 8409 feet will cost 8409 times $\frac{1}{1000}$ of $28.25, which, by multiplying and cutting off three places in the product, we find to be $237.55.

WRITTEN EXERCISES.

What is the cost:

2. Of 897 apples at $2.75 per hundred?

3. Of 84728 bricks at $9.50 per 1000?

4. Of 867 pounds of pork at $6.25 per hundred?

5. Of 94520 feet of stone at $48 a thousand?

6. Of 912 pounds of beef at $9 a hundred?

7. Of 6450 oysters at $3 a hundred? *Ans.* $193.50.

8. What will 8783 feet of hemlock scantling cost at $4.62½ per M.? *Ans.* $40.6213.

9. What will a compositor receive for setting up a book of 430 pages of 784 ems each, if he receive 65 cents per thousand? *Ans.* $219.128.

10. What is the cost of 3816 pounds of wheat flour at $8.25 per C.? *Ans.* $314.82.

CASE III.

131. To find the cost of articles sold by the ton of 2000 pounds.

1. At $15.25 per ton, what will be the cost of 12845 pounds of hay?

OPERATION.

```
2 ) $15.25
    $7.625
    12.845
$97.943125
```

SOLUTION.—Since 1 ton, or 2000 pounds, cost $15.25, ½ ton, or 1000 pounds, will cost $15.25 ÷ 2 = $7.625. If 1000 pounds cost $7.625, 1 pound will cost $\frac{1}{1000}$ of $7.625; and 12845 pounds will cost 12845 times $\frac{1}{1000}$ of $7.625, which is $97.94.

WRITTEN EXERCISES.

What is the cost:

2. Of 5847 pounds of iron at $82.50 a ton?

3. Of 6540 pounds of wool at $580 a ton?

4. Of 3496 pounds of coal at $9.62½ a ton?

5. Of 36847 pounds of meal at $50 a ton?

6. Of 8420 pounds of pig iron at $34½ a ton?

7. Of 27816 pounds of oats at $26.75 a ton?

8. A peddler sold 2915 pounds of rags at $23 a ton; what did he receive? *Ans.* $33.5225.

BILLS.

132. A ***Bill*** is a written statement of the purchase or sale of goods. It should mention the time, the place, the parties concerned, the price of each item, and the entire cost.

Find the amount of each of the following statements:

1. A Student's Expenses.

Oct.	1, '87,	Paid tuition	$20.00
"	4, "	Bought Greek Grammar	1.25
"	6, "	Bought French Reader	1.00
"	6, "	Bought Elementary Algebra	.85
"	15, "	Bought pair boots	6.50
"	24, "	Paid for car fare	2.90

2. A Family's Expenses.

Jan.	8, '86,	Paid cook in full	$30.00
"	10, "	Bought steak	1.80
"	12, "	Bought bread	.95
"	19, "	Bought 3 bushels potatoes	2.25
"	25, "	Bought 2 pounds Java coffee	.70
"	29, "	Paid servants in full	45.50

3. A Farmer's Expenses.

Mar.	1, '85,	Bought a horse	$125.00
"	4, "	Paid balance due Mr. Brown	60.00
"	10, "	Paid 3 years' insurance	84.00
"	12, "	Paid carpenter	24.50
"	16, "	Bought a saddle	14.00
"	20, "	Bought 60 bushels wheat	90.00
"	25, "	Bought 40 pounds soap	3.40
"	26, "	Paid for groceries	27.35
"	28, "	Paid for repairing wagon	10.50
"	30, "	Bought 85 bushels corn	34.00

4. A Painter's Receipts.

Dec.	1,	'84,	Received from W. Clarke on account..	$90.50
"	2,	"	Received from T. Stowe on account...	125.00
"	5,	"	Sold 8 pounds putty.....	.52
"	8,	"	Sold 10 gallons turpentine.....	6.00
"	9,	"	Sold paints and oils.....	79.50
"	11,	"	Received 60 days' painting.....	210.00
"	14,	"	Received from J. Wilkins on account.	83.50
"	20,	"	Sold 120 pounds green paint.....	36.00
"	27,	"	Received 152 days' painting..	532.00

5. A Merchant's Receipts.

June	1,	'86,	Sold 42 yards cloth.....	$105.00
"	4,	"	Sold 75 yards muslin.....	12.00
"	5,	"	Sold 34 yards ribbon.....	17.00
"	8,	"	Received from T. Curran on account	98.50
"	13,	"	Sold 10 yards cloth.....	45.00
"	15,	"	Sold 5 dozen handkerchiefs.....	19.80
"	18,	"	Sold 14 dozen linen collars.....	35.00
"	25,	"	Received from W. Ross on account..	38.25

Find the balance of each of the following accounts.

1. Henry's Cash Account.

Sept.	1,	'87,	Received from mother.....	$5.00
"	2,	"	Bought 4 school books.....	2.75
"	3,	"	Bought necktie.....	.45
"	5,	"	Received from mother.....	1.50
"	7,	"	Bought 1 penknife.....	.65
"	12,	"	Bought 6 handkerchiefs.....	1.50

2. A Laborer's Transactions.

May	2,	'87,	Cash on hand.....	$41.75
"	3,	"	Received for 1 month's work.....	29.50
"	4.	"	Paid 2 weeks' board.....	10.00
"	6,	"	Bought 1 pair shoes.....	3.50
"	8,	"	Bought a hat and linen coat.....	4.25
"	15,	"	Lent a friend.....	15.00

3. A Young Man at College.

Nov.	1,	'83,	Cash on hand	$45.00
"	2,	"	Paid 2d quarter's tuition	12.50
"	3,	"	Paid books and stationery	10.50
"	4,	"	Received from home	15.00
"	5,	"	Bought overcoat and hat	23.50
"	7,	"	Sold 3 books	1.85
"	10,	"	Paid 3 weeks' board	13.50
"	19,	"	Received from home	10.00
"	25,	"	Lost	3.25
"	29,	"	Paid 1 week's board	4.50

4. Find the gain or loss on an investment including the following transactions in raising potatoes:

May	1,	'86	Plowing and harrowing, 8 days	@	$2.00		
"	9,	"	Plowing drills, 1⅛ days	"	1.25		
"	12,	"	Manuring in drills, 4 days	"	2.50		
"	13,	"	Spreading manure, 1½ days	"	.75		
"	15,	"	Seed potatoes, 3½ barrels	"	3.25		
"	16,	"	Cutting and dropping seed, 2 days	"	1.75		
"	18,	"	Covering with light plow, 1½ days	"	1.25		
"	22,	"	Hoeing and levelling, 1¾ days	"	1.50		
June	25,	"	Weeding, 3 days	"	1.00		
July	10,	"	Digging, 5 days	"	1.25		
"	12,	"	Sold 120 bushels	"	.80		
Aug.	15,	"	Took 35 bushels for private use	"	.70		
"	16,	"	Digging, 7 days	"	1.25		
"	24,	"	Sold 90 bushels	"	.60		

5. A Grocer's Transactions—Find gain or loss.

April	2,	'84,	Bought 50 pounds soap	$5.25
"	5,	"	Bought 12 barrels flour	90.00
"	7,	"	Sold 30 pounds soap	3.60
"	8,	"	Bought butter and sugar	75.00
"	11,	"	Sold 6 barrels flour	48.00
"	14,	"	Bought 87 pounds sugar	13.05
"	16,	"	Sold 37 pounds sugar	5.18
"	19,	"	Received from J. Marsh on account	24.00
"	21,	"	Paid store expenses	53.50

6. As a merchant I deal with a butcher. Find the result of the following transactions:

Mar. 1, '87	Bought of H. Rah 10 pounds beef.... @ $0.22	
" 3, "	Bought of him 16 pounds mutton.... " .16	
" 5, "	Sold him 25 yards muslin..... " .13	
" 12, "	Received from him cash, per bill.....($3.25)	
" 15, "	Bought of him 45 pounds pork..... @ .12	
" 16, "	" " 12 " corned beef " .10	
" 19, "	Sold him 15 yards cloth..... .... " 1.75	
" 21, "	Bought of him 20 pounds beef..... " .25	
" 26, "	Sold him 12 yards cambric..... " .16	
" 29, "	Paid him cash, per bill.....($1.20)	
" 31, "	Received from him, cash on account..($20.00)	

Find the amount of each of the following:

1. St. Louis, Sept. 11, 1880.

Mr. J. F. Toner,

Bought of H. W. Williams.

A	48	yards blue broadcloth..@ $3.20				
2168	50	" cassimere.....@ $1.30				
W	2	pieces black broadcloth, each 42 yards.....@ $2.20				

Received payment by note at 30 days,

H. W. Williams.

per G. White.

2. Philadelphia, Aug. 7, 1882.

Mr. H. Connolly,

Bought of W. Broderick & Co.

Aug. 3	12	gallons vinegar.....@ 16¢				
" "	28	" molasses.....@ 40¢				
" 6	30	" kerosene.....@ 17¢				
" "	10	" olive oil.....@ 80¢				

Received payment,

W. Broderick & Co.

per T. Burns.

3. NEW YORK, April 11, 1889.

MR. J. C. REILLY,

Bought of R. SALMON.

2	gallons molasses.....@ 50¢			
12	lb. rice.....@ 9¢			
4	" coffee (Rio).....@ 22¢			
3	" tea.....@ 50¢			
1	" almonds.....@ 27¢			
4	" raisins.....@ 20¢			
1	bbl. XXX flour.....@	6	25	

Received payment,

R. SALMON.

4. BALTIMORE, July 27, 1881.

MR. J. SHEEHAN,

Bought of J. MÜLLER & SON.

May 13	15	gallons port wine....@ $1.10			
" "	30	" brandy.....@ $2.25			
June 17	24	" sherry.....@ $1.67			
July 3	8	dozen champagne...@ $9.00			
" "	16	" ginger ale.....@ $1.20			
		Cr.			
" 7		By cash.....	100	00	
" 12		" "	15	25	
" 14		" "	10	50	

Balance due J. M. & Son.

Received payment,

J. MÜLLER & SON.

5. NEW YORK, May 21, 1888.

MR. JOHN KELLY,

Bought of H. JACKSON.

April 10	120	barrels flour.....@ $6.75			
" 15	75	" " .....@ $7.50			

WRITTEN EXERCISES.

Make out bills, accounts, and invoices, in proper form, from the following statements:

1.

21 pounds dried apples at 9¢.
47 pounds tea at 39¢.
15 pounds coffee at 46¢.
35 pounds barley at 7¢.
28 pounds raisins at 12¢.
19 pounds lard at 9¢.

2.

16 pounds cheese at 8¢.
31 pounds soap at 9¢.
40 pounds sugar at 7¢.
12 pounds butter at 25¢.
26 pounds rice at 10¢.
20 pounds prunes at 18¢.

3.

160 bushels rye at $1.10.
212 bushels turnips at 50¢.
234 bushels onions at 86¢.
311 bushels wheat at $1.35.
452 bushels corn at 80¢.
370 bushels potatoes at 75¢.

4.

122 barrels flour at $8.25.
375 barrels apples at $2.75.
216 barrels cider at $8.25.
324 barrels vinegar at $7.00.
112 bushels pears at $1.25.
340 bushels peaches at $2.25.

5. Mr. Burke ordered June 23, 1883, from Thomas Crogan, the following: 6 lb. cheese @ 14¢; 2 lb. butter @ 30¢; 3 lb. cooking butter @ 18¢; 4 doz. eggs @ 15¢; 3 lb. lard @ 13¢.

6. J. B. Larkin, Philadelphia, sold to Mrs. L. Regan, July 10, 1883, the following articles: 4 lb. soda crackers @ 6¢; 2 lb. egg biscuit @ 10¢; 8 loaves bread @ 6¢; 4 mince pies @ 12¢; 3 lb. sponge cake @ 25¢; 4 lb. ginger snaps @ 8¢.

7. Mr. Philip A. Harvey, Paterson, N. J., presented the following statement to G. B. Grant, April 2, 1882: 4 pair French calf gaiters @ $6.00; 2 pair cloth slippers @ $2.50; half soles and heels on boots @ $1.50; 2 pair children's shoes @ $1.50; 1 doz. shoe laces @ 20¢; 1 razor strop @ 15¢.

8. August 21, 1883, Mr. John F. Sanders, of Hoboken, N. J., purchased of B. J. Murray & Co., N. Y., 32 tons large stove coal @ $7.50; 13 tons nut coal @ $8.00; 15 cords wood @ $5.50; 25 bbl. kindling wood @ $0.50. Make out bill for amount.

9. Make out an appropriate bill on the supposition that you sold a gentleman 4 articles from a hardware store.

10. D. & J. Sadlier, New York, bought of Sower, Potts & Co., Philadelphia, the following articles, June 22, 1883: 20 Quackenbos' Natural Philosophy @ $0.90; 12 Shaw's Literature @ $1.00; 20 Robinson's Geometry @ $0.85; 16 Robinson's University Algebra @ $0.95. They paid $30 in cash, and returned books to the amount of $20. Make out bill showing entire statement.

11. M. J. Fuchs & Co., of New York, purchased of B. J. Vanere, Paris, February 16, 1882, the following: 25 gal. alcohol @ $2; 20 gal. brandy (Cognac) @ $7.50; 20 gal. Scotch gin @ $3.50; 35 gal. Holland gin @ $4.50; 20 gal. Century whiskey @ $4.00. What was the amount of the bill?

12. Chas. H. Schreiber, of Cincinnati, shipped to T. F. Clarkson, of Buffalo, May 31, 1880, the following: 12 bales, 5272 lb., Mid. Upland cotton @ 18¢; 10 bales, 4870 lb., Mid. Gulf cotton @ 20¢; 18 tierces Carolina rice, 10800 lb., @ 6¢. Make out bill, receipted.

13. On October 16, 1883, Mr. F. J. Quirk sold to Mr. L. A. Quigley, of Memphis, the following articles: 8 lb. camphor @ 50¢; 8 oz. quinine @ $3; 6 lb. Epsom salts @ 5¢; 4 oz. morphine @ $5; 7 lb. gum shellac @ 75¢; 5 lb. aqua ammonia @ 8¢; and gave a receipted bill for amount.

14. Make out a bill of ten purchases from a grocery store.

DENOMINATE NUMBERS.

133. A ***Denominate Number*** is a concrete number whose unit is arbitrary.

134. There are two sorts of units; viz., *natural* and *arbitrary*.

135. A ***Natural Unit*** is a unit furnished by the nature of the quantity measured. Thus, in a row of houses, the unit is a house.

136. An ***Arbitrary Unit*** is a unit which is determined by custom or law. Thus, in the length of a road, the unit is the yard.

137. These units or measures produce two sorts of quantity: *discrete* and *continuous*.

138. ***Discrete Quantities*** are those which are composed of similar but distinct objects; as, a regiment of soldiers, a row of trees.

139. ***Continuous Quantities*** are those which do not present distinct parts; as, a surface, a solid, the weight of a body.

140. A ***Compound Number*** is a concrete number composed of different units of the same kind of quantity.

Denominate Numbers treat of:

1. Value.	5. Volume.
2. Weight.	6. Capacity.
3. Length.	7. Time.
4. Surface.	8. Angles.

MEASURES OF VALUE.

141. The ***Value*** of a thing is its price estimated on a piece of metal of determined weight.

142. ***Money*** is the measure by which we estimate the value of things.

143. ***Currency*** is money used as a circulating medium.

UNITED STATES CURRENCY.

144. The ***Unit*** of United States currency is the gold dollar.

TABLE.

10 mills (m.)	= 1 cent	¢ or ct.
10 cents	= 1 dime	d.
10 dimes	= 1 dollar	$.
10 dollars	= 1 eagle	E.

NOTE.—The *gold* coins are the $1, $2½, $3, $5, $10, and $20 pieces; the *silver* coins are the 10¢, 25¢, 50¢, and $1 pieces; the *nickel* coins are the 3¢ and 5¢ pieces; the 1¢ piece is made of *bronze*.

ORAL EXERCISES.

1. How many mills in a cent? In 3¢? In 5¢?

2. How many cents in a dime? In 2 dimes? In 4 dimes? In 10 dimes? In 15 dimes? In 24 dimes?

3. How many mills in 10 cents? Cents in a dime? Mills in a dime? Cents in 8 dimes? Mills in 20 dimes?

4. How many cents in a dollar? In $6? In $8?

5. How many mills in 12¢? In 60¢? In 90¢?

6. How many mills in 100 cents? Cents in a dollar? Mills in a dollar? Mills in 60 cents?

7. How many dimes in a dollar? In $10? In $7?

8. How many dimes in $2.50? In $6.40? In $9.30?

9. How many cents in 90 mills? In 30 m.? In 70 m.?

10. How many cents and mills in 65 m.? In 93 m.?

11. How many dimes in 80¢? In 20¢? In 110¢?

12. How many dimes in 65¢? In 92¢? In 215¢?

13. How many dollars in an eagle? In a double eagle? In 6 eagles? In 9 eagles? In 15 eagles?

14. How many cents in 8 dimes 9 cents? 6 dimes 2 cents? 7 dimes 3 cents? 10 dimes 4 cents?

15. How many cents in the silver coins, taking one of each? 5 of each? 10 of each?

16. How many dollars in the gold pieces, taking one of each? 3 of each? 6 of each?

ENGLISH MONEY.

145. ***English Money*** is the legal currency of Great Britain.

146. The ***Unit*** of English money is the pound sterling.

TABLE.

4 farthings (far.)	= 1 penny	d.
12 pence	= 1 shilling	s.
20 shillings	= 1 pound	£.

Also, 2 shillings = 1 florin; 5 shillings = 1 crown; and 21 shillings = 1 guinea.

NOTE.—The gold coin valued at one £, is called a *sovereign*. Its value in U. S. money is $4.8665. The *gold* coins of Great Britain are the sovereign, half-sovereign, and guinea; the *silver* coins are the crown, half-crown, florin, shilling, six-penny, and three-penny pieces; the *copper* coins are the penny, half-penny, and farthing.

ORAL EXERCISES.

1. How many farthings in a penny? In 6 pence? In 7 pence? In 11 pence? In 9 pence? In 12 pence?

2. How many farthings in 3d. 2 far.? In 4d. 3 far.?

3. How many pence in a shilling? In 4 shillings? In 8 shillings? In 7 shillings? In 11 shillings?

4. How many farthings in 12 pence? Pence in a shilling? Farthings in a shilling?

5. How many farthings in 3s.? In 10s.? In 15s.?

6. How many shillings in a £? In £7? In £13? In £25? In £15? In £20? In £40?

7. How many shillings in a crown? In 6 crowns? In 14 crowns? In 20 crowns? In 9 crowns?

8. How many pounds in 60 shillings? In 100s.? In 80s.? In 480s.? In 360s.? In 640s.?

9. How many pence in 16 far.? In 18 far.? In 36 far.?

10. How many shillings in 72d.? In 96d.? In 240d.?

11. What will 5 lb. of beef cost at 9d. per pound?

12. How many yards of cloth at 6s. a yard can be purchased for £3? For £6? For £11?

13. What cost 36 combs at 9d. apiece? At 3d.?

14. How many bats at 2s. each may be bought for 120d.? For 240d.? For 144d.? For 72d.?

REDUCTION DESCENDING.

1. Reduce £3 11s. 9d. 2 far. to farthings.

OPERATION.

£	s.	d.	far.
3	11	9	2

20

71s.

12

861d.

4

3446 far.

SOLUTION.—Since one £ = 20s., £3 = 60s.; adding 11s. to this, we have 71s. Since 1s. = 12d., 71s. = 852d., which, together with 9d., give 861d. And, since 1d. = 4 far., 861d. = 3444 far., which, plus 2 far., give 3446 far. Hence, £3 11s. 9d. 2 far. = 3446 far.

WRITTEN EXERCISES.

Reduce:

2. £3 8s. 5d. to pence. *Ans.* 821d.
3. £10 6s. 11d. to pence. *Ans.* 2483d.
4. £57 10s. 3d. 2 far. to far. *Ans.* 55214 far.
5. £60 9s. 2d. to farthings. *Ans.* 58040 far.
6. £35 0s. 10d. to farthings. *Ans.* 33640 far.
7. £18 16s. to pence. *Ans.* 4512d.
8. £150 to farthings. *Ans.* 144000 far.
9. 6s. 11d. to pence. *Ans.* 83d.
10. £1 1s. 1d. 1 far. to farthings. *Ans.* 1013 far.
11. £55 to pence. *Ans.* 13200d.
12. £180 18s. to shillings. *Ans.* 3618s.
13. £2860 to shillings. *Ans.* 57200s.

REDUCTION ASCENDING.

1. Reduce 7115 farthings to higher denominations.

OPERATION.

```
4 ) 7115 far.
12 ) 1778 + 3 far.
20 )  148 + 2d.
        £7 + 8s.
£7 8s. 2d. 3 far.
```

SOLUTION.—Since 4 far. = 1d., in 7115 far. there are 1778d. and 3 far. Also, 12d. = 1s.; hence, in 1778d., there are 148s. and 2d. Again, as 20s. = £1, so in 148s. there are £7 and 8s.

WRITTEN EXERCISES.

Change:

2. 6000 farthings to pounds sterling. *Ans.* £6 5s.
3. 23088d. to pounds. *Ans.* £96 4s.
4. 1684 far. to shillings. *Ans.* 35s. 1d.
5. 8152d. to pounds. *Ans.* £33 19s. 4d.
6. 66778 far. to pounds. *Ans.* £69 11s. 2d. 2 far.

7. 23694d. to pounds. *Ans.* £98 14s. 6d.
8. 16860 far. to shillings.
9. 5068d. to crowns. *Ans.* 84 crowns 2s. 4d.
10. 9790 far. to pence.
11. 27210d. to sovereigns. *Ans.* 113 sov. 7s. 6d.
12. 59317s. to pounds. *Ans.* £2965 17s.
13. 1048521d. to pounds.
14. 816 farthings to shillings. *Ans.* 17s.
15. 522315d. to florins. *Ans.* 21763 florins 3d.

MEASURES OF WEIGHT.

147. ***Weight*** is the measure of the earth's attraction.

148. The ***Unit*** of weight is the Troy pound.

149. There are three measures of weight in general use; viz., Troy Weight, Apothecaries' Weight, and Avoirdupois Weight.

TROY WEIGHT.

150. ***Troy Weight*** is used in weighing precious metals, jewels, and also in philosophical experiments.

151. The ***Unit*** of Troy Weight is the pound of 5760 grains.

TABLE.

24 grains (gr.)	= 1 pennyweight.....	pwt.
20 pennyweights	= 1 ounce.....	oz.
12 ounces	= 1 pound.....	lb.

ORAL EXERCISES.

1. How many grains in 1 pwt.? In 4 pwt.? In 6 pwt.? In 10 pwt.? In 20 pwt.? In 3 pwt.?

2. How many pwt. in an ounce? In 3 oz.? In 5 oz.?

3. How many ounces in a lb.? In 5 lb.? In 8 lb.? In 12 lb.? In 15 lb.? In 30 lb.? In 50 lb.?

4. How many lb. in 60 oz.? In 48 oz.? In 288 oz.? In 72 oz.? In 120 oz.? In 144 oz.? In 84 oz.?

5. How many ounces in 60 pwt.? In 480 pwt.? In 180 pwt.? In 220 pwt.?

6. How many pwt. in 72 gr.? In 120 gr.? In 240 gr.?

7. How many grains in 4 pwt. 3 gr.? In 10 pwt. 10 gr.?

8. How many pwt. in 4 oz. 2 pwt.? In 10 oz. 6 pwt.?

9. How many oz. in 3 lb. 4 oz.? In 12 lb. 6 oz.? In 15 lb. 9 oz.? 10 lb. 3 oz.? 20 lb. 5 oz.?

10. At 3 cents a grain what will 2 pwt. of gold cost?

11. How many watches of 2 oz. each can be made from 80 pwt. of gold? From 120 pwt.?

12. What will 2 lb. of silver cost at 6 cents a pwt.?

WRITTEN EXERCISES.

Reduce:

1. 3 lb. 10 oz. 6 pwt. 3 gr. to grains. *Ans.* 22227 gr.

2. 9 lb. 0 oz. 11 pwt. 10 gr. to grains. *Ans.* 52114 gr.

3. 6 oz. 12 pwt. 17 gr. to grains. *Ans.* 3185 gr.

4. 18 lb. 6 oz. 4 pwt. to pwt. *Ans.* 4444 pwt.

5. 200 lb. 9 oz. 0 pwt. to pennyweights.

6. 34 lb. 6 gr. to grains. *Ans.* 195846 gr.

7. 10 oz. 19 pwt. to grains. *Ans.* 5256 gr.

8. 207 lb. to ounces; to pwt.

9. 76 lb. 10 oz. 16 pwt. to pennyweights.

10. 19 lb. 5 oz. to grains. *Ans.* 111840 gr.

11. 23036 gr. to lb. *Ans.* 3 lb. 11 oz. 19 pwt. 20 gr.

12. 3996 pwt. to lb. *Ans.* 16 lb. 7 oz. 16 pwt.

13. 523769 gr. to lb. *Ans.* 90 lb. 11 oz. 3 pwt. 17 gr.

14. 8406 pwt. to lb. *Ans.* 35 lb. 6 pwt.

15. 6004 gr. to ounces. *Ans.* 12 oz. 10 pwt. 4 gr.

16. 167 gr. to pwt. *Ans.* 6 pwt. 23 gr.
17. 2408 pwt. to lb. *Ans.* 10 lb. 8 pwt.
18. 3864 gr. to oz. *Ans.* 8 oz. 1 pwt.
19. 18000 oz. to lb. *Ans.* 1500 lb.
20. 178 pwt. to oz. *Ans.* 8 oz. 18 pwt.

APOTHECARIES' WEIGHT.

152. ***Apothecaries' Weight*** is used in prescribing and in compounding dry medicines.

153. The ***Unit*** of this measure is the Troy pound. This weight differs from Troy Weight only in the divisions of its ounce.

TABLE.

20 grains (gr.)	= 1 scruple	scr., or ℈.
3 scruples	= 1 dram	dr., or ʒ.
8 drams	= 1 ounce	oz., or ℥.
12 ounces	= 1 pound	lb., or ℔.

ORAL EXERCISES.

1. How many grains in a scruple? In 3 scr.? In 5 scr.? In 8 scr.? In 15 scr.? In 30 scr.?

2. How many scruples in 60 grains? In 70 gr.? In 80 gr.? In 40 gr.? In 55 gr.? In 95 gr.?

3. How many scruples in a dram? In 6 dr.? In 8 dr.? In 10 dr.? In 15 dr.? In 24 dr.?

4. How many drams in 6 scruples? In 24 scr.? In 15 scr.? In 36 scr.? In 51 scr.? In 72 scr.?

5. How many drams in an ounce? In 4 oz.? In 11 oz.? In 17 oz.? In 20 oz.? In 30 oz.?

6. How many ounces in 16 drams? In 48 dr.? In 96 dr.? In 24 dr.? In 42 dr.? In 60 dr.?

7. How many ounces in a pound? In 6 lb.? In 7 lb.?

8. How many pounds in 24 oz.? In 36 oz.? In 72 oz.?

9. How many grains in a dram? In 3 dr.? In 4 dr.?

10. How many drams in a lb.? In 3 lb.? In 7 lb.?

11. How many ounces in 3 lb. 2 oz.? In 2 lb. 4 oz.?

12. How many scruples in 3 dr. 2 scr.? In 12 dr. 2 scr.? In 10 dr. 1 scr.? In 15 dr. 4 scr.?

13. How many pills of 2 grains each, may be made from an ounce of quinine?

14. What is their value at 2 cents each?

WRITTEN EXERCISES.

Reduce:

1. 9 lb. 10 oz. 6 dr. 2 scr. to grains. *Ans.* 57040 gr.

2. 11 lb. ℥1 ʒ6 to drams. *Ans.* ʒ1070.

3. ℥16 ʒ4 ℈2 to scruples. *Ans.* ℈398.

4. 1 lb. ℥1 ʒ1 to grains. *Ans.* 6300 gr.

5. 16 lb. 2 oz. to scruples. *Ans.* ℈4656.

6. 3 oz. to grains. *Ans.* 1440 gr.

7. 46 lb. 10 oz. to grains. *Ans.* 269760 gr.

8. 270 lb. ℥9 to drams. *Ans.* ʒ25992.

9. ʒ7 ℈2 13 gr. to grains. *Ans.* 473 gr.

10. ℥11 ʒ2 ℈1 16 gr. to grains. *Ans.* 5436 gr.

11. 57050 grains to lb. *Ans.* 9 lb. ℥10 ʒ6 ℈2 10 gr.

12. 8640 scruples to lb. *Ans.* 30 lb.

13. 34572 grains to lb. *Ans.* 6 lb. 12 gr.

14. 6000 grains to oz. *Ans.* 12 oz. 4 dr.

15. 78406 scruples to oz. *Ans.* ℥3266 ʒ7 ℈1.

16. 8009 drams to lb. *Ans.* 83 lb. ℥5 ʒ1.

17. 4040 grains to drams. *Ans.* ʒ67 ℈1.

18. 16843 scruples to lb. *Ans.* 58 lb. 5 oz. 6 dr. 1 scr.

19. 4444 grains to oz. *Ans.* ℥9 ʒ2 4 gr.

20. 862 drams to oz. *Ans.* ℥107 ʒ6.

AVOIRDUPOIS WEIGHT.

154. ***Avoirdupois Weight*** is used in weighing all common goods.

155. The ***Unit*** of this measure is the pound of 7000 Troy grains.

TABLE.

16 ounces (oz.)	=	1 pound.....	lb.
100 pounds	=	1 hundredweight.....	cwt.
		or cental.....	C.
20 hundredweight	=	1 ton.....	T.

NOTE.—The Long ton contains 2240 pounds; 112 pounds to the hundredweight, and 28 pounds to the quarter. The Long ton is used at the U. S. Custom House, also in weighing ores, coal at the mines, and in ocean freights.

ORAL EXERCISES.

1. How many ounces in a pound? In 3 lb.? In 7 lb.? In 12 lb.? In 15 lb.? In 20 lb.? In 50 lb.?

2. How many pounds in 3 cwt.? In 6 cwt.? In 2 centals? In 6 C.? In 10 C.? In 35 C.? In 47 C.?

3. How many cwt. in a ton? In 3 tons? In 7 tons?

4. How many ounces in 2 lb.? In a cwt.? In a ton?

5. How many pounds in 80 oz.? In 32 oz.? In 96 oz.?

6. How many pounds and ounces in 37 oz.? In 46 oz.? In 89 oz.? In 75 oz.? In 65 oz.?

7. How many centals and pounds in 670 lb.? In 750 lb.? In 1010 lb.? In 2330 lb.?

8. What is the cost of a pig weighing 3 cwt. 50 lb., at 5 cents a lb.? At 6 cents? At 7 cents?

9. At 3 cents an ounce, what will 4 lb. of tea cost? 12 lb.? 30 lb.? 10 lb.?

10. What cost 4 centals of sugar, at 9 cents a lb.?

WRITTEN EXERCISES.

Reduce:

1. 7 T. 9 cwt. 16 lb. to lb. *Ans.* 14916 lb.
2. 1 T. 11 cwt. 70 lb. 15 oz. to oz. *Ans.* 50735 oz.
3. 6 cwt. 9 oz. to ounces. *Ans.* 9609 oz.
4. 8 T. 4 lb. to pounds. *Ans.* 16004 lb.
5. 17 T. 16 cwt. to cwt. *Ans.* 356 cwt.
6. 4 C. 6 lb. to oz. *Ans.* 6496 oz.
7. 25 T. 6 C. 84 lb. 12 oz. to oz. *Ans.* 810956 oz.
8. 25 C. 86 lb. to lb. *Ans.* 2586 lb.
9. 8 T. 2 C. to centals. *Ans.* 162 C.
10. 375 T. 16 C. 90 lb. 14 oz. to oz. *Ans.* 12027054 oz.
11. 470507 oz. to tons. *Ans.* 14 T. 14 cwt. 6 lb. 11 oz.
12. 82880 lb. to tons. *Ans.* 41 T. 8 cwt. 80 lb.
13. 43276 oz. to cwt. *Ans.* 27 cwt. 4 lb. 12 oz.
14. 6780 oz. to lb. *Ans.* 423 lb. 12 oz.
15. 30245 lb. to tons. *Ans.* 15 T. 2 C. 45 lb.
16. 438 cwt. to tons. *Ans.* 21 T. 18 cwt.
17. 16784 oz. to centals. *Ans.* 10 C. 49 lb.
18. 3700 oz. to centals. *Ans.* 2 C. 31 lb. 4 oz.
19. 18000 lb. to tons. *Ans.* 9 T.
20. 5214430 oz. to tons. *Ans.* 162 T. 19 C. 1 lb. 14 oz.

GENERAL WEIGHTS.

Commodities.	lb. per bu.	Commodities.	lb. per bu.	Commodities.	lb. per bu.
Apples (dry)...	28	Timothy seed..	45	Potatoes.....	60
Peaches (dry)..	28	Salt.....	50	Pease.....	60
Oats.....	32	Rye.....	56	Beans.....	60
Malt.....	34	Corn (shelled)..	56	Wheat.....	60
Barley.....	48	Flax seed.....	56	Clover seed....	60
Buckwheat....	48	Onions.....	57	Corn (in ear)..	70

Commodities.	lb.	Commodities.	lb.
Barrel of flour.....	196	Quintal of fish.....	100
" pork.....	200	Keg of nails.....	100
" salt.....	280	Gallon of petroleum....	6½

ORAL EXERCISES.

1. How many lb. in a bushel of dried apples? In 6 bu.? In 11 bu.?

2. How many lb. in 2 bbl. of flour? In 3 kegs of nails? In 4 gallons of petroleum?

3. How many lb. in a bu. of buckwheat? In 3 bu. of potatoes? In 7 bu. of salt?

4. What cost 3 bbl. of pork at 9¢ a lb.? At 11¢ a lb.?

5. What cost 4 bu. of salt at 6¢ a lb.? At 8¢?

6. Which is the heavier, a bushel of potatoes or a bushel of onions?

7. What cost 3 bu. of barley at 5¢ a lb.? At 4¢?

8. What cost 4 quintals of codfish at 10¢ a lb.? At 12¢?

WRITTEN EXERCISES.

1. A farmer sold 20 bushels of oats at 86¢ a bu., and 16 bushels of buckwheat at 3¢ a lb.; how much did he receive for both? *Ans.* $40.24.

2. What cost 9 bbl. of flour at 5¢ a lb., 3 bbl. of salt at 4¢ a lb., 4 bu. of dried peaches at 20¢ a lb.?

Ans. $144.20.

3. What is the weight in tons, of 3 loads of potatoes, averaging 22 bu. each; 1 load of wheat, 19 bu.; and 4 loads of oats, each 25 bu.? *Ans.* 4 T. 3 cwt.

4. Find the cost of 47½ quintals of fish at 5¢ a pound, and 14⅞ gallons of petroleum at 16¢ a gallon.

MEASURES OF LENGTH.

156. ***Measures of Length*** are those standards which enable us to estimate extent considered as a line; as, the *height* of a house, the *depth* of the sea.

157. The ***Unit*** of length is the yard. It is equal to $\frac{360000}{391393}$ of the length of a pendulum vibrating seconds, at London.

TABLE.

12 inches (in.)	= 1 foot	ft.
3 feet	= 1 yard	yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod	rd.
320 rods	= 1 mile	m.

OTHER DENOMINATIONS.

12 lines	= 1	inch.
3 sizes	= 1	inch (shoemaker's).
4 inches	= 1	hand (in measuring horses).
9 inches	= 1	span.
6 feet	= 1	fathom (depth at sea).
3 feet	= 1	pace.
40 rods	= 1	furlong.
8 furlongs	= 1	mile.
$1.152\frac{2}{3}$ miles	= 1	knot, or nautical mile.
60 geographical miles 69.16 statute miles	= 1 degree of	latitude on a meridian. longitude on the Equator.

ORAL EXERCISES.

1. How many inches in a foot? In 3 ft.? In 5 ft.?
2. How many feet in 24 inches? In 72 in.? In 120 in.?
3. How many feet in a yard? In 2 yd.? In 6 yd.?
4. How many yards in 15 feet? In 21 ft.? In 33 ft.?
5. How many feet and inches in 37 inches? In 75 in.?
6. How many yards and feet in 20 feet? In 32 ft.?
7. How many yards in a rod? In 2 rd.? In 7 rd.?
8. How many rods in 11 yd.? In 22 yd.? In 44 yd.?

9. How many feet in a yard? Yards in a rod?
10. How many feet in 3 rods? In 6 rods? In 10 rd.
11. How many inches in a foot? Feet in a yard?
12. How many inches in 6 yd.? In 10 yd.?
13. How many inches in 3 ft. 8 in.? In 10 ft. 6 in.
14. How many feet in 3 yd. 2 ft.? In 4 yd. 1 ft.?
15. How many inches in 1 yd. 1 ft.? In 3 yd. 2 ft.
16. How many feet in 3 rd. 2 yd.? 4 rd. 4 yd.?
17. How many rods in a furlong? In 3 furlongs?
18. How many furlongs in 80 rd.? In 120 rd.?
19. How many furlongs in a mile? In 6 m.? In 9 m.
20. How many miles in 16 fur.? In 40 fur.? 72 fur.
21. How many rods in a mile? In 6 m.? In 4 m.
22. How many rods in 6 fur. 20 rd.? In 7 fur. 16 rd.

WRITTEN EXERCISES.

Reduce:

1. 10 m. 303 rd. 4 yd. 2 ft. 11 in. to inches.
2. 37 rd. 3 yd. 7 in. to inches. *Ans.* 7441 in.
3. 69 m. 270 rd. 1 yd. to feet. *Ans.* 368778 ft.
4. 3 fur. 19 rd. 3 yd. 1 ft. to feet. *Ans.* 2303½ ft.
5. 4 yd. 2 ft. 11 in. to inches. *Ans.* 179 in.
6. 121 m. 280 rd. 5 yd. 9 in. to inches.
7. 3 fur. 28 rd. 4 yd. to yards. *Ans.* 818 yd.
8. 81 m. 3 fur. 38 rd. to rods. *Ans.* 26078 rd.
9. 95936 inches to miles.
10. 553248 in. to miles. *Ans.* 8 m. 234 rd. 1 yd.
11. 29876 ft. to miles. *Ans.* 5 m. 210 rd. 3 yd. 2 ft.
12. 215 inches to yd. *Ans.* 5 yd. 2 ft. 11 in
13. 2640 feet to furlongs. *Ans.* 4 fur
14. 37540 yards to miles.
15. 6552 inches to rods. *Ans.* 33 rd. 1 ft. 6 i
16. 1754 yards to furlongs. *Ans.* 7 fur. 38 rd. 5 y

SURVEYORS' LINEAR MEASURE.

TABLE.

7.92 inches	= 1 link	l.
25 links	= 1 rod	rd.
4 rods or 100 links	= 1 chain	ch.
80 chains	= 1 mile	m.

ORAL EXERCISES.

1. How many inches in a link? In 2 l.? In 10 l.?
2. How many links in a rod? In 2 rd.? In 4 rd.?
3. How many rods in 75 links? In 125 l. In 200 l.?
4. How many rods in a chain? In 6 ch.? In 10 ch.?
5. How many links in 3 ch? In 7 ch.? In 20 ch.?
6. How many chains in 16 rd.? In 30 rd.? In 20 rd.?
7. How many chains in 300 links? In 150 l.?
8. How many chains in a mile? In 3 m.? In 12 m.?
9. How many miles in 160 chains? In 400 ch.?
10. How many miles and chains in 250 ch.? In 90 ch.?

WRITTEN EXERCISES.

Reduce:

1. 3 m. 70 ch. 43 l. to links. *Ans.* 31043 l.
2. 27 ch. 30 l. to links. *Ans.* 2730 l.
3. 13 m. 20 ch. 16 l. to inches. *Ans.* 839646.72 in.
4. 3 rd. 17 l. to links. *Ans.* 92 l.
5. 4 m. 60 ch. to links. *Ans.* 38000 l.
6. 26000 l. to miles. *Ans.* 3 m. 20 ch.
7. 3672 l. to chains. *Ans.* 36 ch. 2 rd. 22 l.
8. 4634 l. to rods. *Ans.* 185 rd. 9 l.
9. 35446 in. to chains. *Ans.* 44 ch. 75 l. 4 in.
10. 174191 l. to miles. *Ans.* 21 m. 61 ch. 91 l.

MEASURES OF SURFACE.

158. ***Measures of Surface*** are those measures by which we estimate extent considered under two dimensions; viz., length and breadth.

159. The ***Unit*** of Surface Measure is the square yard.

TABLE.

144 square inches (sq. in.)	= 1 square foot.....	sq. ft.
9 square feet	= 1 square yard.....	sq. yd.
30¼ square yards	= 1 square rod.....	sq. rd.
160 square rods	= 1 acre.....	A.
640 acres	= 1 square mile.....	sq. m.

ORAL EXERCISES.

1. How many sq. in. in a sq. ft? In 3 sq. ft.?

2. How many sq. ft. in 288 sq. in.? In 576 sq. in.?

3. How many sq. in. in 3 sq. ft. 72 sq. in.? In 2 sq. ft. 19 sq. in.? In 5 sq. ft. 100 sq. in.?

4. How many sq. ft. in a sq. yd.? In 8 sq. yd.?

5. In 27 sq. ft. how many sq. yd.? In 45 sq. ft.? In 81 sq. ft.? In 72 sq. ft.? In 126 sq. ft.?

6. How many sq. ft. in 5 sq. yd. 6 sq. ft.? In 7 sq. yd. 8 sq. ft.? In 6 sq. yd. 10 sq. ft.?

7. How many sq. yd. in 4 sq. rd.? In 8 sq. rd.?

8. How many sq. rd. in an acre? In 4 A.? In 5 A.?

9. How many acres in 320 sq. rd.? In 800 sq. rd.?

10. How many acres in a sq. mile? In 2 sq. m.?

WRITTEN EXERCISES.

Reduce:

1. 3 sq. m. 500 A. 120 sq. rd. 20 sq. yd. 8 sq. ft. 79 sq. in. to sq. in. *Ans.* 15184520431 sq. in.

2. 19 sq. yd. 6 sq. ft. 116 sq. in. to sq. in.

3. 620 A. 88 sq. rd. 25 sq. yd. to sq. yd.
Ans. 3003487 sq. yd.

4. 116 sq. rd. 30 sq. yd. 2 sq. ft. to sq. ft.

5. 18 sq. m. 210 A. 125 sq. rd. to sq. rods.
Ans. 1876925 sq. rd.

6. 17 A. 20 sq. rd. 12 sq. yd. 7 sq. ft. to sq. ft.

7. 1 sq. m. 86 A. 10 sq. yd. to sq. ft.

8. 892 A. to sq. in.

9. 113941 sq. ft. to acres.
Ans. 2 A. 98 sq.rd. 15 sq.yd. 5 sq.ft. 72 sq.in.

10. 322704 sq. in. to sq. rd. *Ans.* 8 sq. rd. 7 sq. yd.

11. 6272640 sq. in. to acres. *Ans.* 1 A.

12. 8028979200 sq. in. to sq. miles. *Ans.* 2 sq. m.

13. 690871 sq. yd. to acres.
Ans. 142 A. 118 sq.rd. 21 sq.yd. 4 sq.ft. 72 sq.in.

14. 269407358 sq. in. to acres.

15. 6920 sq. ft. to sq. rd.

16. 29175 sq. in. to sq. yd.

SURVEYORS' SQUARE MEASURE.

TABLE.

625 square links	= 1 square rod	sq. rd.
16 square rods 10000 square links	= 1 square chain	sq. ch.
10 square chains	= 1 acre	A.
640 acres	= 1 square mile	sq. m.

WRITTEN EXERCISES.

Reduce:

1. 3 sq. m. 120 A. 6 sq. ch. to square links.
Ans. 204060000 sq. l.

2. 300 A. 80 sq. rd. to square links.
Ans. 30050000 sq. l.

3. 20 A. 7 sq. ch. 750 sq. l. to square links.
Ans. 2070750 sq. l.

4. 6 sq. m. 80 sq. ch. 12 sq. rd. to sq. rd.
Ans. 615692 sq. rd.

5. 7386407000 sq. links to square miles.
Ans. 115 sq. m. 264 A. 11 sq. rd. 125 sq. l.

6. 8600300 sq. links to acres. *Ans.* 86 A. 300 sq. l.

7. 36400 sq. rods to acres. *Ans.* 227 A. 5 sq. ch.

8. 240000 sq. links to square chains. *Ans.* 24 sq. ch.

MEASURES OF VOLUME.

160. ***Measures of Volume*** are those measures which serve to estimate extent considered under three dimensions; viz., length, breadth, and thickness.

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot.....	cu. ft.
27 cubic feet	= 1 cubic yard.....	cu. yd.

WOOD MEASURE.

16 cubic feet	= 1 cord foot.....	cd. ft.
8 cord feet	= 1 cord.....	C.

NOTE.—Lumber is measured by board measure, whose unit is a board foot, 1 ft. long, 1 ft. wide, and 1 in. thick, or $\frac{1}{12}$ of a cubic foot.

All kinds of sawed timber, boards, planks, and joists are generally measured by this measure. But hewn and round timber are measured by cubic measure.

ORAL EXERCISES.

1. How many cu. ft. in a cu. yard? In 3 cu. yd.?

2. How many cu. yd. in 54 cu. ft.? In 81 cu. ft.? In 135 cu. ft.?

3. How many cu. ft. in 3 cu. yd. and 8 cu. ft.? In 6 cu. yd. and 7 cu. ft.? In 4 cu. yd. and 3 cu. ft.?

4. How many cu. ft. in a cd. ft.? In 3 cd. ft.? In 18 cd. ft.? In 11 cd. ft.?

5. How many cd. ft. in a cord? In 3 C.? In 8 C.? In 12 C.? In 10 C.? In 25 C.?

6. How many cu. ft. in 5 cords? In 2 C.? In 6 C.? In 9 C.? In 10 C.? In 20 C.?

WRITTEN EXERCISES.

Reduce:

1. 18 cu. yd. 3 cu. ft. 1600 cu. in. to cu. in. *Ans.* 846592 cu. in.

2. 218 cu. yd. 18 cu. ft. to cu. ft. *Ans.* 5904 cu. ft.

3. 13 cu. ft. 1692 cu. in. to cu. in. *Ans.* 24156 cu. in.

4. 8 C. 3 cd. ft. 2 cu. ft. to cu. ft. *Ans.* 1074 cu. ft.

5. 596 cu. yd. to cu. in. *Ans.* 27806976 cu. in.

6. 9 cords to cu. inches. *Ans.* 1990656 cu. in.

7. 921024 cu. in. to cu. yd. *Ans.* 19 cu. yd. 20 cu. ft.

8. 44850 cu. in to cd. feet.

9. 2216 cu. ft. to cords.
Ans. 17 C. 2 cd. ft. 8 cu. ft., or 17 C. 40 cu. ft.

10. 2468784 cu. in. to cords.

MEASURES OF CAPACITY.

161. ***Measures of Capacity*** are those measures employed to measure liquids and dry matters.

LIQUID MEASURE.

162. ***Liquid Measure*** is used in measuring wine, molasses, etc.

163. The ***Unit*** of Liquid Measure is the gallon. It contains 231 cubic inches.

TABLE.

4 gills (gi.)	= 1 pint	pt.
2 pints	= 1 quart	qt.
4 quarts	= 1 gallon	gal.
31½ gallons	= 1 barrel	bbl.
63 gallons	= 1 hogshead	hhd.

OTHER DENOMINATIONS.

36 gallons = 1 barrel of beer.
42 gallons = 1 tierce.
54 gallons = 1 hogshead of beer.

ORAL EXERCISES.

1. How many gills in a pint? In 2 pt.? In 6 pt.? In 13 pt.? In 25 pt.?

2. How many pints in 12 gills? In 16 gi.? In 32 gi.?

3. How many gills in 1 pt. 2 gi.? In 2 pt. 3 gi.? In 4 pt. 1 gi.? In 7 pt. 3 gi.?

4. How many pints in a quart? In 10 qt.? In 15 qt.? In 45 qt.? In 18 qt.?

5. How many qt. in 24 pt.? In 80 pt.? In 72 pt.? In 34 pt.? In 18 pt.?

6. How many pt. in 4 qt. 1 pt.? In 7 qt. 1 pt.?

7. How many qt. and pt. in 87 pt.? In 49 pt.? In 33 pt.? In 17 pt.?

8. How many gills in a quart? In 3 qt.? In 4 qt.? In 11 qt.? In 17 qt.?

9. How many quarts in a gallon? In 15 gal.? In 3 gal.? In 11 gal.?

10. How many quarts in 10 gal. 3 qt.? In 13 gal. 2 qt.? In 27 gal. 1 qt.?

11. How many gal. and qt. in 83 qt.? In 25 qt.? In 33 qt.? In 75 qt.?

12. How many gallons in a barrel? In 2 bbl.? In 6 bbl.? In 8 bbl.? In 10 bbl.?

13. How many gallons in a hhd.? In 2 hhd.? In 4 hhd.? In 10 hhd.?

14. How many hhd. in 126 gal.? In 315 gal.?

15. How many pints in a gal.? In 3 gal.? In 17 gal.? In 20 gal.? In 25 gal.?

16. How many qt. in a hhd.? In 4 hhd.? In 10 hhd.? In 12 hhd.? In 8 hhd.? In 20 hhd.?

17. How many hhd. and gal. in 150 gal.? In 180 gal.? In 240 gal.? In 120 gal.? In 250 gal.?

18. How many gal. in 3 hhd. 40 gal.? In 12 hhd. 1 gal.? In 10 hhd. 50 gal.?

WRITTEN EXERCISES.

Reduce:

1. 6 gal. 3 qt. 1 pt. 3 gi. to gills. *Ans.* 223 gi.
2. 3 hhd. 60 gal. 3 qt. to pints. *Ans.* 1998 pt.
3. 18 hhd. 37 gal. 2 qt. to quarts. *Ans.* 4686 qt.
4. 2 bbl. 30 gal. 2 qt. 1 pt. to pints. *Ans.* 749 pt.
5. 30 bbl. to pints. *Ans.* 7560 pt.
6. 9 gal. 3 gi. to gills. *Ans.* 291 gi.
7. 16 hhd. to pints. *Ans.* 8064 pt.
8. 128 gal. to pints. *Ans.* 1024 pt.
9. 158 gi. to gallons. *Ans.* 4 gal. 3 qt. 1 pt. 2 gi.
10. 4039 pt. to hhd. *Ans.* 8 hhd. 3 qt. 1 pt.
11. 3310 qt. to hhd. *Ans.* 13 hhd. 8 gal. 2 qt.
12. 38609 gi. to gallons. *Ans.* 1206 gal. 2 qt. 1 gi.
13. 6780 pt. to hhd. *Ans.* 13 hhd. 28 gal. 2 qt.
14. 8080 pt. to bbl. *Ans.* 32 bbl. 2 gal.
15. 7824 gal. to hhd. *Ans.* 124 hhd. 12 gal.
16. 372 pt. to gallons. *Ans.* 46 gal. 2 qt.

DRY MEASURE.

164. ***Dry Measure*** is used in measuring articles not liquid; as grain, salt, vegetables, etc.

165. The ***Unit*** of Dry Measure is the bushel. It contains 2150.42 cubic inches.

TABLE.

2 pints (pt.)	= 1 quart	qt.
8 quarts	= 1 peck	pk.
4 pecks	= 1 bushel	bu.

ORAL EXERCISES.

1. How many pints in a quart? In 2 qt.? In 6 qt.?
2. How many qt. in 20 pints? In 14 pt.? In 26 pt.?
3. How many qt. and pt. in 17 pt.? In 31 pt.?
4. How many pt. in 3 qt. 1 pt.? In 19 qt. 1 pt.?
5. How many qt. in a peck? In 9 pk? In 13 pk.?
6. How many pecks in 8 qt.? In 72 qt.? In 36 qt.?
7. How many pints in a peck? In 3 pk.? In 7 pk.?
8. How many pk. in 16 pt.? In 64 pt.? In 128 pt.?
9. How many pk. and qt. in 30 qt.? In 45 qt.?
10. How many pecks in a bushel? In 3 bu.? In 4 bu.?
11. How many bushels in 40 pk.? In 48 pk.? In 68 pk.?
12. How many quarts in a bushel? In 2 bu.? In 3 bu.?

WRITTEN EXERCISES.

Reduce:

1. 16 bu. 3 pk. 2 qt. 1 pt. to pints. *Ans.* 1077 pt.
2. 39 bu. 7 qt. to quarts. *Ans.* 1255 qt.
3. 8 bu. 3 pk. 1 pt. to pints. *Ans.* 561 pt.
4. 7 pk. 7 qt. to pints. *Ans.* 126 pt.
5. 12 bu. 3 pk. to quarts. *Ans.* 408 qt.
6. 1806 pt. to bushels. *Ans.* 28 bu. 7 qt.

7. 8679 qt. to bushels. *Ans.* 271 bu. 7 qt.
8. 176 pt. to pecks. *Ans.* 11 pk., or 2 bu. 3 pk.
9. 18090 qt. to bushels. *Ans.* 565 bu. 1 pk. 2 qt.
10. 2176 pt. to pecks, then to bushels.
Ans. 136 pk.; 34 bu.

MEASURES OF TIME.

166. ***Measures of Time*** are those used to measure periods of duration.

167. The ***Unit*** of this measure is the day.

TABLE.

60 seconds (sec.)	= 1 minute	min.
60 minutes	= 1 hour	hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
4 weeks	= 1 lunar month	l. mo.
365 days	= 1 common year	yr.
366 days	= 1 leap year	l. yr.
100 years	= 1 century	C.

NOTE.—All years divisible by 4, except centennial, and all centennial years divisible by 400 are leap years.

OTHER DENOMINATIONS.

52 weeks 1 day = 1 year.
13 lunar months, 1 day, 6 hours, = 1 year.
12 calendar months = 1 year.

CALENDAR.

1. January (Jan.)	31 days.	7. July	31 days.
2. February (Feb.), 28 or 29	"	8. August (Aug.)	31 "
3. March (Mar.)	31 "	9. September (Sept.)	30 "
4. April (Apr.)	30 "	10. October (Oct.)	31 "
5. May	31 "	11. November (Nov.)	30 "
6. June	30 "	12. December (Dec.)	31 "

ORAL EXERCISES.

1. How many seconds in a minute? In 5 min.? In 8 min.? In 10 min.? In 50 min.? In 40 min.?

2. How many minutes and seconds in 70 sec.? In 90 sec.? In 68 sec.? In 110 sec.?

3. How many seconds in 3 min. 5 sec.? 4 min. 30 sec.?

4. How many minutes in an hour? In 2 hr.? In 5 hr.?

5. How many hours and minutes in 90 min.? In 110 min.? In 85 min.? In 125 min.? In 170 min.?

6. How many hours in a day? In 5 days? In 7 days?

7. How many days in 48 hr.? In 120 hr.? In 96 hr.?

8. How many days in a week? In 30 wk.? In 70 wk.?

9. How many weeks in 14 days? In 56 da.? In 70 da.?

10. How many weeks in a month? In 3 mo.? In 7 mo.?

11. How many months in a year? In 3 yr.? In 9 yr.?

12. How many years in 48 mo.? In 60 mo.? In 132 mo.?

13. How many years and months in 20 mo.? In 37 mo.?

14. How many days in February? In June? In April?

15. How many days in January? In September? In March? In August?

16. How many days in October? In July? In November? In December?

17. How many months have 30 days? Name them.

18. What month has less than 30 days?

19. What months have 31 days?

WRITTEN EXERCISES.

Reduce:

1. 4 da. 7 hr. 20 min. 50 sec. to sec. *Ans.* 372050 sec.

2. 11 da. 31 min. 59 sec. to seconds. *Ans.* 952319 sec.

3. 11 hr. 10 min. 9 sec. to seconds. *Ans.* 40209 sec.

4. 1 yr. 1 da. 1 hr. to minutes. *Ans.* 527100 min.

5. 8 yr. 160 da. 7 hr. to hours.

Ans. Counting two leap years, 73975 hr.

6. 4 wk. 3 da. 1 hr. 10 min. to min. *Ans.* 44710 min.

7. How many seconds in March? *Ans.* 2678400 sec.

8. How many minutes in May? *Ans.* 44640 min.

9. How many hours in June? *Ans.* 720 hr.

Reduce:

10. 86400 seconds to days. *Ans.* 1 da.

11. 3780 minutes to days. *Ans.* 2 da. 15 hr.

12. 18080 hr. to years. *Ans.* 2 yr. 23 da. 8 hr.

13. 78000 seconds to hours. *Ans.* 21 hr. 40 min.

14. 1208 minutes to hours. *Ans.* 20 hr. 8 min.

15. 186078 hours to years. *Ans.* 21 yr. 88 da. 6 hr.

16. Find the number of days from March 21st to May 16th.

OPERATION.

31 da. — 21 da. = 10 da. Mar.
30 da. April.
16 da. May.
Ans. 56 da.

SOLUTION.—Since March has 31 da., and the given date is March 21st, there are ten days to the end of the month. The intervening month, April, has 30 da., which are added to the 10 da. of Mar. The second given date is May 16th; hence, 16 da. more are added to the 10 da. + 30 da., making 56 days, the number of days from March 21st to May 16th.

How many days from:

17. January 17th to March 13th? *Ans.* 55 da.

18. February 16th to May 1st (com. yr.)? *Ans.* 74 da.

19. July 4th to Sept. 3d? *Ans.* 61 da.

20. August 8th to December 23d? *Ans.* 137 da.

21. April 1st to October 27th? *Ans.* 209 da.

22. January 11th to February 9th? *Ans.* 29 da.

23. September 30th to Nov. 30th? *Ans.* 61 da.
24. March 26th to May 3d? *Ans.* 38 da.
25. What is the date for 50 days after March 7th?

OPERATION.

31 da. — 7 da. = 24 da.
50 da. — 24 da. = 26 da.
April 26th.

SOLUTION.—Since from March 7th to March 31st there are 24 da., the date will be 50 da. — 24 da., or 26 days after Mar. 31, which is April 26th.

What is the date for:

26. 63 days after May 4th? *Ans.* July 6th.
27. 45 days after April 19th? *Ans.* June 3d.
28. 16 days after September 28th? *Ans.* Oct. 14th.
29. 33 days after January 26th? *Ans.* Feb. 28th.
30. 210 days after March 17th? *Ans.* Oct. 13th.
31. 69 days after August 11th? *Ans.* Oct. 19th.
32. What date is 87 days before June 6th?

OPERATION.

87 da. — 6 da. = 81 da.
81 da. — 31 da. = 50 da.
50 da. — 30 da. = 20 da.
31 da. — 20 da. = 11 da.

SOLUTION.—Subtracting 6 da. from 87 da. we have 81 da., the number of days before May 31st. Taking 31 da. we have 50 da., the number of days before April 30th. Subtracting 30 da. we have 20 da., the number of days before March 31st. Hence, 87 da. before June 6th is March 11th.

What date is:

33. 47 days before March 3d? *Ans.* Jan. 15th.
34. 83 days before May 17th? *Ans.* Feb. 23d.
35. 93 days before July 6th? *Ans.* April 4th.
36. 60 days before April 19th? *Ans.* Feb. 18th.
37. 180 days before August 1st? *Ans.* Feb. 2d.
38. 308 days before December 21st? *Ans.* Feb. 16th.
39. 201 days before October 11th?
40. 252 days before September 10th?

CIRCULAR MEASURE.

168. ***Circular Measure*** is used to measure angles.

169. The ***Unit*** of Circular Measure is the degree. It is the $\frac{1}{360}$ of the circumference of any circle.

TABLE.

60 seconds (″)	= 1 minute	′
60 minutes	= 1 degree	°
360 degrees	= 1 circumference	C.

OTHER DENOMINATIONS.

30 degrees	= 1 sign.
60 degrees	= 1 sextant.
90 degrees	= 1 quadrant.
4 quadrants	= 1 circumference.
12 signs	= 1 circumference.

ORAL EXERCISES.

1. How many seconds in a minute? In 3′? In 5′?

2. How many minutes in a degree? In 2°? In 4°?

3. How many minutes in 300″? In 120″? In 480″?

4. How many degrees in 180′? In 300′? In 600′?

5. How many degrees in a sign? In 7 signs? In 11 signs? In 15 signs? In 40 signs?

6. How many degrees in a sextant? In 2 sextants?

7. How many degrees in a quadrant? In 3 quad.?

8. How many quadrants and degrees in 170°? In 250°? In 310°? In 205°?

9. How many signs in 60 degrees? In 90 degrees? In 45 degrees? In 70 degrees? In 140 degrees?

10. How many quadrants in a circumference? In 5 circumferences?

WRITTEN EXERCISES.

Reduce :

1. 13° 23′ 45″ to seconds. *Ans.* 48225″.
2. 180° 16″ to seconds. *Ans.* 648016″.
3. 1 quad. 45° 30″ to seconds. *Ans.* 486030″.
4. 270° 45′ to minutes. *Ans.* 16245′.
5. 2° 20′ 22″ to seconds. *Ans.* 8422″.
6. 3 signs to minutes. *Ans.* 5400′.
7. 15° 29′ 37″ to seconds. *Ans.* 55777″.
8. 1963000″ to higher denominations.
9. 96000″ to degrees. *Ans.* 26° 40′.
10. 38000″ to higher denominations.
11. 86078″ to degrees. *Ans.* 23° 54′ 38″.
12. 1897000″ to circumferences.

MISCELLANEOUS TABLES.

COUNTING.	PAPER.
12 units = 1 dozen.	24 sheets = 1 quire.
12 dozen = 1 gross.	20 quires = 1 ream.
12 gross = 1 great gross.	2 reams = 1 bundle.
20 units = 1 score.	5 bundles = 1 bale.

ORAL EXERCISES.

1. How many articles in a dozen? In 3 dozen? In 5 dozen? In 13 dozen?

2. How many articles in a ½ dozen? In 3½ dozen? In 7¼ dozen? In 6½ dozen? In 12 dozen?

3. How many dozen in a gross? In 6 gross? In 15 gross? In 30 gross? In 25 gross?

4. How many gross in 60 dozen? In 48 dozen? In 144 dozen? In 156 dozen? In 288 dozen?

5. How many score in 80? In 60? In 140? In 100? In 75? In 125? In 120?

6. How many years in 4 score 10? In 3 score 5?

7. How many sheets in a quire? In 3 quires? In 7 quires? In 2 quires? In 24 quires?

8. How much paper in 48 sheets? In 96 sheets? In 70 sheets? In 60 sheets?

9. How many quires in a ream? In 30 reams? In 16 reams? In 20 reams? In 25 reams?

10. What cost a ream of paper at 2 cents a sheet?

WRITTEN EXERCISES.

1. How many pencils in a box containing 2 great gross? *Ans.* 3456 pencils.

2. What cost 27 boxes of writing ink, each containing 2½ dozen bottles, at 9 cents a bottle? *Ans.* $72.90.

3. How many reams of paper in 4678 sheets?

4. What will 7 reams of legal cap cost at 35 cents a quire? *Ans.* $49.

5. What cost 9 boxes of fancy pen-holders, each containing ½ gross, at 2½ cents apiece? *Ans.* $16.20.

MENTAL REVIEW.

1. What cost 2 oz. of gold, if 3 pwt. cost $2.70?

2. What will 2 quarts of kerosene cost at 40 cents a gallon? At 20 cents a gallon? At 50 cents a gallon?

3. What will 3 quarts of tomatoes cost at $1.60 a bushel? 10 quarts? 25 quarts?

4. How many feet high is a horse 16 hands high?

5. What is the difference between two square feet and two feet square?

6. At 8 cents a peck, how many bushels of apples can be bought for $6.00?

7. If 25 lb. of flour cost $1.25, what will 2 cwt. cost?

8. How many half-pint bottles may be filled from $2\frac{1}{2}$ gallons of wine?

9. What will 7 quires of paper cost at $3.20 a ream?

10. What will 8 eggs cost at 18 cents a dozen?

11. If 6 oz. of tea cost 36 cents, what will 3 lb. cost?

12. What will a gallon of molasses cost at 5 cents a pint? At 8 cents a pint? At 15 cents a pint?

13. At 8 shillings a pair, how many pairs of shoes can be purchased for 2 sovereigns?

14. At what price must $\frac{1}{3}$ dozen of chairs, worth $15.00 a dozen, be sold in order to gain 50 cents apiece?

15. How much will a peddler gain by selling 3 dozen combs worth 30 cents a dozen, at 5 cents apiece?

16. What will $\frac{5}{8}$ of a lb. of candy cost at 2 cents an oz.? $\frac{3}{4}$ of a lb.? $\frac{1}{2}$ of a lb.?

17. How many tablespoons each weighing 2 oz., can be made from 1 lb. 8 oz. of silver?

18. How many leap years in a century?

19. How many pills of 5 grains each, can be made from $\frac{1}{2}$ an ounce of quinine?

20. If a gallon of wine cost $5.00, what will 3 pt. cost? $\frac{1}{2}$ of a pint? 3 quarts?

21. What will it cost to paint a ceiling 12 ft. by 20 ft., at 25 cents a square yard?

22. How many yards of carpeting, a yard wide, will cover a floor 20 ft. long and 21 ft. wide?

23. How many quarts of milk will a boy drink in a week, if he drink a pint a day?

24. At 5 cents a quart, how many bushels of chestnuts can I get for $3.20?

25. If the wages of 8 men for 5 days is $90, what does each man receive per day?

WRITTEN REVIEW.

1. What will it cost to ship 75 T. 8 cwt. 70 lb. of freight, at 6 cents a pound? *Ans.* $9052.20.

2. How many farms of 75 acres each, in a tract of land 6 miles long and 5 miles wide? *Ans.* 256 farms.

3. What is the height in feet of a horse 17½ hands high? *Ans.* 5 ft. 10 in.

4. What will be the cost of a cask of sugar weighing 9 cwt. 68 lb., at 6¼ cents a pound? *Ans.* $60.50.

5. A dozen silver tablespoons weighed 3 lb. 2 oz.; what did they cost at $2.75 per ounce? *Ans.* $104.50.

6. How much will 11 barrels of flour cost, at the rate of 5¼ cents a pound? *Ans.* $113.19.

7. If your age be 15 years and 7 months, how many minutes old are you, counting 3 leap years, and supposing you were born on Jan. 1st?

8. What cost 13 bushels of clover seed at 13½ cents a pound? *Ans.* $105.30.

9. How many minutes in the summer months?

10. What will 5 bushels of potatoes cost at 15 cents per half-peck? *Ans.* $6.00.

11. How many times will a wheel 8 ft. 3 inches in circumference, revolve in going 45 miles?

Ans. 28800 times.

12. How many half-pint, pint, and quart bottles, of each an equal number, can be filled from 2 barrels of wine?

13. How many rods of fence will inclose a farm a mile square? *Ans.* 1280 rods.

14. In what time will a person walk 8 miles, if he take a step 2 feet 8 inches long every second?

Ans. 4 hr. 24 min.

15. How many yards of carpeting $1\frac{1}{2}$ yards wide, will be required to carpet a room 35 feet long, by 16 feet wide? *Ans.* $41\frac{13}{17}$ yards.

16. In a pile of wood 50 feet long, 18 feet high, and 6 feet deep, how many cords? *Ans.* $42\frac{3}{16}$ cords.

17. What cost 3 T. 6 cwt. 78 lb. of iron at 11 cents a pound? *Ans.* $734.58.

18. A barrel of vinegar containing $31\frac{1}{2}$ gallons, was retailed at 12 cents a quart. How much was received for it? *Ans.* $15.12.

19. How much time will a person gain in 35 years by rising 35 minutes earlier than his usual time every day? *Ans.* 310 da. 16 hr. 45 min.

20. A farmer sold a load of corn weighing 2296 lb. at 56 cents a bushel, and a load of wheat weighing 2400 lb. at $1.15 per bushel. What did he receive for both? *Ans.* $68.96.

21. How long will it take a man to dig a plot of 2 acres, if he dig 80 sq. yd. a day? *Ans.* 121 da.

22. What will it cost to carpet a room 18 feet long by 16 feet wide, at $1.25 per sq. yd? *Ans.* $40.

23. A pile of wood is 8 feet high, 4 feet deep, and 188 feet long; what is its value at $6.50 a cord?

24. During a storm at sea a ship changed her course 280 geographical miles; how many degrees and minutes did she change? *Ans.* 4° 40′.

25. What will be the cost of plastering a room 24 feet long, 16 feet wide, and 12 feet high, at 25 cents a square yard, allowing 84 sq. ft. for windows and a door? *Ans.* $35.

26. How many days of 12 working hours each, will it take a person to count 1600000, at the rate of 80 in a minute? *Ans.* 27 da. 9 hr. 20 min.

27. How many demijohns containing 1 gal. 1 qt. 2 gi. can be filled from a hhd. of wine?

28. What will it cost to dig a cellar 25 feet long, 19 feet wide, and 7 feet deep at 23 cents a cu. yd.?

29. A grocer sold 4 hhd. of syrup, containing 60 gal. 1 qt. each, at 18 cents a quart. What did he gain dishonestly, if his quart measure was too small by one gill?
Ans. $21.69.

30. What will be the cost of excavating a ditch 4 miles long, with an average depth of 6 feet, and width of 4 feet, at 40 cents a cubic yard? *Ans.* $7509.33⅓.

31. A boy gathered 5 bu. 2 pk. of apples, which he sold at the rate of 5 for 12 cents. If each peck contained 2⅓ dozen apples, how much did he receive for the entire quantity? *Ans.* $14.78⅔.

32. If a brick measure 64 cubic inches, how many such bricks will it take to build a wall 60 feet long, 2 feet thick, and 12 feet high? *Ans.* 38880 bricks.

33. The front of a hall is 48½ feet wide and 24 feet high. What will it cost to paint it at the rate of 33 cents per square yard, allowing for 6 windows each 8 feet by 3 feet, and one door 14 ft. by 6 ft.? *Ans.* $34.32.

34. What will it cost to glaze 6 windows of 12 panes each, and each pane 14 inches by 10 inches; the glass being 35 cents per square foot? *Ans.* $24.50.

35. A garden 380 ft. long and 200 ft. wide is enclosed by a flagged walk 8 ft. in breadth. What is the area of the walk? *Ans.* 35 sq. rd. 7 sq. ft. 36 sq. in.

36. How many sods 16 in. by 9 in., will be required to turf a plot 64 ft. by 80 ft.? *Ans.* 5120 sods.

37. How many paving stones 8 in. by 5 in. will be required to pave a street 10 rods long by 60 feet wide?
Ans. 35640 stones.

38. What is a person's income for Jan., Feb., and Mar., at the rate of 3 cents a minute? *Ans.* $3888.

39. A man buys a barrel of beer (36 gal.) for $9.50, and retails it at 14 cents a quart. How much does he gain?

DENOMINATE FRACTIONS.

170. A ***Denominate Fraction*** is a fraction whose unit is denominate.

CASE I.

171. To reduce denominate fractions to integers of lower denominations.

ORAL EXERCISES.

1. How many ounces in $\frac{3}{4}$ of a pound avoirdupois?

SOLUTION.—Since in one pound there are 16 ounces, in $\frac{3}{4}$ of a pound there are $\frac{3}{4}$ of 16 ounces, or 12 ounces.

2. How many days in $\frac{3}{7}$ of a week? In $\frac{2}{3}$ of a week?

3. How many quarts in $\frac{7}{8}$ of a peck? $\frac{3}{4}$ of a peck?

4. How many quarts in $\frac{1}{2}$ of a gallon? In $\frac{1}{8}$ of a gallon?

5. How many pwt. in $\frac{3}{5}$ of an ounce? $\frac{1}{4}$ of an ounce?

6. How many cwt. in $\frac{3}{4}$ of a ton? $\frac{2}{5}$ of a ton? $\frac{3}{10}$ of a ton? $\frac{1}{4}$ of a ton? $\frac{4}{5}$ of a ton?

7. How many inches in .5 of a foot?

SOLUTION.—Since in 1 foot there are 12 inches, in .5 of a foot there are .5 of 12 inches, or 6 inches.

8. How many sq. rd. in .3 of an acre? .5 of an acre?

9. What is the value of .25 of an hour? .125 of an hour? .75 of an hour? $\frac{5}{12}$ of an hour?

10. How many pwt. and gr. in .3 of an oz.? In .7 of an oz.?

WRITTEN EXERCISES.

1. Reduce $\frac{3}{5}$ of a bushel to lower denominations.

OPERATION.

$\frac{3}{5}$ bu. $\times 4 = \frac{12}{5} = 2\frac{2}{5}$ pk.
$\frac{2}{5}$ pk. $\times 8 = \frac{16}{5} = 3\frac{1}{5}$ qt.
$\frac{1}{5}$ qt. $\times 2 = \frac{2}{5}$ pt.
$\frac{3}{5}$ bu. $= 2$ pk. 3 qt. $\frac{2}{5}$ pt.

SOLUTION.—Since 4 pk. make a bu., in $\frac{3}{5}$ of a bu. there are $\frac{3}{5} \times 4$ pk. $= \frac{12}{5}$ pk., or $2\frac{2}{5}$ pk. There being 8 qt. in a pk., $\frac{2}{5}$ of a pk. $= \frac{2}{5} \times 8$ qt., or $3\frac{1}{5}$ qt. As there are 2 pt. in a qt., in $\frac{1}{5}$ of a qt. there is $\frac{1}{5}$ of 2 pt., or $\frac{2}{5}$ of a pt. Hence, $\frac{3}{5}$ of a bushel $= 2$ pk. 3 qt. $\frac{2}{5}$ pt., *Ans.*

2. Reduce .375 of a yard to lower denominations.

SOLUTION.—Since there are three feet in a yard, .375 of a yard equals .375 times 3 ft. $= 1.125$ ft. To reduce .125 ft. to inches, multiply it by 12: .125 ft. $\times 12 = 1.5$ in. Hence, .375 yd. $= 1$ ft. $1\frac{1}{2}$ in., *Ans.*

Reduce to lower denominations:

3. $\frac{5}{8}$ of a rd. *Ans.* 3 yd. 1 ft. $3\frac{3}{4}$ in.
4. $\frac{3}{7}$ of a degree. *Ans.* 25′ $42\frac{9}{7}$″.
5. $\frac{8}{9}$ of a mo. *Ans.* 26 da. 16 hr.
6. $\frac{5}{6}$ of an A. *Ans.* 133 sq. rd. 10 sq. yd. 108 sq. in.
7. $\frac{1}{8}$ of a bbl. *Ans.* 3 gal. 3 qt. 1 pt. 2 gi.
8. $\frac{3}{5}$ of a cu. yd. *Ans.* 16 cu. ft. $345\frac{3}{5}$ cu. in.
9. $\frac{8}{9}$ of a lb. (Troy). *Ans.* 10 oz. 13 pwt. 8 gr.
10. $\frac{5}{6}$ of a lb. (Apoth.) *Ans.* 10 oz.
11. .214 of a T. *Ans.* 4 cwt. 28 lb.
12. .6 of a lb. (Troy). *Ans.* 7 oz. 4 pwt.
13. .3125 of a great gross. *Ans.* 45 doz.
14. .375 of a sign. *Ans.* 11° 15′.
15. .4267 of a bu. *Ans.* 1 pk. 5 qt. 1.308 pt.
16. .66$\frac{2}{3}$ of an A.
Ans. 106 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in.
17. .625 of a yr. *Ans.* 228 da. 3 hr

Case II.

172. To reduce a compound number to a fraction of a higher denomination.

ORAL EXERCISES.

1. What fraction of a pound in 12 ounces (avoir.)?

Solution.—Since there are 16 ounces in a pound, 12 ounces are $\frac{12}{16}$, or $\frac{3}{4}$ of a pound.

2. What part of an hour is 20 minutes? 45 min.? 10 min.? 50 min.? 24 min.? 15 min.?

3. What part of a bu. is 3 pk.? 6 qt.? 2 pk.?

4. What part of a foot is 3 in.? 9 in.? 7 in.? 11 in.?

5. What part of a £ is 4 s.? 9 s.? 12 s.? 15 s.?

6. What part of a gallon is 3 qt.? 2 pt.? 1 pt.?

7. What fraction of 6 lb. (Troy) is 18 oz.? 24 oz.? 9 oz.?

8. What fraction of an acre is 75 sq. rd.? 80 sq. rd.?

9. What part of 3 yd. is 8 ft.? 6 ft.? 5 ft.?

10. What part of a bu. is 2 pk. 3 qt.? 1 pk. 7 qt.?

WRITTEN EXERCISES.

1. Reduce 37 min. 30 sec. to the fraction of an hour.

OPERATION.

37 min. 30 sec. = 2250 sec.

1 hr. = 3600 sec.

$\frac{2250}{3600} = \frac{5}{8}$ hr., *Ans.*

Or,

30 sec. $= \frac{30}{60}$ min. $= \frac{1}{2}$ min.

$37\frac{1}{2}$ min. $= \frac{\overset{5}{\cancel{75}}}{\underset{4}{2 \times \cancel{60}}} = \frac{5}{8}$ hr., *Ans.*

Solution.—37 min. 30 sec. reduced to seconds give 2250 seconds; whereas in 1 hr. there are 3600 seconds; hence, 37 min. 30 sec. equal to $\frac{2250}{3600}$ hr. $= \frac{5}{8}$ hr. Or,

Since 60 sec. = 1 min., 30 sec. $= \frac{30}{60}$ min., or $\frac{1}{2}$ min.; which added to 37 gives $37\frac{1}{2}$ min. And since 60 min. = 1 hr., $37\frac{1}{2}$ min. $= \frac{75}{2 \times 60}$ hr. $= \frac{5}{8}$ hr., *Ans.*

2. Reduce 6 sq. ft. 108 sq. in. to a fractional sq. yd

OPERATION.

108 sq. in. ÷ 144 = .75 sq. ft.
6.75 sq. ft. ÷ 9 = .75 sq. yd., *Ans.*

SOLUTION.—Dividing 180 sq. in. by 144, the number of sq. in. in a sq. ft., we obtain .75 of a sq. ft.; which added to 6 sq. ft. gives 6.75. This divided by 9, the number of sq. ft. in a sq. yd., gives .75 sq. yd., *Ans.*

NOTE.—The fractional part may be obtained by either of the solutions in problem 1, and may then be reduced to the required decimal.

Reduce:

3. 8s. 8d. to the fraction of a £. *Ans.* $\frac{13}{30}$ of a £.
4. 9 oz. 15 pwt. to the fraction of a lb.
5. 1 ft. 4 in. to the fraction of a yd. *Ans.* $\frac{4}{9}$ of a yd.
6. 13 rd. 10 ft. 8 in. to the fraction of a mile.
7. 14s. 6d. to the decimal of a £. *Ans.* .725 of a £.
8. 18 lb. 10 oz. to the decimal of a cwt.

ADDITION.

173. ***Addition of Compound Numbers*** is the process of finding the sum of two or more similar compound numbers.

1. Find the sum of 5 T. 6 cwt. 73 lb.; 8 cwt. 80 lb.; 15 T. 3 cwt. 11 oz.; and 10 T. 8 cwt. 49 lb. 10 oz.

OPERATION.

T.	cwt.	lb.	oz.
5	6	73	0
	8	80	0
15	3	0	11
10	8	49	10
31	7	3	5

SOLUTION.—Arrange the quantities so that units of the same denominations may stand in the same column. Beginning with the lowest denomination, we find that 10 oz. + 11 oz. = 21 oz. (16 oz. = 1 lb.), or 1 lb. 5 oz. Place the 5 oz. in the ounce column, and add the 1 lb. to the lb. of the next column. 1 lb. + 49 lb. + 80 lb. + 73 lb. = 203 lb., (100 lb. = 1 cwt.) or 2 cwt. 3 lb. Writing the 3 lb. in the proper

column, we add the 2 cwt. to the column of cwt.; 2 cwt. + 8 cwt. + 3 cwt. + 8 cwt. + 6 cwt. = 27 cwt. (20 cwt. = 1 T.), or 1 T. 7 cwt. Placing 7 cwt. in the cwt. column, add 1 T. to the tons of the next column. 1 T. + 10 T. + 15 T. + 5 T. = 31 T. Having no higher denomination, we put down the entire amount. The sum then is 31 T. 7 cwt. 3 lb. 5 oz.

WRITTEN EXERCISES.

2. Find the sum of 45° 55′ 43″; 16° 45′ 54″; and 103° 57′ 8″. *Ans.* 166° 38′ 45″.

3. Find the sum of 17 da. 14 hr. 35 min.; 39 da. 8 hr. 50 min.; and 53 da. 13 hr. 25 min.

4. A London clothier's receipts for 5 weeks were as follows: £44 18s. 9d.; £99 19s. 10d. 2 far.; £39 8s. 8d.; £29 10s. 9d. 2 far.; £10 4s. 3d. 1 far. What was the entire amount? *Ans.* £224 2s. 4d. 1 far.

5. Find the sum of 18 T. 16 cwt. 50 lb. 8 oz.; 20 T. 10 cwt. 40 lb. 10 oz.; 67 lb. 11 oz.; 13 T. 60 lb. 15 oz.; 60 T. 7 cwt.; 103 T. 7 cwt. 78 lb. 9 oz.

Ans. 216 T. 2 cwt. 98 lb. 5 oz.

6. How much wine in four casks containing, respectively, 8 gal. 2 qt. 1 pt.; 10 gal. 3 qt. 3 gi.; 3 gal. 1 pt. 2 gi.; and 15 gal. 3 qt. 1 pt. 3 gi.?

7. Add 20 bu. 3 pk. 5 qt. 1 pt.; 18 bu. 2 pk. 7 qt.; 27 bu. 1 pt.; 6 qt. 1 pt.; 1 bu. 1 pk. 1 qt. 1 pt.; and 3 pk. 1 pt. *Ans.* 68 bu. 3 pk. 5 qt. 1 pt.

8. Add 8° 27′ 45″; 12° 30′ 17″; 90° 10′ 30″; 25° 14′ 35″; 190° 50′ 58″; and 10′ 45″.

9. Find the sum of 14 hhd. 30 gal. 2 qt.; 50 gal. 2 qt. 1 pt.; 18 hhd. 37 gal. 3 qt.; 10 hhd. 25 gal.; 1 hhd. 36 gal.; and 16 hhd. 13 gal. 4 qt.

10. Find the sum of 8 yr. 3 mo. 16 da.; 4 yr. 11 mo. 29 da.; 10 yr. 8 mo. 10 da.; 1 yr. 1 mo. 6 da.; and 9 mo. 11 da. *Ans.* 25 yr. 10 mo. 12 da.

11. Add 8 da. 3 hr. 4 min. 50 sec.; 16 hr. 4 min. 18 sec.; 10 da. 3 hr. 20 min.; and 20 da. 6 hr. 3 min. 30 sec. *Ans.* 39 da. 4 hr. 32 min. 38 sec.

12. Add 3 T. 4 cwt. 70 lb.; 3 cwt. 60 lb. 10 oz.; 3 T. 18 lb.; 14 cwt. 8 oz.; 8 T. 9 cwt. 16 lb. 14 oz.; and 18 lb. 12 oz. *Ans.* 15 T. 11 cwt. 84 lb. 12 oz.

13. A farmer sold 16 lb. 3 oz. of butter on Monday; 27 lb. 10 oz. on Tuesday; 10 lb. 9 oz. on Wednesday; 40 lb. 6 oz. on Thursday; 48 lb. 9 oz. on Friday; and 30 lb. 10 oz. on Saturday; how much butter did he sell in that week? *Ans.* 1 cwt. 73 lb. 15 oz.

14. Find the sum of 10 lb. 10 oz. 3 pwt. 8 gr.; 11 oz. 16 gr.; 3 lb. 8 oz. 16 pwt.; 9 oz. 6 pwt. 14 gr.; 10 lb. 8 oz. 10 pwt.; and 13 lb. 6 oz. 10 pwt. 18 gr.

Ans. 40 lb. 6 oz. 7 pwt. 8 gr.

15. Add 8 m. 80 rd. 5 yd. 1 ft.; 2 m. 17 rd. 3 yd. 2 ft.; and 11 m. 13 rd. 2 yd. 2 ft.

16. A pile of wood contains 50 cd. 16 cu. ft.; another 90 cd. 75 cu. ft.; a third 100 cd. 100 cu. ft.; and a fourth 75 cd. and 110 cu. ft.; how much wood in the four piles?

Ans. 317 cd. 45 cu. ft.

17. How much land in 5 farms containing, respectively, 186 A. 3 sq. rd.; 100 A. 40 sq. rd.; 163 A. 130 sq. rd.; 42 A. 16 sq. rd.; and 150 A. 100 sq. rd.?

Ans. 1 sq. m. 2 A. 129 sq. rd.

18. Find the sum of 31 cu. yd. 10 cu. ft. 378 cu. in.; 444 cu. yd. 19 cu. ft. 520 cu. in.; 258 cu. yd. 16 cu. ft. 300 cu. in.; and 173 cu. yd. 25 cu. ft. 1000 cu. in.

19. A farmer raised 10 bu. 3 pk. 5 qt. of corn in one field; 30 bu. 3 pk. 6 qt. in a second field; 25 bu. 7 qt. in a third field; and 70 bu. 6 pk. in a fourth field. How much corn did he raise in the four fields?

Ans. 138 bu. 2 pk. 2 qt.

SUBTRACTION.

174. ***Subtraction of Compound Numbers*** is the process of finding the difference between two similar compound numbers.

1. What is the difference between 90° 2′ 30″ and 5′ 38″?

OPERATION.

°	′	″
90	2	30
	5	38
89	56	52

SOLUTION.—The numbers are so arranged that units of the same denomination stand in the same column. As we cannot subtract 38″ from 30″ we add to this latter number 1 unit of the next higher denomination, which gives 30″ + 60″ = 90″; then 90″ minus 38″ leaves 52″. In order that the difference may not be changed, having "borrowed" 1′, it is necessary to add 1′ to 5′; but as 6′ cannot be subtracted from 2′, add 1° or 60′ to the 2′, which gives 62′. 62′ less 6′ equals 56′; and lastly, the 1° "borrowed," is taken from 90°, leaving a remainder of 89°, which completes the work.

WRITTEN EXERCISES.

2. From £50 6s. 8d. take £45 7s. 5d.

3. From a barrel of molasses, containing 59 gal. 2 qt. 1 pt., a grocer sold 37 gal. 3 qt. 1 pt.; how much had he left? *Ans.* 21 gal. 3 qt.

4. A tree 16 yd. 2 ft. 3 in. high, was struck by lightning and broken off 6 yd. 1 ft. 10 in. from the top. What was the height of the part left standing?

5. How much remains of 50 bushels of corn, after selling 36 bu. 2 pk. 5 qt.? *Ans.* 13 bu. 1 pk. 3 qt.

6. From a barrel of wine containing 31 gal. 2 qt., there leaked out 11 gal. 3 qt.; how much remained?

7. How much remains of a farm of 175 A. 16 sq. rd. 20 sq. yd., after selling 90 A. 27 sq. rd. 9 sq. yd.?

DIFFERENCE BETWEEN DATES.

8. Find the difference in time from October 27th, 1860, to March 31st, 1883.

OPERATION.

yr.	mo.	da.
1883	3	31
1860	10	27
22	5	4

SOLUTION.—Observing that October is the 10th month, and March is the 3d, we arrange the denominations in order, and subtract, 30 days being counted to a month and 12 months to a year.

Find the difference in time:

9. From May 10th, 1876, to November 10th, 1876.
10. From April 19th, 1775, to July 4th, 1776.
11. From July 4th, 1776, to October 19th, 1781.
12. From April 14th, 1861, to April 26th, 1865.
13. From May 7th, 1873, to March 11th, 1878.
14. From June 6th, 1880, to Sept. 2d, 1882.
15. From Sept. 17th, 1879, to Feb. 8th, 1883.
16. From Jan. 11th, 1864, to May 7th, 1881.
17. From July 4th, 1880, to March 4th, 1883.
18. From Feb. 9th, 1860, to April 7th, 1883.
19. From Oct. 11th, 1863, to June 9th, 1882.

LONGITUDE AND LATITUDE

175. ***The Longitude*** of a place is its distance east or west of a given meridian.

176. ***The Latitude*** of a place is its distance north or south of the equator.

177. Longitude and Latitude are reckoned in degrees, minutes, and seconds. The highest degree of longitude is 180 degrees; and the highest degree of latitude is 90 degrees.

TABLE

Of the Longitude (meridian of Greenwich) of a few important places.

1. Albany, 73° 44′ 50″ W.
2. Boston, 71° 3′ 30″ W.
3. Berlin, 13° 23′ 45″ E.
4. Cincinnati, 84° 29′ 31″ W.
5. Chicago, 87° 37′ 45″ W.
6. Constantinople, 28° 59′ 0″ E.
7. Jerusalem, 35° 32′ 0″ E.
8. London, 0° 51′ 38″ W.
9. Mexico, 99° 5′ 0″ W.
10. Montreal, 73° 25′ 0″ W.
11. New York, 74° 0′ 3″ W.
12. New Orleans, 90° 2′ 30″ W.
13. Paris, 2° 20′ 22″ E.
14. Philadelphia, 75° 10′ 0″ W.
15. Pekin, 116° 23′ 45″ E.
16. Rome, 12° 27′ 14″ E.
17. S. Francisco, 122° 26′ 45″ W.
18. St. Louis, 90° 15′ 15″ W.
19. Washington, 77° 0′ 15″ W.
20. St. Petersburg, 30° 19′ 0″ E.

TABLE

Showing the Latitude of a few places.

1. Buenos Ayres, 34° 3′ S.
2. Cape of Good Hope, 34° 22′ S.
3. Cape Horn, 55° 58′ 40″ S.
4. Charleston, S.C., 32° 46′ 33″ N.
5. Copenhagen, 55° 40′ N.
6. Cape Town, 34° 56′ S.
7. Dublin, 53° 21′ N.
8. Mozambique, 15° 2′ S.
9. New York, 40° 42′ 43″ N.
10. New Orleans, 29° 58′ N.
11. Paris, 48° 50′ N.
12. Philadelphia, 39° 57′ N.
13. Rome, 41° 54′ N.
14. St. Louis, 38° 27′ 28″ N.

1. Find the difference in longitude between Albany and Boston.

OPERATION.

Albany, 73° 44′ 50″ W.
Boston, 71° 3′ 30″ W.
Ans. 2° 41′ 20″

SOLUTION.—Arranging the denominations in order, we subtract, both places being on the same side of the meridian.

NOTES.—1. If the places be on different sides of the meridian, take their sum.

2. To obtain the difference of Latitude subtract, if both places are in the same direction from the equator; if in different directions, add.

3. When the difference of either latitude or longitude is found by adding, and the sum be greater than 180°, subtract it from 360°.

WRITTEN EXERCISES.

Find the difference in longitude between:

2. Boston and Cincinnati; Chicago and Mexico.
Ans. 13° 26′ 1″; 11° 27′ 15″.

3. Montreal and New York; Philadelphia and Washington. *Ans.* 35′ 3″; 1° 50′ 15″.

4. Constantinople and Jerusalem; Paris and Pekin.
Ans. 6° 33′; 114° 3′ 23″.

5. Berlin and London; Paris and New York.
Ans. 14° 15′ 23″; 76° 20′ 25″.

6. Rome and St. Louis; San Francisco and St. Petersburg. *Ans.* 102° 42′ 29″; 152° 45′ 45″.

Find the difference in latitude between:

7. Charleston, S. C., and Copenhagen; New Orleans and New York. *Ans.* 22° 53′ 27″; 10° 44′ 43″.

8. St. Louis and Mozambique; Dublin and Paris.
Ans. 53° 29′ 28″; 4° 31′.

9. Buenos Ayres and Cape Town; Cape Horn and Rome. *Ans.* 53′; 97° 52′ 40″.

10. Philadelphia and Charleston, S. C.; Philadelphia and Cape of Good Hope. *Ans.* 7° 10′ 27″; 74° 19′.

MULTIPLICATION.

178. ***Multiplication of Compound Numbers*** is the process of finding the product when the multiplicand is a compound number.

1. What are the contents of 5 cisterns, each containing 50 gal. 2 qt. 1 pt.?

OPERATION.

gal.	qt.	pt.
50	2	1
		5
253	0	1

SOLUTION.—Commence at the right. 5 times 1 pt. gives 5 pt. (2 pt. = 1 qt.), or 2 qt. and 1 pt. Write 1 pt. and carry 2 qt.; 5 times 2 qt. gives 10 qt. and 2 qt. carried make 12 qt. (4 qt. = 1 gal.), or 3 gal. Having no remainder, place 0 in the place of qt.; 5 times 50 gal. = 250 gal., and 3 gal. carried = 253 gal.; which completes the product.

WRITTEN EXERCISES.

2. Multiply £137 18s. 4½d. by 25.

Ans. £3447 19s. 4½d.

3. Multiply 8 da. 6 hr. 7 min. 48 sec. by 84.

Ans. 1 yr. 328 da. 10 hr. 55 min. 12 sec.

4. Multiply 8 yd. 2 ft. 10 in. by 18. *Ans.* 161 yd.

5. Multiply £12 13s. 10d. by 180. *Ans.* £2284 10s.

6. Multiply 3 lb. 6 oz. 7 dr. 4 scr. by 16.

Ans. 57 lb. 4 oz. 5 dr. 1 scr.

7. Multiply 16° 9′ 8″ by 20. *Ans.* 323° 2′ 40″.

8. Multiply 80° 30′ 30″ by 4. *Ans.* 322° 2′.

9. What are the contents of 6 casks of wine each containing 67 gal. 3 qt. 1 pt. 2 gi.?

Ans. 407 gal. 2 qt. 1 pt.

10. How much coal in 3 cars, each containing 6 T. 8 cwt. 75 lb.? *Ans.* 19 T. 6 cwt. 25 lb.

11. If a spoon contain 3 oz. 6 pwt. 20 gr. of silver, what will 12 spoons weigh? *Ans.* 3 lb. 4 oz. 2 pwt.

12. If a man travel 35 m. 101 rd. 1½ ft., consecutively, for 6 days, how far has he travelled?

Ans. 211 m. 286 rd. 9 ft.

13. What is the total weight of 32 sheep, having an 'ge weight of 69 lb. 11 oz.? *Ans.* 22 cwt. 30 lb.

How far will a railroad train go in 16 hr. at the f 35 m. 103 rd. 1½ yd. per hr.?

Ans. 565 m. 52 rd. 2 yd.

15. How much wine in 12 casks, each containing 37 gal. 3 qt. 1 pt.? *Ans.* 454 gal. 2 qt.

16. If a steam-boat run a mile in 5 min. 26 sec., how long will it take to make a trip of 98 m.?

Ans. 8 hr. 52 min. 28 sec.

17. If a meteor move 6° 27′ 50″ in a second, how far will it move in 25 seconds? *Ans.* 161° 35′ 50″.

18. How much hay in 13 loads, of 1 T. 3 cwt. 48 lb. each? *Ans.* 15 T. 5 cwt. 24 lb.

19. In 12 bins, with a capacity of 200 bu. 3 pk. 6 qt. each, how much corn? *Ans.* 2411 bu. 1 pk.

20. How much land in 37 farms of 16 A. 6 sq. rd. 20 sq. yd. each? *Ans.* 593 A. 86 sq. rd. 14 sq. yd.

DIVISION.

179. ***Division of Compound Numbers*** is the process of finding the quotient when the dividend is a compound number.

1. Divide £113 1s. 10d. 2 far. by 21.

OPERATION.

```
      £   s.  d.  far. £  s.  d. far.
21 ) 113  1   10   2 ( 5,  7,  8,  2.
     105
      8                21 ) 178 ( 8
      20                     168
21 ) 161 ( 7                  10
     147                       4
      14               21 ) 42 ( 2
      12                     42
      178
```

SOLUTION.—Dividing £113 by 21 we obtain £5, and a remainder of £8, which, reduced to shillings, and added to 1s. of the dividend, gives 161s. This divided by 21 gives 7s. and a remainder of 14s. Reducing this last to pence and adding it to 10d. gives 178d.; which divided by 21 gives 8d. and a remainder of 10d. 10d. = 40 far. And 40 far. added to 2 far. gives 42 far. This divided by 21 gives 2 far. and no remainder.

WRITTEN EXERCISES.

2. Divide 309° 27′ 30″ by 25. *Ans.* 12° 22′ 42″.

3. Divide 71 lb. 0 oz. 8 pwt. by 16.
Ans. 4 lb. 5 oz. 5 pwt. 12 gr.

4. Divide £650 1s. 4d. by 196. *Ans.* £3 6s. 4d.

5. Divide 524 gal. 2 qt. 1 pt. 2 gi. by 365.
Ans. 1 gal. 1 qt. 1 pt. 2 gi.

6. Divide 32 bu. 3 pk. 6 qt. by 24.
Ans. 1 bu. 1 pk. 3 qt. $1\frac{5}{8}$ pt.

7. Divide 19 cwt. 88 lb. 12 oz. by 10.
Ans. 1 cwt. 98 lb. 14 oz.

8. Divide £183 10s. $1\frac{1}{2}$d. by 18.
Ans. £10 3s. $10\frac{3}{4}$d.

9. 11 lb. 1 oz. 4 pwt. of silver were used in making 18 goblets; how much silver was used for each?
Ans. 7 oz. 8 pwt.

10. How many packages, each containing 5 oz., can be filled from 95 lb. of cinnamon? *Ans.* 304 packages.

NOTE.—When both dividend and divisor are compound numbers reduce them to the same denomination and divide as in simple numbers.

11. A vessel sailed 258 m. 196 rd. 9 ft. in 21 hours; how far was that per hour? *Ans.* 12 m. 100 rd. 13 ft.

12. Divide 1275 A. 96 sq. rd. 22 sq. yd. 8 sq. ft. 32 sq. in. equally among 32 persons.
Ans. 39 A. 138 sq. rd. 6 sq. ft. 64 sq. in.

If a town 6 miles square be divided into 165 farms, any acres will each farm contain?
Ans. 139 A. 101 sq. rd. 24 sq. yd. 6 sq. ft. 108 sq. in.

How many bins, each containing 65 bu. 1 pk. 7 qt., will be required to contain 785 bu. 3 pk. 2 qt.?
Ans. 12 bins.

15. How far will a meteor move in a second, if it move 242° 26′ 15″ in 45 seconds? How far will it move in 25 seconds?

16. How many loads of wood in 69 cd. 12 cu. ft., each load containing 2 cd. 12 cu. ft.? *Ans.* 33 loads.

17. If a man travel 3 m. 108 rd. in an hour, how long will it take him to travel 36 m. 228 rd.?

Ans. 11 hours.

18. Divide 114 cwt. 30 lb. 8 oz. by 8.

Ans. 14 cwt. 28 lb. 13 oz.

19. The aggregate weight of 67 hhd. of sugar is 28 T. 18 cwt. 21 lb. What is the average weight of a hhd.?

Ans. 8 cwt. 63 lb.

20. From 10 lb. 8 oz. 6 dr. 1 scr. of camphor, how many packages can be put up, each containing 3 dr. 2 scr.?

Ans. 281 packages.

LONGITUDE AND TIME.

180. As there are 360° in a circumference, and the earth makes a revolution on its axis once every 24 hours, it follows that in every hour the earth proceeds 15°; in a minute $\frac{1}{60}$ of 15°, or 15′; and in a second $\frac{1}{60}$ of 15′, or 15″. Hence, also, the

TABLE.

1 hour of time = 15° of longitude.
1 minute of time = 15′ of longitude.
1 second of time = 15″ of longitude.

NOTE.—The *Standard Time* of the United States and Canada is divided into the Eastern, Central, Mountain, and Pacific time. It is reckoned from the four meridians, 75°, 90°, 105°, and 120° west of Greenwich. As 15° of longitude equal 1 hour of time, Eastern time is 5 hours slower than London time.

Case I.

181. To find the difference in time between two places when the difference in longitude is given.

1. The difference of longitude between two places is 70°; what is the difference in time?

OPERATION.

15) 70° 0′

4 hr. 40 min.

Solution.—Since 15° of longitude are equal to one hour of time, 70° of longitude are equal to 4 hours, and a remainder of 10°, or 600′. 600′ are equal to 40 minutes, since 15′ of longitude are equal to 1 minute. Hence, the difference in time is 4 hr. 40 min.

WRITTEN EXERCISES.

Find the difference in time between two places whose difference of longitude is:

2. 18° 40′. *Ans.* 1 hr. 14 min. 40 sec.
3. 150°. *Ans.* 10 hr.
4. 28° 18′ 15″. *Ans.* 1 hr. 53 min. 13 sec.
5. 30° 40′ 30″. *Ans.* 2 hr. 2 min. 42 sec.
6. 100° 20′. *Ans.* 6 hr. 41 min. 20 sec.
7. 80° 45′. *Ans.* 5 hr. 23 min.

Find the difference in time between:

8. Albany and Montreal; Constantinople and Jerusalem. *Ans.* 1 min. 19 sec.; and 26 min. 12 sec.

9. Mexico and London; New Orleans and San Francisco. *Ans.* 6 hr. 32 min. 53 sec.; 2 hr. 9 min. 37 sec.

10. St. Louis and Berlin; Philadelphia and Pekin. *Ans.* 6 hr. 54 min. 36 sec.; 12 hr. 46 min. 15 sec.

11. Washington and Paris; St. Petersburg and Paris. *Ans.* 5 hr. 17 min. 22 sec.; 1 hr. 51 min.

12. New York and Cincinnati; Chicago and Boston.

Ans. 41 min. 57 sec.; and 1 hr. 6 min. 17 sec.

13. What change must be made in our watches, if we travel from Berlin to Paris?

Ans. Set back 44 min. 13 sec.

NOTE.—To find the time of a place *east*, add the difference of time between the two places to the given time. But to find the time of a place *west*, subtract the difference of time between the two places from the given time.

14. What time is it in Boston when it is 4 o'clock P.M. in Mexico? *Ans.* 52 min. 6 sec. past 5 P.M.

15. When it is noon at Greenwich, what time is it at the first two places given in the table?

Ans. To first, 5 min. $\frac{2}{3}$ sec. past 7 A.M.

16. When it is half past seven A.M. in New York, what time is it in Philadelphia? New Orleans? Cincinnati? Paris? London?

Ans. To last, 22 min. $33\frac{2}{3}$ sec. past 12 noon.

17. What change would a person have to make in his watch, in going from Washington to London?

Ans. Set ahead, 5 hr. 4 min. 34 sec.

CASE II.

182. To find the difference of longitude when the time is given.

1. The difference in time between two places is 4 hr. 10 min.; what is the difference of longitude?

OPERATION.

4 hr. 10 min.
15
62° 30′

SOLUTION.—Since an hour of time equals 15° of longitude, and a minute of time equals 15′ of longitude, 15 times the number of hours and minutes will equal the difference of longitude in degrees and minutes. Hence, 15 times 4 hr. 10 min. equals 62° 30′.

WRITTEN EXERCISES.

Find the difference in longitude of two places whose difference of time is:

2. 3 hr. 20 min. *Ans.* 50°.

3. 36 min. *Ans.* 9°.

4. 4 hr. 22 min. *Ans.* 65° 30′.

5. 8 hr. 16 min. 24 sec. *Ans.* 124° 6′.

6. 6 hr. 6 min. 30 sec. *Ans.* 91° 37′ 30″.

7. It is 6 hr. 52 min. 40 sec. A.M. in Harrisburg, Penn., when it is noon at Greenwich; what is its longitude? *Ans.* 76° 50′.

8. When it is 8 o'clock A.M. in New Orleans, it is 9 hr. 5 min. 11 sec. in Albany; what is the difference of longitude? *Ans.* 16° 17′ 45″.

TEMPERATURE.

183. A ***Thermometer*** is an instrument for measuring variations of heat and temperature.

There are three methods of registering heat and temperature by means of thermometers.

Fahrenheit's, used in the United States, England, and Holland; Réaumer's, used in Russia, Spain, and Turkey; and Celsius's, called the Centigrade, used in France, Germany, Sweden, and some other countries of Europe.

In Fahrenheit's (F.) the freezing point is marked 32°, and the boiling point 212°.

In Réaumer's (R.) the freezing point is zero, and the boiling point 80°.

The Centigrade (C.) has the freezing point zero, and the boiling point 100°.

184. To reduce from one scale to another.

RULE I.—*To express degrees Fahrenheit in Réaumer, deduct 32 and take $\frac{4}{9}$ of the remainder; conversely, to change Réaumer to Fahrenheit, multiply by $\frac{9}{4}$ and add 32 to the product.*

Formulas: $R. = \frac{4}{9}(F. - 32°)$. $F. = \frac{9}{4}R. + 32°$.

ILLUSTRATIONS.

1. Express 59° F. in Réaumer's scale.

2. Express 24° R. in Fahrenheit's scale.

OPERATION *1.*	OPERATION *2.*
$59° - 32° = 27°$ F.	$\frac{9}{4} \times \overset{6}{\cancel{24}}°$ R. $= 54°$.
$\frac{4}{\cancel{9}} \times \overset{3}{\cancel{27}}°$ F. $= 12°$ R., *Ans.*	$54° + 32° = 86°$ F., *Ans.*

WRITTEN EXERCISES.

3. Express in Réaumer, 68° F.; 95° F.; 77° F.; 140° F.
Ans. 16° R.; 28° R.; 20° R.; 48° R.

4. Express in Fahrenheit, 32° R.; 76° R.; 8° R.; 50° R.
Ans. 104° F.; 203° F.; 50° F.; $144\frac{1}{2}$° F.

5. Express 47° F. in R.; 59° R. in F.; 67° R. in F.; 14° F. in R.; 10° R. in F.
Ans. $6\frac{2}{3}$° R.; $164\frac{3}{4}$° F.; $182\frac{3}{4}$° F.; -8° R.; $54\frac{1}{2}$° F.

6. Express 5° F. in R.; -4° R. in F.; 252° R. in F.
Ans. -12° R.; 23° F.; 599° F.

RULE II.—*To express degrees Fahrenheit in Centigrade, subtract 32 and take $\frac{5}{9}$ of the remainder; conversely, to change Centigrade to Fahrenheit, multiply by $\frac{9}{5}$ and add 32 to the product.*

Formulas: $C. = \frac{5}{9}(F. - 32°)$. $F. = \frac{9}{5}C. + 32°$.

ILLUSTRATIONS.

1. Express in Centigrade 77° F.

2. Express in Fahrenheit 75° C.

OPERATION *1.*	OPERATION *2.*
$77° - 32° = 45°$ F.	$\overset{15}{\cancel{75}}° \times \frac{9}{\cancel{5}} = 135°$.
$\frac{5}{\cancel{9}} \times \overset{5}{\cancel{45}}° = 25°$ C., *Ans.*	$135° + 32° = 167°$ F., *Ans.*

WRITTEN EXERCISES.

3. Express in Centigrade, 95° F.; 131° F.; 41° F.; 203° F. *Ans.* 35° C.; 55° C.; 5° C.; 95° C.

4. Express in Fahrenheit, 15° C.; 90° C.; 130° C.; 30° C. *Ans.* 59° F.; 194° F.; 266° F.; 86° F.

5. Express in C. 14° F. In C. 71° F. In F. 18° C. In F. 91° C. *Ans.* —10° C.; 21.67° C.; 64.4° F.; 195.8° F.

RULE III.—*To express degrees Réaumer in Centigrade, multiply by* $\frac{5}{4}$; *and, conversely, to change Centigrade to Réaumer, multiply by* $\frac{4}{5}$.

Formulas: $C. = \frac{5}{4}R.$ $R. = \frac{4}{5}C.$

ILLUSTRATIONS.

1. Express in Centigrade 60° R.

2. Express in Réaumer 50° C.

OPERATION *1.*	OPERATION *2.*
$\frac{5}{\cancel{4}} \times \overset{15}{\cancel{60}}° = 75°$ C., *Ans.*	$\overset{10}{\cancel{50}}°$ C. $\times \frac{4}{\cancel{5}} = 40°$ R., *Ans.*

WRITTEN EXERCISES.

3. Express in Centigrade 36° R.; 80° R.; 120° R.; 40° R. *Ans.* 45° C.; 100° C.; 150° C.; 50° C.

4. Express in Réaumer 80° C.; 160° C.; 320° C.; 60° C. *Ans.* 64° R.; 128° R.; 256° R.; 48° R.

PRACTICAL MEASUREMENTS.

185. These ***Practical Measurements*** include measures of surface, volume, capacity, lumber, wood, stone and brick work, shingles, laths, plastering, painting, and carpeting.

186. A ***Line*** is that which has only one dimension, *length*.

187. A ***Surface*** is that which has two dimensions, *length* and *breadth*. When it does not change its direction, it is called *plane*.

188. A ***Polygon*** is a portion of a plane surface bounded by straight lines.

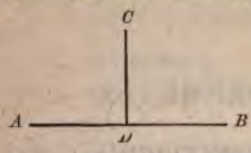
189. The ***Perimeter*** of a polygon is the sum of all its bounding lines, or sides.

190. The ***Area*** of a polygon is the surface included within its sides.

191. An ***Angle*** is the opening or inclination of two straight lines meeting each other in a common point, called the *vertex;* as *BAC*.

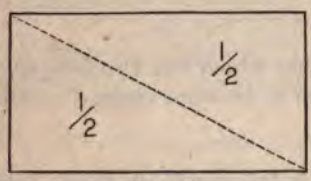
Angles are either *right, acute,* or *obtuse*. A *Right Angle* is an angle of 90 degrees. An *Acute Angle* is less than a right angle; and an *Obtuse Angle* is greater than a right angle.

192. A ***Perpendicular Line*** is a line drawn to meet another in such a manner as to incline no more to one side than to the other. The two adjacent angles thus formed are equal, and are called *Right Angles.*



In the figure, the line CD is perpendicular to AB, and the angles ADC and BDC are equal, and right angles.

193. A ***Rectangle*** is a polygon bounded by four straight lines and having four right angles.

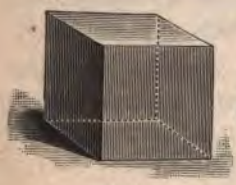


Rectangle.

Square.

194. A ***Square*** is a rectangle which has all its sides equal.

195. A ***Solid,*** or volume, is that which has length,



Cube.

Rectangular Solid.

breadth, and thickness, or height. The plane surfaces bounding a solid are called *Faces.*

196. A ***Cube*** is a solid bounded by six equal squares.

197. A ***Rectangular Solid,*** or parallelopipedon, is a volume bounded by rectangular faces; as, a box, a room, a block of marble, a load of wood.

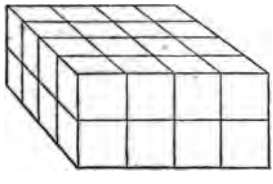
198. The ***Measurement*** of a line, of a surface, or of a solid consists in finding how many times it contains another magnitude of the same kind taken as a *unit of measure.* Thus, to say that a line is 6 feet long, is the same as to say that it contains the *unit of measure,* 1 foot, 6 times.

199. The ***Measuring Unit*** of a solid is a small cube, or solid, whose side is one inch, one foot, or any other convenient length. The number of times such a unit is contained in any solid, gives the *volume* or *solid contents* of that solid.

Fig. 1.

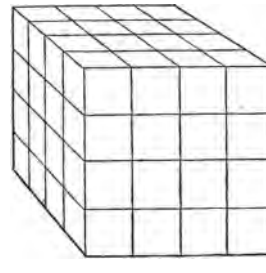
1 cu. ft.

Fig. 2.



32 cu. ft.

Fig. 3.



64 cu. ft.

Thus, the *measuring unit* of the solid represented in the margin, Fig. 3, is a small cube, Fig. 1, which is 1 ft. long, 1 ft. wide, and 1 ft. thick. Its solidity is equal to the product of its three dimensions, or 1 cu. ft.

The given solid, Fig. 3, may be divided into 2 equal parts, each of which is 4 ft. long, 4 ft. wide, and 2 ft. thick, as shown in Fig 2. These parts may be subdivided into 16 smaller solids, whose dimensions are the same as those of the unit of measure. Hence, the measure of the given solid equals 4 ft. × 4 ft. × 4 ft. = 64 cu. ft.

WRITTEN EXERCISES.

SURFACE.

1. How many trees planted 10 ft. apart, will be required to go round a quarter-section of land?

Ans. 1056 trees.

NOTE.—A Section of land is one mile square.

2. What is the cost of a piece of oil-cloth 30 ft. long by 18 ft. 6 in. wide, at 60¢ a square yard? *Ans.* $37.

3. How many telegraph poles, placed 20 yd. apart, will be required for a distance of 600 rods?

Ans. 166 poles.

4. Find the cost of 121000 feet of plank, at $28.25 per thousand feet. *Ans.* $3418.25.

5. Find the cost of 150 miles of telegraph wire at 35¢ a pound, reckoning 20 ft. to a pound. *Ans.* $13860.

6. What will it cost to kalsomine a ceiling 48 ft. by 25 ft. 4 in., at 8¢ a square yard? *Ans.* $10.81.

7. What will 5400 sq. rods of land cost, at $2.50 per square chain? *Ans.* $843.75.

8. How many garden plots, each containing 12 sq. yd., can be made out of a two-acre lot, allowing $\frac{1}{8}$ of the lot for walks? *Ans.* $705\frac{5}{6}$ plots.

9. What is the value of a township of land, at $140 a half-section? *Ans.* $10080.

NOTE.—A Township is six miles square.

10. How many quarter-sections are there in a piece of land 32 miles long by 5 miles wide?

Ans. 640 quarter-sections.

11. How many sq. rods in three quarter-sections of land? *Ans.* 76800 sq. rd.

12. How many sq. yd. in the walls and ceiling of a room 26 ft. long, 18 ft. wide, and 13 ft. high?

13. How much will it cost to pave a sidewalk 145 ft. long and 8 ft. wide, at 75¢ a sq. yd.?

14. How many posts, placed 8 ft. apart, are required for a fence round a garden 236 yards by 124 yards?

15. In No. 14, find the cost of the posts, at 15¢ apiece.

16. How many $2\frac{1}{2}$-inch pickets, placed 2 in. apart, will be required to fence a field 60 rods wide and 20 rods deep?

Ans. 7040 pickets.

17. What will it cost to enclose a lot, 130 rods by 50 rods, with 6 6-inch boards, at $8\frac{1}{2} per M.?

Ans. $151.47.

18. If the perimeter of a rectangular field be 220 yd., and the difference between its length and breadth 50 yd., what is the area? *Ans.* 2400 sq. yd.

19. How many scantlings, 15 ft. long, would be required for the two stringers of a fence around the field in No. 18? *Ans.* 88 scantlings.

20. How much lumber will be required to fence the field in No. 18, with a 12-inch base-board and 4 6-inch boards? How many posts placed 12 ft. apart?

Ans. 55 posts.

21. Find the total cost of the fence in No. 18, supposing the lumber to be $10\frac{3}{4} per M., the posts 12¢ each, and the labor $144. *Ans.* $171.89.

22. If an acre is cut from a field 30 rods long and 10 rods wide, what is the length of the part remaining?

Ans. 14 rods.

23. How many sq. yd. of block pavement in a street $\frac{3}{4}$ of a mile long and $1\frac{1}{4}$ chains wide?

24. What will it cost to slate a roof 50 ft. long by 20 ft. wide, at $4.25 per 100 sq. ft.? *Ans.* $42.50.

VOLUME AND CAPACITY.

25. How many cu. yd. of earth will be removed by digging a cellar 45 ft. long, 28 ft. wide, and 14 ft. deep?

Ans. $653\frac{1}{3}$ cu. yd.

26. What will be the cost of digging the cellar in No. 25, at 80¢ per cu. yd., and 60¢ a load for hauling?

Ans. $914.67.

NOTE.—A load of earth is equal to a cu. yd.

27. How many cu. ft. of air in a room 40 ft. 6 in. long, 28 ft. 4 in. wide, and 16 ft. 8 in. high?

Ans. 19125 cu. ft.

28. Find the contents of a bin of grain, which is 25 ft. long, 14 ft. wide, 3 ft. deep, and $\frac{3}{5}$ full.

29. How many cu. yd. of air for each of 32 students in a room 52 ft. long, 36 ft. wide, and 12 ft. high?

30. How many building bricks can be piled in a shed 125 ft. long, 70 ft. wide, and 48 ft. high?

NOTE.—Bricks used in building are $8\frac{1}{2}$ in. by 4 in. by $2\frac{1}{2}$ in.

31. How many loads of gravel, spread 10 in. deep, will be required for a road $2\frac{1}{4}$ miles long, and 12 ft. wide?

Ans. 4400 loads.

32. How many pounds of water in a tank $7\frac{1}{2}$ ft. long, 5 ft. wide, and $6\frac{1}{2}$ ft. deep, reckoning $62\frac{1}{2}$ pounds to the cubic foot? *Ans.* $15234\frac{3}{8}$ lb.

33. If a bin is 40 ft. long, 10 ft. wide, and 8 ft. deep, how many bushels of shelled corn will it contain?

NOTE.—A bushel of grain contains about $1\frac{1}{4}$ cu. ft. In heaped measure it contains about $1\frac{1}{2}$ cu. ft.

34. What must be the capacity of a bin that will hold 5400 bushels of potatoes? *Ans.* 6750 cu. ft.

35. How many bushels of unshelled corn in a wagon-box 8 ft. long, $3\frac{1}{2}$ ft. wide, and $1\frac{1}{2}$ ft. deep?

NOTE.—Two bushels of unshelled corn make one bushel of shelled corn.

36. How many bushels of turnips, heaped measure, can be stowed in a cellar $22\frac{1}{2}$ ft. long, 16 ft. wide, and $7\frac{3}{4}$ ft. deep? *Ans.* 1860 bu.

37. What must be the height of a school-room that is 20 ft. long by 16 ft. wide, to allow 140 cu. ft. of air to each of 32 pupils? *Ans.* 14 ft.

38. A tank is $2\frac{3}{4}$ ft. long, $1\frac{5}{6}$ ft. wide, and 21 inches deep; how many gallons of water will it contain?

39. At $62\frac{1}{2}$¢ a cu. yard, what will it cost to build a wall 84 ft. long, 15 ft. high, and 3 ft. 6 in. thick?

LUMBER AND WOOD

40. What is the cost of a pile of lumber, 24 ft. long, 4 ft. 6 in. wide, and 3 ft. 4 in. high, at $3 per cord?

41. How many cords of wood can be piled in a shed 64 ft. long, 28 ft. wide, and 20 ft. high?

Ans. 280 cords.

42. Find the cost of a pile of wood 244 ft. 5 in. long, 24 ft. wide, and 5 ft. 4 in. high, at $4.25 a cord, and $1.00 a load for hauling. *Ans.* $1147.40.

NOTE.—Reckon $2\frac{1}{4}$ cords to a load.

43. If a cord of 2-foot wood is 5 ft. high, what is its length? *Ans.* $12\frac{4}{5}$ ft.

44. What is the value of a pile of tan-bark 60 ft. long, 24 ft. wide, and 12 ft. high, at $2.00 a cord?

45. At $16 per thousand, what will it cost to cover a bridge 220 yd. long, and 22 ft. wide, with plank $1\frac{3}{4}$ inches thick? *Ans.* $406.56.

46. How many board-feet in a plank 15 ft. long, 16 in. wide, and $1\frac{1}{2}$ in. thick? How many cu. ft.?

NOTES.—1. A *board-foot* is reckoned 1 foot square and 1 inch thick, and is equal to $\frac{1}{12}$ of a cubic foot.

2. Even when the lumber is less than an inch in thickness, it is considered inch lumber.

3. In measuring the width, when a fraction is less than a half inch, it is rejected; but when it is greater than a half, it is counted a half inch.

47. How many feet of lumber in:

24 scantlings 12 ft. long, 6 in. wide, and $3\frac{1}{2}$ in. thick?

10 beams 16 ft. long, and 14 in. square?

30 joists 20 ft. long, 10 in. wide, and $1\frac{3}{4}$ in. thick?

60 pieces siding 8 ft. long, $5\frac{1}{4}$ in. wide, and 1 in. thick?

48. At $15.25 per thousand, find the total cost of:

7 boards 10 ft. long, 16 in. wide, and $2\frac{1}{2}$ in. thick.

3 boards 9 ft. long, $14\frac{3}{4}$ in. wide, and $3\frac{1}{4}$ in. thick.

5 boards 16 ft. long, 13 in. wide, and $\frac{7}{8}$ in. thick.

Ans. $6.50.

49. Find the cost of the $\frac{7}{8}$-inch flooring required for a room 36 ft. long, and 20 ft. wide, at 16\frac{1}{2}$ per thousand.

50. A garden is 15 rods long, and 12 rods wide; what will be the cost of a $2\frac{1}{2}$-in. plank sidewalk, 3 ft. wide, around the outside of it, at $18 per thousand?

51. A bridge is 300 yd. long, and 40 ft. wide; how many feet of 2-inch plank will be required for its two foot-paths, each of which is 8 ft. wide?

52. At $12 per thousand, find the cost of covering the whole bridge in No. 51, supposing the plank on the roadway to be $4\frac{1}{2}$ in. thick. *Ans.* $1512.

53. How many 4 x 5 rafters, 30 ft. long, and placed 21 in. apart, are required for a double roof 70 ft. long?

Ans. 82 rafters.

54. Find the cost of the rafters required for 3 roofs, having the same dimensions as the one in No. 53, at $14.50 per M. *Ans.* $178.35.

55. Find the cost of a close board fence, 75 rods long, 10 ft. high, and $1\frac{1}{2}$ in. thick, at $22.50 per M.

STONE AND BRICK WORK

56. How many perches of stone are there in a pile 325 ft. long, 50 ft. wide, and 16 ft. high?

NOTE.—A perch of stone = $16\frac{1}{2}$ ft. × $1\frac{1}{2}$ ft. × 1 ft. = $24\frac{3}{4}$ cu. ft., and a part of a perch is counted a whole one.

57. Find the cost of the stone required for a wall 25 ft. long, 12 ft. high, and 3 ft. thick, at $1.20 per perch.

58. A cellar is 32 ft. long, 20 ft. wide, and 15 ft. deep; what will be the cost of the stone required for its 20-in. walls, at $1.30 per perch? *Ans.* $137.80.

NOTE.—It is customary to measure around the outside of the wall.

59. How many cords of stone in the pile in No. 56, reckoning 128 cu. ft. to the cord? *Ans.* $2031\frac{1}{4}$ cd.

60. If a cord of stone will make 100 cu. ft. of wall, how many cords will be required for the walls in No. 58?

61. A stone house is 60 ft. front, 125 ft. deep, and 100 ft. high; the cellar walls are 15 ft. high, and 3 ft. thick. What did they cost, at 85¢ a cu. yd.?

62. Find the cost of the remainder of the stone work in No. 61, at 75¢ a cu. yd., supposing one half of it to be $2\frac{1}{2}$ ft. thick, and the other half $1\frac{1}{2}$ ft. thick.

63. Stone-masons reckon a cu. yd. of sand and 3 bu. of lime to a cord of stone. How much of each will be required for the cellar walls in No. 61?

64. Find the number of bricks required for a wall 80 ft. long, 10 ft. high, and $1\frac{1}{2}$ ft. thick.

NOTES.—1. The dimensions of a brick are about 8½ in. by 4 in. by 2½ in.

2. To a cu. ft. we commonly reckon 27 bricks laid dry, or 20 bricks laid in mortar.

65. At $10.50 per thousand, what is the value of a pile of bricks 70⅚ ft. long, 48 ft. wide, and 15 ft. high?

Ans. $14458.50.

66. How many bricks, laid 8 in. by 4 in., will pave a walk 90 ft. long, and 15 ft. wide, with no allowance for edges or waste? *Ans.* 6075 bricks.

67. How many bricks are required for a wall 32 ft. long, 16 ft. high, and half a brick thick?

NOTE.—In *half-brick* walls, each brick has an external surface of 9 in. by 3 in.; in *single-brick* walls, 4½ in. by 3 in., or 14 in. of surface; in *a-brick-and-a-half* walls, 10 in. of surface; and in *double-brick* walls, 7 in. of surface.

68. At $8 per thousand, find the cost of the bricks required for the double-brick walls of a basement, 75 ft. long, 40 ft. wide, and 12 ft. high. *Ans.* $454.22.

69. How much less would the bricks in No. 68 cost, if the walls were a brick and a half thick?

70. How many bricks will be required for a single brick wall 60 ft. long, and 84 ft. high?

71. The front wall of a house is 125 ft. long, 75 ft. high, and 2 bricks thick. Find the cost of the bricks required, at $9 per M., allowing for 2 doors and 24 windows, each 8 ft. by 4 ft. *Ans.* $1581.674.

72. Find the number of bricks required for one course of a chimney with a flue one foot square.

Ans. 8 bricks.

NOTE.—The walls of an ordinary chimney are 4 inches thick, the bricks being laid on their broadest face.

73. A chimney is 50 ft. high, and its flue $7\frac{1}{2}$ in. by 4 in.; how many bricks were used in its construction, reckoning a mortared brick 3 in. thick?

74. Find the cost of the bricks for a chimney 30 ft. high, and having a 10-inch flue, at $12 per M.

SHINGLES AND LATHS.

75. How many bunches of shingles are required for a double roof 30 ft. long, with 16-ft. rafters?

NOTE.—1000 shingles make 4 bunches, and cover 100 sq. ft. of surface. They are 4 in. wide, are generally laid 4 in. to the weather, and are sold only in whole bunches.

76. Find the number of shingles required for a double roof 25 ft. long, with 20-foot rafters.

77. Find the cost of the shingles in No. 76, at $4.25 per thousand. *Ans.* $42.50.

78. A roof is $87\frac{1}{2}$ ft. long, and 30 ft. wide; what will it cost to shingle it, at $4 per M.?

79. How many bunches of shingles are required for the roof in No. 78? *Ans.* 105 bunches.

80. In No. 78, how much, more or less, would it cost to use metallic shingles, at $5.25 per thousand, if each shingle covers 20 sq. in. of surface? *Ans.* $5.77 less.

81. A room is 16 ft. long, 14 ft. wide, and 12 ft. high; how many bunches of laths are required for its walls and ceiling? *Ans.* 35 bunches.

NOTE.—Laths are 4 ft. long, $1\frac{1}{2}$ in. wide, and are laid $\frac{3}{8}$ of an inch apart. A bunch contains 50, and will cover three sq. yd. of surface.

82. How many bunches of laths are required for a room 75 ft. long, 56 ft. wide, and 16 ft. high, allowing for 3 doors and 12 windows, each 14 ft. by 6 ft.?

NOTE.—Deduct only one half of the surface of the openings.

83. Find the cost of the laths in No. 82, at 25¢ a bunch. *Ans.* $72.

84. At 40¢ a bunch for laths, 5¢ a sq. yd. for labor, and 8¢ a pound for nails, allowing a half-pound to a bunch, what will it cost to lath a hall 124 ft. long, 60 ft. wide, and 50 ft. high? *Ans.* $565.08.

85. What would be the difference in the cost of the laths in No. 84, if they crossed each other?

PLASTERING, PAINTING, CARPETING, ETC.

86. Find the cost of plastering the walls and ceiling of a room 87½ ft. long, 54 ft. wide, and 15 ft. high, at 21¢ per sq. yd. *Ans.* $209.30.

87. A room is 40 ft. long, 25 ft. wide, and 14 ft. high. It has 2 doors, and 10 windows, each 12 ft. by 6 ft. What will it cost to plaster it, at 18¢ per sq. yd.?

NOTE.—Allow for only half the surface of the openings.

88. What will it cost to plaster 3 rooms, each of which is 18 ft. square, and 12 ft. high, @ 22½¢ per sq. yd.?

89. At 16¢ a sq. yd., find the cost of painting the walls in No. 88. *Ans.* $46.08.

90. What will it cost to kalsomine the ceilings in No. 88, @ 8½¢ a sq. yd.? *Ans.* $9.18.

91. Find the cost of painting a double roof 56 ft. by 35 ft., @ 18¢ a sq. yd. *Ans.* $78.40.

92. How many rolls of paper are required for the ceiling of a room 23 ft. long and 16 ft. wide?

NOTE.—A roll of wall-paper is 24 ft. long, and 1½ ft. wide. A double roll is 48 ft. long. Dealers will not sell a part of a roll.

93. Find the number of strips of paper required for the walls of a room 15 ft. long, 12 ft. wide, and 12 ft. high, allowing for 4 windows and 1 door.

NOTE.—In estimating, paper-hangers divide the distance around the room, after deducting 3 ft. for each opening, by $1\frac{1}{2}$ ft., the width of a roll.

94. How many rolls will paper a room 42 ft. long, 30 ft. wide, and 16 ft. high, with 10 windows and 4 doors? *Ans.* 46 rolls.

95. Find the cost of the paper in No. 94, at 65¢ per double roll. *Ans.* $14.95.

96. In No. 94, how much would be gained by using single rolls, at 25¢ per roll? *Ans.* $3.45.

97. Find the cost of papering a room 22 ft. long, 18 ft. wide, and 9 ft. high, with 5 openings and a cornice, at 50¢ per double roll. *Ans.* $4.00.

NOTE.—An allowance of one foot is made for a cornice.

98. In No. 97, find the cost of a border, @ 8¢ a yd.

99. At 15¢ a roll, and 5¢ a yd. for border, what will it cost to paper the walls of a hall 85 ft. long, 62 ft. wide, and 13 ft. high, with a cornice and 8 double openings?

100. A room is 26 ft. long and 18 ft. wide; how should the breadths of a carpet, one yard wide, be laid in order to have the least possible waste? *Ans.* 26 ft. long.

101. In No. 100, how many yd. would be required if the "breadths" were laid 18 ft. long? *Ans.* 54 yd.

102. A room is 35 ft. long, and 24 ft. wide; find the cost of carpeting it with carpet 27 in. wide, @ 80¢ a yd.

103. Find the cost of carpeting a room 16 ft. by 15 ft., with carpet 27 in. wide, at $1.25 a yard.

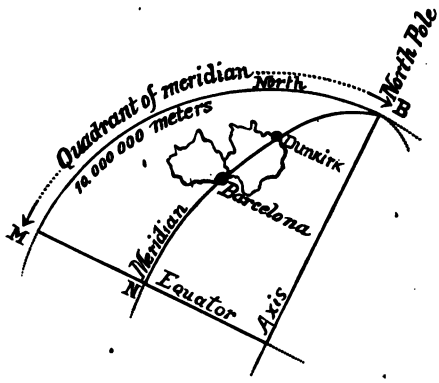
104. Allowing a waste of 6 in. per breadth in matching, find the cost of carpeting 4 rooms having the same dimensions as No. 103.

105. In No. 103, find the value of the carpet wasted by laying the breadths 15 ft. long. *Ans.* $5.83.

METRIC SYSTEM.

200. The ***Metric System*** is a series of measures whose units are derived from a fixed standard called the *Meter*.

201. The ***Meter*** is the *unit of length*. It is equal to the *ten-millionth* part of the quadrant of the terrestrial meridian, or 39.37 in.



This system of weights and measures is expressed on the decimal scale of notation. Having established the *unit* of each measure, we derive the other denominations by taking decimal multiples and divisions of it. Thus, there are five

PRINCIPAL UNITS.

The ***Meter*** (meter),	for measures	of	length.
The ***Are*** (air),	"	"	surface.
The ***Stere*** (stair),	"	"	volume.
The ***Liter*** (leeter),	"	"	capacity.
The ***Gram*** (gram),	"	"	weight.

The higher denominations of each measure are expressed by placing, successively, before the name of its unit the following Greek words:

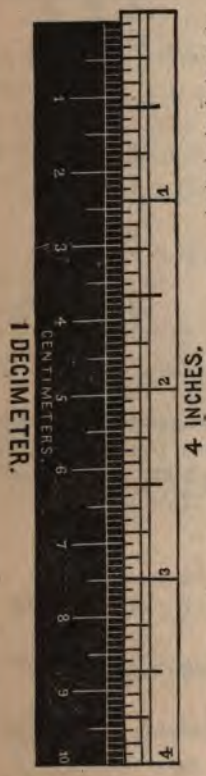
Deca	*Hecto*	*Kilo*	*Myria*
10,	100,	1000,	10,000,

The lower denominations, by the Latin prefixes,

Deci, $\frac{1}{10}$, *Centi*, $\frac{1}{100}$, *Milli*, $\frac{1}{1000}$.

Hence, the formation of the tables may be illustrated by the following diagram:

	Length.	*Surface.*	*Volume.*	*Capacity.*	*Weight.*
Milli					
Centi					
Deci					
——	METER.	AR.	STERE.	LITER.	GRAM.
Deca					
Hecto					
Kilo					
Myria					



MEASURES OF LENGTH.

202. In measures of length the principal unit is the ***Meter.*** It is equivalent to 39.37 inches, or 1.0936 + yards.

TABLE.

10 millimeters (mm.)	= 1 centimeter...	cm.
10 centimeters	= 1 decimeter.....	dm.
10 decimeters	= 1 ***meter***.....	m.
10 meters	= 1 decameter....	Dm.
10 decameters	= 1 hectometer...	Hm.
10 hectometers	= 1 kilometer.....	Km.
10 kilometers	= 1 myriameter...	Mm.

The meter is used for measuring short distances; as, the length of a street, the height of a house, etc.

For great distances, as, the length of a canal, a railroad, etc., the unit is the *kilometer* or the *myriameter*.

To show the correspondence between the Metric System of nota-

tion and that of United States money, the table for linear measure may be arranged as follows:

U. S. MONEY.....	Ten Thous.-dols.	Thous.-dollars.	Hundred-dollars.	Ten-dollars.	Dollars.	.	Dimes.	Cents.	Mills.
METRIC SYSTEM....	Tens of thous.	Thousands	Hundreds	Tens.	Units.	.	Tenths	Hundredths	Thousandths
	5	6	4	2	8	.	3	1	7
	Mm.	Km.	Hm.	Dm.	M.		dm.	cm.	mm.

The unit, *meter*, corresponds to the unit, *dollar*, decimeter to dimes, centimeter to cents, etc.

The number is read 56428.317 *meters*. To express the same number in any other denomination, place the decimal point at the right of the required denomination. Thus, 564.28317 Hm. is read 564.28317 *hectometers;* 564283.17 dm., 564283.17 *decimeters.*

ORAL EXERCISES.

1. How many meters in a kilometer? In a hectometer? In a decameter?

2. How many decameters in a hectometer? In a myriameter?

3. What part of a meter is a decimeter? What part of a decimeter is a millimeter?

4. How many meters in 3 hectometers? In 5 kilometers?

5. How many mm. in 4 m.? In 6 m.? In 12 cm.?

6. How many m. in 100 dm.? In 2000 cm.?

7. Read each of the following: 6.5 dm.; 45 cm.; 251 Hm.; 12.64 Km.; 725.8 m.

8. Read 1283.14 m. in the different denominations that compose it.

WRITTEN EXERCISES.

1. Write 7 Dm. 5 m. 2 dm. in one number as meters.
Ans. 75.2 m.

In like manner, express:

2. 2 Km. 4 Hm. 5 Dm. 3 m. as decameters.
Ans. 245.3 Dm.

3. 8 Mm. 3 Km. 1 Hm. 4 dm. as meters.
Ans. 83100.4 m.

NOTE.—As in decimal notation, all vacant intervening orders are filled with ciphers.

4. 9 Hm. 3 dm. 8 mm. as meters. *Ans.* 900.308 m.

5. 3 Km. 7 Dm. 4 cm. as myriameters.
Ans. .307004 Mm.

6. Reduce 9.25 Km. to meters. *Ans.* 9250 m.

7. Reduce 1.2436 Hm. to decimeters.
Ans. 1243.6 dm.

8. Reduce 8956 m. to hectometers. *Ans.* 89.56 Hm.

9. Reduce 103.94 dm. to kilometers.
Ans. .010394 Km.

10. Reduce 342.75 Hm. to myriameters.
Ans. 3.4275 Mm.

11. In .208006 myriameters how many kilometers?

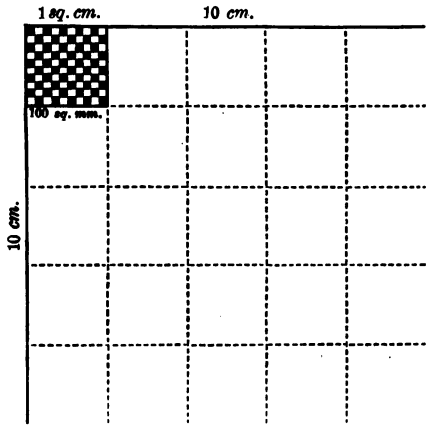
12. How many decimeters in 73.1804 hectometers?

13. In 89.725 myriameters how many kilometers?

14. Reduce 4 Hm. 3 Dm. 2 cm. to myriameters.

MEASURES OF SURFACE.

203. In measures of surface the principal unit is the ***Square Meter.*** It is divided into 100 square decimeters, and is equal to 1.196 sq. yd., or 10.764 sq. ft.



The accompanying diagram represents one fourth of a square decimeter. One hundred such decimeters equal 1 square meter.

As indicated by the lines of division, a square decimeter contains 100 square centimeters, and each of the latter is equal to 100 sq. millimeters.

10 cm. × 10 cm. = 100 sq. cm. = 1 sq. dm.
10 dm. × 10 dm. = 100 sq. dm. = 1 sq. m.

TABLE.

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter..	sq. cm.
100 sq. centimeters	= 1 sq. decimeter...	sq. dm.
100 sq. decimeters	= 1 ***sq. meter***.....	sq. m.
100 sq. meters	= 1 sq. decameter...	sq. Dm.
100 sq. decameters	= 1 sq. hectometer..	sq. Hm.
100 sq. hectometers	= 1 sq. kilometer...	sq. Km.

LAND MEASURE.

100 centares (ca.)	= 1 ***are***.....	a.
100 ares	= 1 hectare.....	Ha.

For surfaces of great extent, as the area of a country or state, the unit is the *square kilometer,*

In measuring land, the square meter is called the *centare;* the square decameter, an *are;* and the square hectometer, a *hectare.*

The standard unit in land measure is the *are.* For convenience, it is derived immediately from the decameter, instead of from the meter. It contains 100 sq. meters, or nearly 119.6 square yards.

ORAL EXERCISES.

1. How many sq. meters in a sq. decameter? In a sq. hectometer? In a sq. kilometer?

2. What part of a sq. centimeter is a sq. millimeter? What part of a sq. meter is a sq. decimeter?

3. How many centares in a hectare? Ares in a hectare?

4. What part of a hectare is an are? What part of an are is a centare?

WRITTEN EXERCISES.

1. Write 40 sq. Dm. 8 sq. m. as sq. meters.

SOLUTION.—As each denomination in square measure occupies two places, we must fill the vacant *tens'* place of sq. meters by a cipher. Hence, 40 sq. Dm. and 8 sq. m. equal 4008 sq. m.

2. Write 10 sq. Hm. 7 sq. Dm. 15 sq. m. 3 sq. dm. as square meters. *Ans.* 100715.03 sq. m.

3. Write 7 sq. Dm. 9 sq. m. 14 sq. dm. 5 sq. cm. as square centimeters. *Ans.* 7091405 sq. cm.

4. Write 5 Ha. 3 a. 4 ca. as ares. *Ans.* 503.04 a.

5. Express 540.32 a. as hectares. *Ans.* 5.4032 Ha.

NOTE.—When the *are* is the unit, move the point only as many places as indicated, but when the *square meter* is the unit, move it twice as many.

6. How many sq. meters in 8.23 hectares?

Ans. 82300 sq. m.

7. How many sq. dm. in 72.05 sq. Dm.?

Ans. 720500 sq. dm.

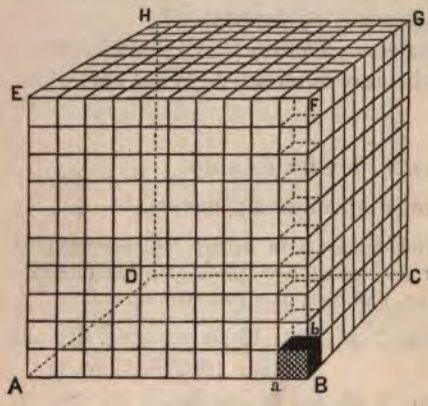
8. How many ares in 49630 ca.? *Ans.* 496.30 a.

9. Change 6.1824 sq. kilometers to hectares.
Ans. 618.24 Ha.

10. Reduce 7585.40 hectares to centares.

MEASURES OF VOLUME.

204. In measures of volume the primary unit is the ***Cubic Meter.*** It is equal to 1.30799 cu. yd.



Let *ABCD–G* represent a cubic meter. Suppose it to be divided into ten equal parts of the same dimensions as the solid *ABCD*. Each of these parts is one meter square, and one decimeter thick, and contains one tenth of a cubic meter.

Secondly, each part may be subdivided into ten smaller solids, each of which being a decimeter square, and a meter long, will be .01 of a cubic meter.

Lastly, each of the smaller solids may also be divided into 10 cubic decimeters, such as *ab*, each being .001 of the cubic meter.

TABLE.

1000 cu. mm. = 1 cu. cm. = .061 cu. in.
1000 cu. cm. = 1 cu. dm. = 1 liter (capacity) = .0353 cu. ft.
1000 cu. dm. = 1 ***cu. m.*** = 1 stere = 35.3166 cu. ft.

WOOD MEASURE.

10 decisteres (ds.) = 1 ***stere.***(s.) = 35.3166 cu. ft.
10 steres = 1 decastere (Ds.) = 2.759 cords.

In cubic measure the higher units are not used. The cubic decimeter and cubic centimeter serve chiefly as a basis for the measures of capacity and weight. When used to measure wood or stone, the cubic meter is called a *stere* (*stair*).

As 1000 units of a lower denomination, in this measure, make one unit of the next higher denomination, each denomination must have three places. Thus, 10 cu. m. 6 cu. dm. 47 cu. cm., written as cubic meters, is 10.006047 cu. m., and may be read 10 cu. m. and 6047 cu. cm.

WRITTEN EXERCISES.

1. Express 39 cu. m. 70 cu. dm. 2 cu. cm. as cu. meters.
Ans. 39.070002 cu. m.

2. Reduce 15 cu. m. 12 cu. cm. to cu. m.
Ans. 15.000012 cu. m.

3. How many cu. dm. in 56 cu. m. 25 cu. dm.?
Ans. 56025 cu. dm.

4. How many steres in 17 Ds. 9 s. 4 ds.?
Ans. 179.4 steres.

NOTE.—In wood measure, each denomination occupies only one place.

5. Express 235 Ds. 3 s. 5 ds. as decisteres.
Ans. 23535 ds.

6. Reduce 7186 decisteres to decasteres.
Ans. 71 Ds. 8 s. 6 ds.

7. Reduce 90357054 cu. cm. to cu. m.
Ans. 90.357054 cu. m.

8. Express 710.3486 cu. dm. as cu. meters.
Ans. .7103486 cu. m.

9. How many cu. dm. in 12 steres? In 6 cu. m.?

10. How many decasteres in 678 steres?

11. In 45.178527 cu. m. how many cu. cm.?

12. Reduce 11767.5 decisteres to decasteres.
Ans. 117.675 Ds.

MEASURES OF CAPACITY.

205. The unit of capacity is the ***Liter.*** It is equal in volume to 1 cu. decimeter, equal to 1.05671 qt. Liquid, or .9081 qt. Dry measure.

The *liter* is equal to the capacity of a box, or cubical vessel, that is one tenth of a meter, or a decimeter, in length, breadth, and depth. The capacity of such a vessel may be divided into 1000 cubic centimeters, each of which, in measures of capacity, is called a *milliliter.*

TABLE.

				Dry M.	Liq. M.
10 milliliters	=	1 centiliter....	cl.	.61 cu. in.	.388 fl. oz.
10 centiliters	=	1 deciliter....	dl.	6.10 "	.845 gi.
10 deciliters	=	1 ***liter*.....**	l.	.908 qt.	1.0567 qt.
10 liters *	=	1 decaliter....	Dl.	9.081 qt.	2.64175 gal.
10 decaliters	=	1 hectoliter...	Hl.	2.837 bu.	26.4175 "
10 hectoliters	=	1 kiloliter....	Kl.	28.37 bu. 1.308 cu. yd.	264.175 "
10 kiloliters	=	1 myrialiter..	Ml.	283.72 bu.	2641.75 "

The *liter* is used in measuring liquids; but for measuring large quantities of grain, fruit, and liquids in casks, the hectoliter is employed. The latter is equal to 26½ gallons, or 2⅝ bushels.

ORAL EXERCISES.

1. How many liters in a hectoliter? How many decaliters in a kiloliter? In a hectoliter?

2. How many centiliters in a liter? How many milliliters in a liter?

3. What part of a liter is a deciliter? What part of a hectoliter is a liter?

4. How many liters in $4\frac{1}{2}$ Dl.? How many kiloliters in 2000 l.?

5. What part of a hectoliter is a dl.? What part of a myrialiter is a Kl.?

WRITTEN EXERCISES.

1. Express 9 Hl. 7 Dl. 4 l. 8 dl. as liters.

2. Express 15 Kl. 3 Dl. 6 dl. as hectoliters.

3. How many milliliters in 25.8 deciliters?

4. Change 718.12 l. to centiliters. *Ans.* 71812 cl.

5. Reduce 50637 l. to kiloliters. *Ans.* 50.637 Kl.

6. Write 123.4 dl. as hectoliters. *Ans.* .1234 Hl.

7. In 16 Dl. 7 dl. how many centiliters?
Ans. 16070 cl.

MEASURES OF WEIGHT.

206. The unit of weight is the ***Gram.*** It is the weight of a cubic centimeter of distilled water at its maximum density, 39.2° F., or 4° C.; and is equal to .03527 oz. avoir., or 15.432 Troy gr.

A vessel having the capacity of a cubic decimeter, and filled with pure water at its greatest density, weighs one thousand grams, and is called a kilogram. A gram, therefore, is equal to the weight of the water contained in a small vessel having the capacity of a cubic centimeter.

TABLE.

10 milligrams	= 1 centigram.....	cg.	.1543	gr. Tr.
10 centigrams	= 1 decigram.....	dg.	1.543 +	"
10 decigrams	= 1 ***gram***.....	g.	15.432 +	"
			.03527 +	oz. av.
10 grams	= 1 decagram.....	Dg.	.3527 +	"
10 decagrams	= 1 hectogram.....	Hg.	3.5274 +	"
10 hectograms	= 1 kilogram, or kilo....	Kg.	2.6792	lb. Tr.
			2.2046	lb. av.
10 kilograms	= 1 myriagram.....	Mg.	22.046	"
100 kilos	= 1 quintal.....	Q.	220.46 +	"
10 quintals	= 1 ***ton***.....	T.	2204.62 +	"

The *kilogram* is used for weighing groceries and common articles; the *gram* for weighing letters, gold, and medicines. For articles of great weight, as coal and hay, the *metric ton* is the standard.

The *kilogram*, or *kilo*, weighs about $2\frac{1}{5}$ pounds, and the *metric ton* about $1\frac{1}{10}$ common tons.

NOTE.—The U. S. nickel five-cent piece is 2 centimeters in diameter, and weighs 5 grams. The lawful weight of a letter for a single postage of 2¢, is 15 grams, or the weight of three nickels.

ORAL EXERCISES.

1. How many grams in a decagram? In a hectogram? In a kilogram?

2. How many decigrams in 40 centigrams? How many grams in 1000 milligrams?

3. What part of a decagram is a decigram? What part of a kilogram is a decagram?

4. How many metric tons in 3000 kilograms? How many hectograms in 400 grams?

5. How many grams in 75 decagrams? How many kilograms in 5 metric tons?

6. How many kilos in 4 quintals? How many tons in 40 quintals?

WRITTEN EXERCISES.

1. Express 16 Kg. 7 Dg. 8 g. as grams.
2. Write 45 Dg. 2 g. 9 dg. as grams. *Ans.* 452.9 g.
3. In 82.13 g., how many decigrams?
4. How many milligrams in 15 g. 4 dg.?
5. Change 837.52 cg. to grams. *Ans.* 8.3752 g.
6. Reduce 703.49 g. to hectograms.
7. How many metric tons in 9860.50 kilograms?
Ans. 9.8605 T.

WRITTEN REVIEW.

1. Find the sum of 71 m. 2.5 Hm. and 6 dm.
Ans. 321.6 m.

NOTE.—The ordinary operations in metric measures, are performed according to the principles of decimals.

2. What is the difference between 127 Km. 3 Dm., and 274 Hm. 4 dm.? *Ans.* 99629.6 m.

3. Find the cost of 16.25 m. of cloth, at $2.50 per meter. *Ans.* $40.62½.

4. How much did I receive for 63.75 Dl. of milk, at 6¢ a liter? *Ans.* $38.25.

5. A man bought 36.42 hectares of land, at $1.25 per are. What was the cost of the land? *Ans.* $4552.50.

6. What is the value of a cubical bin of grain whose edge is 3 meters, at $1.25 per hectoliter?

7. A grocer bought 542 Kg. of raisins, at 20¢ a kilo, and sold them at 25¢ a pound. How much did he gain?

8. A room is 120 dm. long, and 75 dm. wide. How much will it cost to plaster the ceiling, at 22½¢ per square meter? *Ans.* $20.25.

9. At 60¢ a bu., what will 80 hectoliters of corn cost?
Ans. $136.176.

10. If I buy 130 yd. of carpet, at 85¢ a meter, and sell it at the same price per yard, how much will I gain?
Ans. $9.46.

11. What will be the gain on 200 steres of wood, bought at 60¢ a stere, and sold at $2.50 a cord?
Ans. $17.95.

12. How many cubic meters in a block of marble 11 ft. long, 5.351 ft. wide, and 3 ft. thick?
Ans. 5 cu. m.

13. What must be the height of a pile of wood which is 8.5 m. long, 6.3 m. wide, and contains 289.17 steres?
Ans. 5.4 m.

14. How many 5¢ pieces can be made of a bar of metal weighing 86 hectograms? *Ans.* 1720 pieces.

15. What will be the profit on 2150 liters of molasses, bought at 4½¢ per liter, and sold at 28¢ a gallon?

16. How many miles will an engine run in 8 hours, at the rate of 28.5 Km. an hour? *Ans.* 141.68 miles.

17. From a barrel, containing 76 Kg. of sugar, were taken three parcels of 42.50 Hg. each, and four parcels of 97.50 Hg. each. How much sugar was left in the barrel? *Ans.* 24.25 Kg.

18. How many jets can be fed by a gasometer holding 36000 cu. m., if each jet burn 150 l. in an hour, and is used for 6 hours every evening? *Ans.* 40000 jets.

19. Find the cost of papering a room 13 m. square, and 2.5 m. high, with paper 20 cm. wide, and 6.25 m. in a roll, at 80¢ a roll? *Ans.* $83.20.

20. A lady bought 66.1628 yd. of silk in New York. What would she have paid for it in Paris, at $5¼ per meter? *Ans.* $317.63.

PART II.

PART II.

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COMMERCIAL ARITHMETIC.

PERCENTAGE.

1. ***Percentage*** is a method of computation having 100 for its unit or base.

The sign of Percentage is the symbol, %, called *per cent.*, which is an abbreviation of the Latin *per centum*, and signifies *by the hundred.* Thus, 8% means 8 of every hundred. It may refer to any denomination, and may denote 8 cents of every hundred cents, 8 dollars of every hundred dollars, 8 bushels of every hundred bushels, etc.

2. The elements of percentage are the *Base*, the *Rate*, and the *Percentage.*

3. The ***Base*** is the number of which the percentage is taken.

4. The ***Rate*** is the number denoting how many *hundredths* of the base are to be taken.

5. The ***Percentage*** is the result obtained by taking a certain number of hundredths of the base.

6. The ***Amount*** is the sum of the *base* and the *percentage.*

7. The ***Difference*** is the remainder obtained by subtracting the *percentage* from the *base.*

In the statement 8% of \$400 is \$32, the *base* is \$400; the *rate*, 8%, or .08; and the *percentage*, \$32. The *amount* is \$400 + \$32 = \$432; and the *difference*, \$400 − \$32 = \$368.

8. One per cent. of any number is *one hundredth* of it; four per cent. is *four hundredths*, and, in general, any per cent. of a number may be expressed as so many decimal or fractional parts of that number; thus:

RATE.		DECIMAL.		FRACTIONAL.	RATE.		DECIMAL.		FRACTIONAL.
1%	=	.01	=	$\frac{1}{100}$	$\frac{1}{2}$%	=	.005	=	$\frac{1}{200}$
5%	=	.05	=	$\frac{1}{20}$	$\frac{3}{4}$%	=	.0075	=	$\frac{3}{400}$
10%	=	.10	=	$\frac{1}{10}$	50%	=	.50	=	$\frac{1}{2}$
$12\frac{1}{2}$%	=	$.12\frac{1}{2}$	=	$\frac{1}{8}$	$66\frac{2}{3}$%	=	$.66\frac{2}{3}$	=	$\frac{2}{3}$
25%	=	.25	=	$\frac{1}{4}$	75%	=	.75	=	$\frac{3}{4}$
$33\frac{1}{3}$%	=	$.33\frac{1}{3}$	=	$\frac{1}{3}$	125%	=	1.25	=	$1\frac{1}{4}$

ORAL EXERCISES.

1. How many *hundredths* of a number is 9% of it? 11%? 16%? 25%? 18%? 50%? 64%? 80%? 90%? $6\frac{1}{4}$%? $8\frac{1}{3}$%?

2. What part of a number is 25% of it? 10%? 20%? 50%? 75%? 15%? $12\frac{1}{2}$%? $16\frac{2}{3}$%? $33\frac{1}{3}$%? $6\frac{2}{3}$%?

3. What *per cent.* of a number is $\frac{1}{4}$ of it? $\frac{1}{2}$? $\frac{3}{8}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{7}{8}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{9}$? $\frac{5}{6}$? $\frac{3}{10}$? $\frac{7}{20}$?

4. What *per cent.* of a number is .03 of it? .05? .11? .46? .225? .1875? .065? .0625? $.7533\frac{1}{3}$? $.666\frac{2}{3}$?

WRITTEN EXERCISES.

1. Express 8% as a decimal and as a common fraction.

OPERATION.

$8\% = .08 = \frac{8}{100} = \frac{2}{25}$

SOLUTION.—Since 8% signifies 8 on the hundred, it may be expressed decimally, .08; or, as a common fraction, $\frac{8}{100} = \frac{2}{25}$.

Express in the same manner:

2. 2%; 6%; 20%; 12%; 40%; 15%; 55%; 98%.
3. 142%; 118%; $6\frac{1}{4}$%; $8\frac{1}{3}$%; $37\frac{1}{2}$%; $40\frac{2}{5}$%; $22\frac{3}{5}$%.
4. $112\frac{1}{2}$%; $116\frac{2}{3}$%; $118\frac{2}{5}$%; $\frac{1}{4}$%; $\frac{1}{2}$%; $\frac{1}{3}$%; $\frac{3}{4}$%.
5. $\frac{3}{8}$%; $\frac{5}{8}$%; $\frac{7}{8}$%; $\frac{4}{5}$%; $\frac{3}{5}$%; $\frac{5}{6}$%; $\frac{3}{16}$%; $\frac{1}{6}$%.

Change to expressions with *per cent.* sign:

6. .04; .27; .45; .375; .$66\frac{2}{3}$; .05; .5; .0625.
7. $\frac{2}{5}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{3}{16}$; $\frac{1}{12}$; $\frac{1}{5}$; $\frac{7}{8}$; $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{8}$; $\frac{3}{10}$.

Case I.

9. The base and rate being given, to find the percentage, or amount, or difference.

ORAL EXERCISES.

1. What is 20% of 80?

Solution.—Since 20% = $\frac{20}{100}$, or $\frac{1}{5}$, 20% of 80 is equal to $\frac{1}{5}$ of 80, or 16. Hence, 20% of 80 is 16.

How much is:	What is:
2. 9% of 300?	*7.* 50% of 60 sheep?
3. 25% of 160?	*8.* 75% of 40 bbl.?
4. 30% of 120?	*9.* $16\frac{2}{3}$% of 36 hhd.?
5. $12\frac{1}{2}$% of 480?	*10.* 20% of 350 eggs?
6. $33\frac{1}{3}$% of 600?	*11.* $37\frac{1}{2}$% of \$64?

12. What are my expenses, if I earn \$800 a year and expend 30% of it? 60%? 25%? 15%? 27%?

13. What is the amount of 40 increased by 5% of itself? 80%? $12\frac{1}{2}$%? $66\frac{2}{3}$%? 40%?

14. How much is \$16 increased by 25%? \$60? \$250? \$900? \$1000? \$24000?

15. What is the remainder after decreasing 600 by 20%? 15%? 90%? 60%? 75%? $6\frac{1}{4}$%?

16. What remains after deducting 3% from 200 gal.? 16% from 50 gal.? 40% from 70 gal.?

17. The base is $680 and the percentage is $160.40; what is the amount? The difference?

18. The base is 800 and the rate 75%; what is the percentage? The amount? The difference?

19. The base is 400 sheep and the rate 20%; what is the amount? The difference?

WRITTEN EXERCISES.

1. What is 35% of 140 lb.?

OPERATION.

```
  140
  .35
 ----
  700
 420
 ----
49.00 lb.
```

SOLUTION.—Since 35% = .35, then 35% of 140 lb. = 140 lb. × .35, or 49 lb.

RULE.—*To find the percentage, multiply the base by the rate.*

Formula: $P = B \times R$.

What is:

2. 26% of $828? *Ans.* $215.28.

3. 24% of $25.50? *Ans.* $6.12.

4. 84% of 3276 mi.? *Ans.* 2751.84 mi.

5. $66\frac{2}{3}$% of 4860 bu.? *Ans.* 3240 bu.

6. $40\frac{2}{5}$% of 675 gal.? *Ans.* 272.7 gal.

7. 112% of 435? *Ans.* 487.2.

8. $72\frac{1}{2}$% of 1608? *Ans.* 1165.8.

9. $44\frac{1}{2}$% of 1200 sheep? *Ans.* 534 sheep.

10. 125% of $8500? *Ans.* $10625.

11. What is 116⅔% of 1250 bu.? *Ans.* 1458⅓ bu.

12. A lawyer having collected $2460, charged a fee of 6%; what was his fee? *Ans.* $147.60.

13. Sold at a loss of 24%, a horse that cost $250; what was the loss? *Ans.* $60.

14. If goods purchased for $1050, are sold at a gain of 8⅓%, how much is gained? *Ans.* $87.50.

15. Which is the greater, and how much, 6¼% of 52250, or 3½% of 96470? *Ans.* The second, 110.825.

16. A drover having 3200 head of cattle, sold 25% of them at one time, and 50% of the remainder at another time; how many had he left? *Ans.* 1200.

17. In a freight of 360 bbl. of apples, 15% were damaged; what was the value of the remainder at $2.75 a bbl.? *Ans.* $841.50.

18. Six years ago a city had a population of 138450, which has increased 18%; what is its present population? *Ans.* 163371.

19. The base is $13770, and the rate 33⅓%; what is the percentage? the amount? the difference? *Ans.* To first, $4590.

20. Bought a house for $8764, and sold it at a gain of 8½%; how much did I receive for it? *Ans.* $9508.94.

21. What is 40% of 6 gal. 3 qt. 1 pt.? *Ans.* 2 gal. 3 qt.

22. What is 12½% of 6 yd. 2 ft. 8 in.? *Ans.* 2 ft. 7 in.

23. What is 2% of £123 16*s.* 9*d.*? *Ans.* £2 9*s.* 6.42*d.*

24. What is the difference between 3¼% of $18000, and 2½% of $19000? *Ans.* $110.

25. If the base is $25000, and the rate 37½%, what is the amount? *Ans.* $34375.

CASE II.

10. The base and the percentage being given, to find the rate.

ORAL EXERCISES.

1. What per cent. of 8 is 4?

SOLUTION.—Since 4 is $\frac{1}{2}$ of 8, it is $\frac{1}{2}$ of 100% of 8, or 50% of 8.

2. What per cent. of 72 is 36? 24? 18? 6? 8?

3. What % of 144 is 36? 24? 18? 6? 8?

4. What % of $40 are $5? $20? $16? $25? $30?

5. 18 is what % of 36? 54? 24? 18? 6?

6. 2 ft. are what % of 1 yd.? 8 oz., of a pound (avoirdupois)?

7. What % of a gal. are 3 qt.? 6 qt.? Of a bu. are 3 pk.? 7 qt.?

8. What % of a hhd. are 35 gal.? 40 qt.? $31\frac{1}{2}$ gal.?

9. The base is 45 and the percentage 9; what is the rate?

10. The amount is 96 and the percentage 6; what is the base? The rate?

11. The difference is 80 and the percentage 20; what is the base? The rate?

12. Bought coffee for 27 cents and sold it for 30 cents; what was the gain %?

13. What part of 25 is $12\frac{1}{2}$? What %?

14. What part of 50 is $6\frac{1}{4}$? What %?

15. What part of 20 is $6\frac{2}{3}$? What %?

16. What part of 100 is $33\frac{1}{3}$? What %?

17. What part of 100 is $8\frac{1}{3}$? What %?

18. What part of 125 is $62\frac{1}{2}$? What %?

19. What part of 149 is $37\frac{1}{4}$? What %?

WRITTEN EXERCISES.

1. What per cent. of \$625 are \$25 ?

OPERATION.

$\frac{25}{625} = \frac{1}{25}$

$\frac{1}{25} \times 100\% = 4\%$

Or,

625) 25.00 (.04
25.00
.04 = 4%

SOLUTION.—Since \$625 are 100% of \$625, and \$25 are $\frac{25}{625}$, or $\frac{1}{25}$ of \$625, \$25 are $\frac{1}{25}$ of 100%, or 4% of \$625.

Or, since \$25 are the product of \$625 multiplied by the rate, the rate must be the quotient of \$25 divided by \$625, which equals .04, or 4%.

RULE.—*Divide the percentage by the base.* Or, *Take such a part of 100% as the percentage is of the base.*

Formula: $R = P \div B$.

What % of :

2. 800 is 40 ? *Ans.* 5%.
3. \$5 are 50¢ ? *Ans.* 10%.
4. 63 gal. are 9 gal.? *Ans.* $14\frac{2}{7}$%.
5. 1850 sheep are 518 sheep ? *Ans.* 28%.
6. 372 bu. are $55\frac{4}{5}$ bu.? *Ans.* 15%.
7. $\frac{7}{8}$ is $\frac{1}{4}$? *Ans.* $28\frac{4}{7}$%.
8. \$8.50 are \$1.70 ? *Ans.* 20%.
9. 7884 is 5256 ? *Ans.* $66\frac{2}{3}$%.
10. $43\frac{1}{2}$ is $7\frac{1}{4}$? *Ans.* $16\frac{2}{3}$%.
11. 4 cwt. 40 lb. are 44 lb.? *Ans.* 10%.
12. Find what per cent. 12*s.* 9*d.* are of £2 10*s.* *Ans.* $25\frac{1}{2}$%.
13. A man having \$37840, purchased a house for \$7851.80; what per cent. of his money did it cost ? *Ans.* $20\frac{3}{4}$%.

14. An agent charged $45 for collecting a bill of $1500; what per cent. did he charge? *Ans.* 3%.

15. What rate % is charged by an auctioneer who requires $60 for selling $4000 worth of furniture?
Ans. $1\frac{1}{2}$%.

16. What per cent. is gained by selling for $230 a horse that cost $180? *Ans.* $27\frac{7}{9}$%.

17. The amount is 325 and the base 250; what is the rate? *Ans.* 30%.

18. What per cent. of 72 lb. are 2 lb. 4 oz. (avoir.)?
Ans. $3\frac{1}{8}$%.

19. The difference is 2700 and the base 3000; what is the rate? *Ans.* 10%.

Case III.

11. The rate and the percentage being given, to find the base.

ORAL EXERCISES.

1. 8 is 25% of what number?

Solution.—Since 8 is 25% of a certain number, it is $\frac{25}{100}$, or $\frac{1}{4}$ of it. Hence, if $\frac{1}{4}$ of the number is 8, $\frac{4}{4}$, or the number, is 32. Therefore 8 is 25% of 32.

Of what number:

2. Is 4, 20%? 80%?
3. Is 10, 10%? 125%?
4. Are $16, 5%? 12%?
5. Are 80 gal., 14%? $133\frac{1}{3}$%?
6. Is 1, $\frac{1}{2}$%? $12\frac{1}{2}$%?
7. Is $\frac{1}{2}$, 50%? 10%?
8. Is 25, $6\frac{1}{4}$%? $8\frac{1}{3}$%?
9. Is $3\frac{3}{4}$, 20%? 120%?

10. The percentage is $40 and the rate $6\frac{2}{3}$%; what is the base?

11. The percentage is 45 gal. and the rate 20%; what is the amount? The difference?

12. A man sold 360 sheep, which was 15% of his flock; how many sheep had he?

13. Having lost 30% of my money, I had $700 left; how much had I at first?

14. 12% of 200 is 5% of what number?

15. Of what are 2 ft. 4 in., 25%?

WRITTEN EXERCISES.

1. Of what number is 468, 65%?

OPERATION.

$468 \div .65 = 720$

Or,

$65\% = \frac{65}{100} = \frac{13}{20}$

$\frac{13}{20} = 468$

$\frac{1}{20} = 36$

$\frac{20}{20} = 720$

SOLUTION.—Since 468 is the percentage, or product of the base multiplied by .65, the base is equal to the quotient of 468 divided by .65, which is 720.

Or, since 468 is 65% of the base, it is $\frac{65}{100}$ or $\frac{13}{20}$ of the base. Then if $\frac{13}{20} = 468$, $\frac{1}{20}$ will equal 36, and $\frac{20}{20}$, or 100% = 720.

RULE.—*To find the base, divide the percentage by the rate.*

Formula: $B = P \div R$.

Of what are:

2. $954, 53%? *Ans.* $1800.

3. $1683, 27%? *Ans.* 6233\frac{1}{3}$.

4. $3000, 37$\frac{1}{2}$%? *Ans.* $8000.

5. 525 hhd., 62$\frac{1}{2}$%? *Ans.* 840 hhd.

6. 120 lb., 16%? *Ans.* 750 lb.

7. $6300, 15%? *Ans.* $42000.

8. 16\frac{2}{3}$, 25%? *Ans.* 65\frac{1}{2}$.

9. $$\frac{1}{4}$, 37$\frac{1}{2}$%? *Ans.* $2.

10. 460 ft. 10 in., 20%? *Ans.* 2304 ft. 2 in.

11. How many acres in a farm of which 736 acres are 24%? *Ans.* 3066$\frac{2}{3}$ A.

12. Mr. Olwell sold 45% of his property for $174438.90; what was his property worth? *Ans.* $387642.

13. A clerk spends yearly $405, which is 18% of his salary; what is his salary? *Ans.* $2250.

14. Mr. Feldhaus bought flour for $8500, which was $33\frac{1}{3}$% of what he was worth; what was he worth? *Ans.* $25500.

15. A man having 50% interest in a steamboat, sold $18\frac{3}{4}$% of his share for $7968.75; what was the value of the steamboat? *Ans.* $85000.

16. A workman after having his wages increased 8%, received $2.16 per day; what were his daily wages before? *Ans.* $2.

CASE IV.

12. Given the amount or the difference, and the rate, to find the base.

ORAL EXERCISES.

1. What number increased by 10% of itself equals 55?

SOLUTION.—Since 55 is the number increased by 10% of itself, it is $\frac{110}{100}$, or $\frac{11}{10}$ of the number; hence the number is 10 times the $\frac{1}{11}$ of 55, or 50.

2. What number diminished by 20% of itself equals 16?

SOLUTION.—Since 16 is the number diminished by 20% of itself, it is $\frac{80}{100}$, or $\frac{4}{5}$ of the number; hence the number is 5 times the $\frac{1}{4}$ of 16, or 20.

What increased by:	What diminished by:
3. 30% is 260?	*8.* 40% is 120?
4. 25% is 5?	*9.* 20% is 28?
5. $6\frac{1}{4}$% is 34 lb.?	*10.* $33\frac{1}{3}$% is 46?
6. $8\frac{1}{3}$% is 65?	*11.* 8% is £69?
7. 50% is 81 yd.?	*12.* 50% is $2\frac{1}{2}$?

13. The amount is 240, and the rate 20%; what is the base?

14. The difference is 140, and the rate 30%; what is the base?

15. Sold coffee for 30¢, which was a gain of 20%; what was the cost?

16. After using 60% of a piece of wire, James had 30 ft. of it left; how many feet were there in the piece?

WRITTEN EXERCISES.

1. What number increased by 35% of itself equals 108?

OPERATION.

$1.00 + .35 = 1.35$
$108 \div 1.35 = 80$

SOLUTION.—Since the number increased by 35% of itself, is equal to 108, then $108 \div 1.35$, or 80, equals the number.

Or,

$.35 = \frac{35}{100} = \frac{7}{20}$
$\frac{20}{20} + \frac{7}{20} = \frac{27}{20}$
$\frac{27}{20} = 108$
$\frac{1}{20} = 4$
$\frac{20}{20} = 80$

Or, if the number be increased by 35% of itself, it will then be equal to $\frac{20}{20} + \frac{7}{20} = \frac{27}{20}$ of itself. Hence $\frac{27}{20}$ of the number $= 108$; and $\frac{1}{20}$ of it $= 4$; and $\frac{20}{20}$, or the number itself $= 80$.

2. What number diminished by 8% of itself equals 736?

OPERATION.

$1.00 - .08 = .92$
$736 \div .92 = 800$

SOLUTION.—Since 736 is 8% less than the number, it is 92% of the number; then $736 \div .92 = 800$, or the number.

Or,

$.08 = \frac{8}{100} = \frac{2}{25}$
$\frac{25}{25} - \frac{2}{25} = \frac{23}{25}$
$\frac{23}{25} = 736$
$\frac{1}{25} = 32$
$\frac{25}{25} = 800$

Or, $8\% = \frac{2}{25}$ of the number. Hence if $\frac{25}{25}$, the number itself, be diminished by $\frac{2}{25}$, the remainder $= \frac{23}{25} = 736$; and $\frac{1}{25} = 32$; and $\frac{25}{25} = 800$.

RULE.—*To find the base, divide the amount by 100% plus the rate, or the difference by 100% minus the rate.*

Formulas: (1) $B = A \div (100\% + R)$. (2) $B = D \div (100\% - R)$.

What number increased:

3. By 30% = $975? *Ans.* $750.
4. By $8\frac{1}{3}$% = 760.5 yr.? *Ans.* 702 yr.
5. By $12\frac{1}{2}$% = $20\frac{1}{4}$ oz.? *Ans.* 18 oz.
6. By 18% = 1180 mi.? *Ans.* 1000 mi.

What number diminished:

7. By $16\frac{2}{3}$% = $1080.30? *Ans.* $1296.36.
8. By 40% = 2925 t.? *Ans.* 4875 t.
9. By $3\frac{1}{3}$% = 310 bales? *Ans.* 320 bales.
10. By $37\frac{1}{2}$% = $687\frac{1}{2}$ bbl.? *Ans.* 1100 bbl.

11. Goods are sold at a loss of 5%, for $7125; what was the cost? *Ans.* $7500.

12. A partner having 23% interest in a clothing firm, sold 40% of his share for $2760; what was the value of the portion that remained to him? *Ans.* $4140.

13. The average daily attendance of a school is 483, which is 8% less than the number on the roll; how many on the roll? *Ans.* 525.

14. A tradesman having drawn 40% of his bank deposit, purchased for 80% of this amount, a house valued at $5120; what was the entire amount remaining? *Ans.* $10880.

15. A government clerk's salary having been decreased 15%, is now $1700; how much did he receive at first? *Ans.* $2000.

PROFIT AND LOSS.

13. ***Profit and Loss*** are terms indicating the sum gained or lost in business transactions.

14. The quantities in Profit and Loss are the *Cost*, the *Rate*, the *Gain* or *Loss*, and the *Selling Price*, corresponding to the base, the rate, the percentage, and the amount or difference in Percentage.

CASE I.

15. To find the gain, the loss, or the selling price, when the cost and the rate are given.

ORAL EXERCISES.

1. Paid $20 for a stove and sold it at a gain of 5%; how much did I gain? What was my selling price?

SOLUTION.—If I sold the stove at a gain of 5%, I must have sold it for 105% of the cost. $105\% = \frac{105}{100} = \frac{21}{20}$. $\frac{20}{20} = \$20$, the cost; $\frac{1}{20} = \frac{1}{20}$ of $\$20 = \$1$, the gain; and $\frac{21}{20}$, the selling price, = 21 times $1, or $21.

2. The buying price is $45, and the gain 8%; how much is gained?

3. What is the gain, at 3%, on goods bought for $300?

4. What is lost, at 4%, on carpet costing $65?

5. The cost of an article is $224, the loss 20%; what is the loss? The selling price?

6. Bought a grand piano for $400, and sold it at a gain of 15%; what was the gain? The selling price?

7. A horse cost $340 and was sold at a loss of $12\frac{1}{2}$%; what was the selling price?

8. The cost is $450, the gain 6%; required the gain.

9. A farm that cost $8200, was sold at a gain of 10%; required the selling price.

10. How much was gained by selling at an advance of 25%, a hat that cost $4 ?

11. A watch that cost $50, was sold at a loss of 10%; what was the loss ? The selling price ?

12. What was received for a horse costing $150, and sold at a loss of 33⅓%?

WRITTEN EXERCISES.

1. Bought goods to the amount of $4000, and sold them so as to gain 30% on the cost; what was my gain?

OPERATION.

$4000
.30
$1200.00

SOLUTION.—The goods were bought for $4000, and sold at a gain of 30%, or .30 × $4000 which is $1200.

RULE.—*I. To find the gain or loss, multiply the cost by the rate.*

II. To find the selling price, multiply the cost by 100% plus the rate of gain, or by 100% minus the rate of loss.

Formulas: $\begin{cases} G \text{ or } L = C \times R. \\ SP = C \times (100\% + RG), \text{ or } C \times (100\% - RL). \end{cases}$

2. A merchant bought 64 bbl. of flour @ $6.50 per bbl., and sold them at a loss of 25%; how much did he lose ? *Ans.* $104.

3. Mr. Baker sold a house and lot which cost $16000, at a gain of 12½% ; what was his gain ? *Ans.* $2000.

4. A gentleman entered into a speculation in which he invested $15000, and realized 18% on his investment; what was his gain ? *Ans.* $2700.

5. I bought a horse for $300, and sold him to A at a loss of 6%; what did A pay for him? *Ans.* $282.

6. A dealer having $2674 worth of cotton on hand, lost 60% of it by fire; how much had he remaining? *Ans.* $1069.60.

7. Samuel Smith bought 40 bbl. of sugar at $13 per bbl., and 15 bags of salt at 3\frac{1}{3}$ per bag; he sold the whole so as to gain 15% on the entire cost; what was his gain? *Ans.* $85.50.

8. A wine merchant paid $450 for wine and $265 for brandy. In selling them he gained 15% on the wine and 10% on the brandy; what was his entire gain? *Ans.* $94.

9. A jeweller sold a watch which cost him $45, for 70% of its value; what was his selling price? *Ans.* $31.50.

10. Bought cloth at $3 a yard and sold it so as to gain 20% on the cost; what was the selling price? *Ans.* $3.60.

CASE II.

16. To find the rate, when the cost and the gain or loss, or the selling price are given.

ORAL EXERCISES.

1. Bought a horse for $160 and sold him at a gain of $40; what was the gain %?

SOLUTION.—Since the horse was sold at a gain of $40 on $160, there was gained $\frac{40}{160}$, or $\frac{1}{4}$ of the cost; and $\frac{1}{4}$ of 100% = 25%. Hence there was a gain of 25%.

2. Buying price $350, gain $50; required the gain %.

3. Buying price $219, loss $73; required the loss %.

SOLUTION.—Since there is a loss of $73 on $219, the loss is equal to $\frac{1}{3}$ of the cost; and $\frac{1}{3}$ of 100% = 33$\frac{1}{3}$%. Hence there is a loss of 33$\frac{1}{3}$%.

4. Buying price $160, selling price $240; required the gain %.

5. Buying price $200, selling price $170; required the loss %.

6. Selling price $160, gain $60; what is the buying price? The gain %?

7. Selling price $10, loss $2; required the loss %.

8. Loss $4, buying price $40; required the loss %.

9. Gain $19, buying price $76; required the gain %.

10. Loss $24, selling price $216; required the loss %.

11. What per cent. is lost by selling goods for $\frac{3}{5}$ of their cost? For $\frac{3}{4}$? For $\frac{4}{5}$? For $\frac{5}{6}$? For $\frac{1}{4}$? For $\frac{1}{2}$? For $\frac{1}{3}$?

12. What per cent. is gained by selling for $18 what cost $15?

13. From a flock of 30 geese, 6 were lost; what per cent. was that of the flock?

14. A horse costing $280, was sold for $240; what was the loss %?

15. Sold a coat for $\frac{4}{7}$ of its cost; what % did I lose?

WRITTEN EXERCISES.

1. A merchant bought goods to the amount of $400, and sold them so as to gain $120; what was his gain %?

OPERATION.

$120 \div 400 = .30,$

Or,

$120 \div 400 = \frac{3}{10}$ of $100\% = 30\%$.

SOLUTION:—Since the gain, $120, is a certain % of $400, that per cent., or the rate, must be the quotient of $120 ÷ 400 = .30, or 30%.

RULE.—*To find the rate, divide the gain or loss by the cost.*

Formula: $R = G \div C$, *or* $L \div C$.

2. Thomas bought a watch for $80, and sold it for $90; what % did he gain? *Ans.* $12\frac{1}{2}$%.

3. What would be the rate % profit on goods bought for $920 and sold for $1150? *Ans.* 25%.

4. Bought 420 bbl. flour at $6 per bbl. and sold them at $8 per bbl.; what was my gain %? *Ans.* $33\frac{1}{3}$%.

5. A factory valued at $10000 and insured for $8000, was destroyed by fire; what was the loss %?

Ans. 20%.

6. A wine merchant bought 236 bbl. of wine for $13452 and sold them at $75 per bbl.; did he gain or lose, and what %? *Ans.* Gained $31\frac{11}{15}$%.

7. What % do I gain by selling sugar at 11¢, that cost me 8¢? *Ans.* $37\frac{1}{2}$%.

8. A man bought a house and lot for $12200; after paying $300 for improvements, he sold the whole so as to gain $250; what was his selling price, and what his gain %? *Ans.* Gained 2%.

9. A vessel loaded with 34000 bu. of wheat was wrecked and only 8000 bu. were saved; what was the loss %? *Ans.* $76\frac{8}{17}$%.

10. What % does a grocer lose by selling $\frac{1}{2}$ of a barrel of sugar for what $\frac{3}{8}$ of a barrel cost him? *Ans.* 25%.

11. Mr. Curtis bought pine-apples for 20% more than 10¢ each, and sold them for 15% less than 10¢ each; what did he lose per cent.? *Ans.* $29\frac{1}{6}$%.

12. Mr. Gray sold 2 horses for $150 each. On one he gained $45, and on the other he lost $25; what was his gain %? *Ans.* $7\frac{1}{7}$%.

13. A speculator invested $2400 in business. The first year he gained $500, the second $300, and the third year he lost $450. What was his net gain %?

Ans. $14\frac{7}{12}$%.

14. I bought cloth at $2.50 a yard; the marked price shows a gain of 40%; what will be my gain, if I sell it for $\frac{1}{5}$ less than my asking price? Gain %? *Ans.* 12%.

15. A farmer sold a cow to B for $\frac{5}{7}$ of its cost, who sold it to C for $12\frac{1}{2}$% more than its original cost; what was B's gain %? *Ans.* $31\frac{1}{4}$%.

CASE III.

17. Given the rate and the gain or loss, to find the cost.

ORAL EXERCISES.

1. I gained $20 by selling goods at an advance of 5% on the cost; what was the cost?

SOLUTION.—Since the gain, $20, is 5% of the cost, then .05 or $\frac{1}{20}$ = $20, and $\frac{20}{20}$, or 100%, = 20 × 20, or $400.

2. I gained $25 by selling a quantity of pig-iron at an advance of 4%; what was the cost?

3. An advance of 5% on cloth increases its value $40; what was its cost?

4. A man gained $20 by selling a cow for 40% more than it cost; what was the cost?

5. Lost $36 by selling goods at 12% below cost; required the cost.

6. By selling a house 4% below cost, a gentleman lost $148; what did it cost him?

7. The gain is 328 and the rate $12\frac{1}{2}$% advance; what is the cost?

8. The loss is $75 and the rate 25% of the cost; what is the cost?

9. The rate is 10% advance, and the gain $80; find the cost.

10. The rate is $33\frac{1}{3}\%$ of the cost; and the loss $\$266\frac{2}{3}$; required the cost.

11. Find the cost of a pound of sugar, on which 8% was lost by selling it 1 cent per pound below cost.

12. Sold a pair of pants at a gain of $1, which was $12\frac{1}{2}\%$ above cost; what did it cost?

WRITTEN EXERCISES.

1. By selling a quantity of hay a man gained $180, which was 20% of the cost; required the cost.

OPERATION.

$\$180 \div .20 = \$900.$

Or,

$20\% = \frac{1}{5}$ of the cost.
$\frac{1}{5} = \$180$, gain.
$\frac{5}{5} = \$900$, cost.

SOLUTION.—Since the gain, $180, is .20 times the cost, then the cost must be as much as .20 is contained times in $180, which is $900.

Or, if the gain is 20% of the cost, and he gained $180, 20%, or $\frac{1}{5}$, must be equal to $180; and $\frac{5}{5}$, or the cost, = 5 times $180 = $900.

RULE.—*To find the cost, divide the gain or loss by the rate.*

Formula: $C = G \div R$, *or* $L \div R$.

2. Mr. Simpson sold his horse and buggy so as to gain $75, which was $12\frac{1}{2}\%$ of the cost; what was the cost of the horse and buggy? *Ans.* $600.

3. A sold a cargo of sugar at 15% profit and gained $7462.50; what was the value of the cargo?

Ans. $49750.

4. Mr. Hoadly bought 250 bbl. apples and sold them so as to gain $125, which was 20% of the cost; what was the cost per bbl.? *Ans.* $2.50.

5. A druggist bought a quantity of quinine, and sold

it at 37½% profit, thereby gaining $21.60. What did he pay for it, and what was his selling price?

Ans. Selling price, $79.20.

6. A grocer bought 42 chests of tea and sold them so as to gain 30% on the cost. His profits amounted to $945. What was the cost price per chest? *Ans.* $75.

7. Mr. Fay sold his house at a loss of 36%; what was the value of the house if his loss amounted to $7560?

Ans. $21000.

8. Two capitalists invested equal sums in business: one gained 7⅓% and the other 9⅔%; what amount did each invest, if the gain of the second was $186 more than that of the first? *Ans.* $9000.

9. A farmer lost 1½% of his sheep by wolves, and 4½% by accident, losing altogether 24; how many sheep had he at first? *Ans.* 400 sheep.

10. Mr. Townsend offered his house for sale at an advance of 40%, but afterwards sold it for 5% less than his asking price, thereby gaining $660; what was the value of the property? *Ans.* $2000.

11. By selling 24 bags of coffee I gained $45, which was 15% of what they cost; what was my selling price per bag? *Ans.* $14.37½.

12. A farmer sold 13½% of his hay to C, 37½% to D, and 300 tons to E, which last was equal to what D received more than C. How many tons had he at first?

Ans. 1250 tons.

13. If by selling an article at an advance of 40¢, I gain 14⅔%, what was the cost? *Ans.* $2.80.

14. By selling his country residence at a gain of $25000, a gentleman realized 15% more than if he had gained only $10600; what did the residence cost him?

Ans. $96000.

CASE IV.

18. Given the rate and the selling price, to find the cost.

ORAL EXERCISES.

1. The selling price of an article is $510, and the gain 2%; what is the cost price?

SOLUTION.—If he gained 2%, he must have sold it for 102%. 102% = $\frac{102}{100}$ = $\frac{51}{50}$ = $510. $\frac{1}{50}$ of the cost = $\frac{1}{51}$ of $510 = $10; and $\frac{50}{50}$, or the cost, = 50 times $10, or $500.

2. The selling price is $84, and the loss is 20%; what is the cost price?

SOLUTION.—If the loss be 20%, the selling price must be 80%. 80% = $\frac{4}{5}$ of the cost = $84. $\frac{1}{5}$ of the cost = $\frac{1}{4}$ of $84 = $21; and $\frac{5}{5}$, or the cost, = 5 times $21, or $105.

3. Mr. Williams sold flour for $530, which was a gain of 6%; what did it cost him?

4. By selling a span of horses for $666, a gentleman gained 11% on their cost; what was their cost?

5. Selling price $175, loss 12½%; required the cost price.

6. Selling price $800, gain 33⅓%; required the cost price.

7. Gain 10%, selling price $440; required the cost and gain.

8. Loss 10%, selling price $450; required the cost and loss.

9. Selling price $324, gain 8%; required the cost and gain.

10. Loss 7%, selling price $279; required the cost and loss.

11. Beef sold for \$15, was an advance of 10%; what was its cost?

12. Sold wheat at \$1.32$\frac{1}{3}$ per bu., thereby losing 5% on my purchase price; what had it cost me?

WRITTEN EXERCISES.

1. Sold a horse for \$210, thereby losing 30%; what was the cost?

OPERATION.

100% − 30% = 70%
\$210 ÷ .70 = \$300.

Or,

100% − 30% = 70%
70% = $\frac{7}{10}$ of the cost.
$\frac{7}{10}$ = \$210, selling price.
$\frac{1}{10}$ = \$30
$\frac{10}{10}$ = \$300, cost.

SOLUTION.—Since by selling the horse for \$210, 30% is lost, the selling price is .70 times the cost. Therefore the cost will be as many dollars as .70 is contained times in \$210, which is \$300.

Or, if the loss was 30%, the selling price is equal to 100% − 30% = 70% of the cost. 70% = $\frac{7}{10}$ = \$210. $\frac{1}{10}$ of the cost = $\frac{1}{7}$ of \$210 = \$30; and $\frac{10}{10}$, or the cost, = 10 times \$30, or \$300.

RULE.—*To find the cost, divide the selling price by 100% plus the rate, or by 100% minus the rate.*

Formula: $C = SP \div (100\% + R)$, *or* $SP \div (100\% - R)$.

2. A merchant sold 40 bales of cotton at \$60 per bale, thereby gaining 60%; how much did it cost?

Ans. \$1500.

3. By selling sugar at 10¢ a lb., I lose 20%; what is it worth per lb.? *Ans.* 12$\frac{1}{2}$¢.

4. Sold flour at \$9 per barrel, which was a gain of 33$\frac{1}{3}$%; required the cost per bbl. *Ans.* \$6.75.

5. Mr. Wilkins sold a horse at a gain of 12$\frac{1}{2}$%, and received \$877.50; what had he paid for the horse?

Ans. \$780.

6. Mr. Gorman sold a horse and buggy for $640, thereby gaining 33⅓%; what did the horse cost him, supposing the buggy to be worth 1½ times as much as the horse? *Ans.* $192.

7. A and B formed a partnership and at the end of the year their joint investment amounted to $10605, showing a gain of 25%; how much did each invest, supposing A's share to be equal to ¾ of B's?

Ans. A's, $3636; B's, $4848.

8. A quantity of corn was sold for $837, which was 35% more than its cost; how many bushels were sold, if it cost 50¢ per bu.? *Ans.* 1240 bu.

9. Mr. West sold two horses for $320 each; on one he gained 33⅓%, and on the other he lost 33⅓%; did he gain or lose, and how much? *Ans.* Lost $80.

10. If I sell a sewing machine for $48 and gain 20%, would I have gained or lost by selling it at $35, and what per cent.? *Ans.* Lost $5; 12½%.

11. Sold tea at ¾ of a dollar a pound, thereby gaining 50%; required the cost per pound. *Ans.* 50 cents.

12. Loaned a sum of money for one year at 6%, at the end of which time it amounted to $1007; what was the amount loaned? *Ans.* $950.

13. Sold a farm for $2400, losing 14⅔%; what did the farm cost me, and how much did I lose?

Ans. Cost $2800; Lost $400.

14. Sold two houses for $13500 each; on one I gained 12½%, and on the other I lost 20%; did I gain or lose, and how much? *Ans.* Lost $1875.

15. If my profits equal 20% of my selling price, what per cent. of the cost is my gain? *Ans.* 25%.

16. A farm was sold for $76500, the gain on the cost being 112½%; what was the cost? *Ans.* $36000.

TRADE DISCOUNT.

19. ***Trade Discount*** is an allowance made on the fixed price of certain kinds of merchandise.

20. The fixed price is called the *List Price;* and prices subject to discount on account of early payment, are called *Time Prices.*

1. If the list price is $60, and there is a discount of 30% and 10%, what is the net price ?

OPERATION.

30% of $60 = $18
$60 — $18 = $42
10% of $42 = $4.20
$42 — $4.20 = $37.80, net price.

SOLUTION.—30% of $60, the list price, = $18, the first discount. Subtracting and, in like manner, taking 10% of the remainder, we obtain $4.20 as the second discount. Hence $42 — $4.20 = $37.80, the net price.

NOTE.—The several discounts are generally taken in the order given. But changing the order does not change the result.

WRITTEN EXERCISES.

2. The list price of a refrigerator is $60, and the rates of discount are 25% and 5%; what is the net price ?
Ans. $42.75.

3. What is the net amount of a bill, the gross amount of which is $37.62½, and the rate of discount 20% ?
Ans. $30.10.

4. What is the value of 76 pairs of rubber boots at $2.75 per pair, less 30% and 10% ? *Ans.* $131.67.

5. What is the net price of a piano valued at $375, less 13⅓% ? *Ans.* $325.

6. What is to be paid on a bill of 17 tons white ash coal at $6.75, and 6 cords of wood at $4.25, the rate of discount being 8%? *Ans.* $129.03.

7. At what price should potatoes at $2.60 per bbl., be marked to allow a reduction of 20% and still sell at cost? *Ans.* $3.25.

8. What is the difference between a discount of 30% on a bill of $740, and a discount of 25% and 5% on the same? *Ans.* $9.25.

9. What is the net value of a bill of hardware amounting to $2460 at a discount of 30%, $16\frac{2}{3}$%, and 8%? *Ans.* $1320.20.

10. Bought goods, March 19, 1883, to the amount of $623.50, on the following terms: "60 days, or less 2% if paid in 15 days." How much would settle the bill, March 31, 1883? *Ans.* $611.03.

11. Sold groceries amounting to $2846.80, April 13, '84, on the following terms: "90 days, or less 2% in 30 days, or less 5% in 10 days." How much would settle the bill, April 18? May 11?

Ans. $2704.46; and $2789.86.

12. What direct discount is equivalent to a discount of 20% and $16\frac{2}{3}$%? $37\frac{1}{2}$% and 10%? 40% and 5%? 25% and 10%? 10% and 5%? *Ans.* First, $33\frac{1}{3}$%.

13. The net amount of a bill was $432, and the rates of discount were 25% and 10%; what was the gross amount of the bill? *Ans.* $640.

SOLUTION.—1.00 − .25 = .75; $432 ÷ .75 = $576. 1.00 − .10 = .90; $576 ÷ .90 = $640.

14. The net amount of a bill of glassware is $999.60, the discounts are 30% and 15%; what is the gross amount? *Ans.* $1680.

15. At what price must I mark goods valued at $3.00 per yard, so as to allow a reduction of 30% on the asking price, and still gain 12%? *Ans.* $4.80.

SOLUTION.—$3.00 × 1.12 = $3.36; 100% − 30% = 70%; $3.36 ÷ .70 = $4.80.

16. At what price must coal valued at $5.00 a ton, be marked so as to allow a margin of 15% and a gain of 5%? *Ans.* $6.18.

NOTE.—In answers 5 mills and over are reckoned 1 cent; less than 5 are not regarded.

17. What must be the asking price for a quantity of wheat that cost $240, so that 10% may be deducted and a profit of 20% made? *Ans.* $320.

18. Bought land at $252 per acre; what should be my asking price per acre, that I may deduct 12½% and still gain 15% on the cost price? *Ans.* $331.20.

19. Mr. Browne received a bill of merchandise at the following rates of discount: $64.40 at 30% and 5%; $57.50 at 50% and 2%; $86 at 11%; and $32.40 at 12½% and 8%; what was the net amount of the bill? *Ans.* $173.62.

COMMISSION.

21. ***Commission*** is an allowance made an agent for transacting business.

22. An ***Agent*** is a person who transacts business for another. He is also called *Commission Merchant, Factor,* or *Broker.*

23. A ***Consignment*** is a quantity of goods sent to an agent for sale. The party who sends the goods is

called the *Consignor,* and the party to whom they are sent, the *Consignee.*

24. The ***Net Proceeds*** of a consignment is the balance due the consignor after deducting the commission and other charges. The actual amount of the sale or purchase is called the *Base.*

CASE I.

25. To find the commission, or net proceeds, when the base and the rate are given.

MENTAL EXERCISES.

1. An agent received 25% for collecting a bill of $20; what was his commission? The net proceeds?

SOLUTION.—If an agent received 25% commission for collecting a bill of $20, he must have received 25% = $\frac{1}{4}$ of $20 = $5. Hence his commission was $5; and the net proceeds equal $20 − $5 = $15.

What is the commission on:

2. $300 worth of goods, at 20%? 10%? 25%? 30%?

3. $250 worth of coal at 40%? 20%? 60%? 80%?

4. $40 worth of brandy at 10%? 15%? 35%? 70%?

5. 700 lb. of sugar at 6¢, at $33\frac{1}{3}$%? $14\frac{2}{7}$%? $16\frac{2}{3}$%?

6. 30 lb. of butter at 25¢, at $8\frac{2}{3}$%? $6\frac{2}{3}$%? $66\frac{2}{3}$%?

7. $\frac{1}{4}$ of $200 at $16\frac{2}{3}$%? $12\frac{1}{2}$%? 50%?

8. What is the commission on the sale of 100 yd. of cloth, at $3.50 per yd., 6% being paid for selling?

9. What is received by an auctioneer who sells $2500 worth of furniture at a commission of 4%?

10. How much will a lawyer receive for collecting $33\frac{1}{3}$% of a bill of $900, at a commission of 18%?

11. The base is $1400, and the rate 15%; what is the commission? The net proceeds?

12. What is the commission for collecting \$4000 at $3\frac{1}{4}\%$?

13. What are the net proceeds on the sale of \$60 worth of goods, commission $33\frac{1}{3}\%$?

14. An agent receives $12\frac{1}{2}\%$ on a sale worth \$96; what are the net proceeds?

15. How much will I receive from a bill of \$50, after deducting a commission of 10%?

WRITTEN EXERCISES.

1. What is the commission at $6\frac{2}{3}\%$, on the sale of 240 bushels of wheat at 75 cents a bushel? The proceeds?

OPERATION.

$240 \times .75 = \$180$, value.
$\$180 \times .06\frac{2}{3} = \$12$, commission.
$\$180 \times (1.00 - .06\frac{2}{3}) = \$168$, proceeds.

SOLUTION.—The value of 240 bushels at 75 cents a bushel = \$180. And the commission = $.06\frac{2}{3}$ times \$180 = \$12. Hence the commission is \$12. To find the proceeds we multiply the base, \$180, by 100% minus the rate and obtain \$168.

RULE.—*I. To find the commission, multiply the base by the rate.*

II. To find the net proceeds, multiply the base by 100% minus the rate.

Formulas: (1) $C = B \times R$. (2) $NP = B \times (100\% - R)$.

2. An agent purchased goods to the amount of \$1296; what was his commission at 5%? *Ans.* \$64.80.

3. What is a lawyer's fee for collecting a bill of \$3325 at a commission of 7%? *Ans.* \$232.75.

4. How much do I owe my agent for purchasing for me 250 bbl. of flour at \$4.75 per bbl., his charges being $2\frac{1}{2}\%$? *Ans.* \$29.69.

5. A merchant instructs his agent to buy wheat to the amount of $3483. How much must he remit to settle the bill, the agent's commission being $3\frac{1}{3}\%$, and the freight $17? *Ans.* $3616.10.

6. What is a real estate agent's fee, at 8%, for selling a house for $14680, and a lot for $7620? *Ans.* $1784.

7. How much will a commission merchant receive for selling 360 bbl. of pork, at $11.25 per bbl., rate of commission 16¢ per bbl.? *Ans.* $57.60.

8. A lawyer having a bill of $670 to collect, agrees to settle for 90%; what is his commission at $3\frac{1}{2}\%$? *Ans.* $21.11.

9. What are the net proceeds on the sale of 6700 lb. of leather at 25¢ per lb., storage 12¢ per 100 lb., and commission 5%? *Ans.* $1583.21.

10. What are the net proceeds of a sale of $7000, after allowing a commission of 6%? The commission? *Ans.* $6580; $420.

11. A freight broker procures transportation for 350 tons of merchandise at $3.50 per ton; what is his commission at $\frac{1}{2}\%$? *Ans.* $6.12\frac{1}{2}$.

CASE II.

26. To find the base, when the rate and the commission or the net proceeds or the remittance are given.

MENTAL EXERCISES.

1. If an agent received $20 for selling goods, at the rate of 10%, what amount did he sell?

SOLUTION.—Since his commission was 10%, he received $\frac{1}{10}$ of the sales. Hence $\frac{1}{10}$ of the sales = $20; and $\frac{10}{10}$, the whole amount sold = 10 times $20 = $200.

What amount of sales will give:

2. \$24 commission, at 4%? \$30 at 10%?
3. \$12 commission, at $\frac{1}{4}$%? \$25 at 40%?
4. \$150 commission, at $1\frac{1}{2}$%? \$120 at 2%?
5. \$320 commission, at 5%? \$500 at 4%?
6. \$240 commission, at 10%? \$300 at 3%?
7. \$350 commission, at 25%? \$200 at 6%?
8. \$500 commission, at 20%? \$1000 at $2\frac{1}{2}$%?
9. \$450 commission, at $33\frac{1}{3}$%? \$2500 at $12\frac{1}{2}$%?
10. \$600 commission, at $\frac{1}{6}$%? \$3000 at 3%?

11. How much of a remittance of \$78 may an agent invest and retain his commission at 4%?

SOLUTION.—Since the commission is 4%, it is equal to $\frac{1}{25}$ of the investment. $\frac{25}{25}+\frac{1}{25}=\frac{26}{25}$ = the remittance. Since $\frac{26}{25}$ = \$78, $\frac{1}{25}$ = \$3; and $\frac{25}{25}$ = \$75, the investment. Or, since \$1 of the investment = \$1.04 of the remittance, he can invest as many dollars as \$78 ÷ \$1.04 = \$75.

Find investment from:

12. \$63 remittance, at 5%. \$88 remittance, at 10%.
13. \$104 remittance, at 4%. \$225 remittance, at $12\frac{1}{2}$%.
14. \$350 remittance, at $16\frac{2}{3}$%. \$550 remittance, at $37\frac{1}{2}$%.
15. \$510 remittance, at 2%. \$209 remittance, at $4\frac{1}{2}$%.
16. \$434 remittance, at $8\frac{1}{2}$%. \$309 remittance, at 3%.

WRITTEN EXERCISES.

1. A merchant sent his agent in New Orleans \$18540 to be invested in cotton, after deducting his commission at 3%; what was the investment?

OPERATION.

\$1.00 + .03 = \$1.03
\$18540 ÷ \$1.03 = \$18000

SOLUTION.—For every dollar invested the agent must receive \$1.03. Hence the investment = \$18540 ÷ \$1.03 = \$18000.

2. A commission merchant received \$647.25 for selling a quantity of wheat at a commission of $7\frac{1}{2}\%$; what was the amount invested?

OPERATION.

$\$647.25 \div .07\frac{1}{2} = \$8630$

Or,

$7\frac{1}{2}\% = \frac{15}{200} = \frac{3}{40}$

$\frac{3}{40} = \$647.25$

$\frac{1}{40} = \$215.75$

$\frac{40}{40} = \$8630$

SOLUTION.—At a commission of $7\frac{1}{2}\%$, $.07\frac{1}{2}$ times the cost of the goods equals the commission. Hence, the commission, \$647.25, $\div$ $.07\frac{1}{2}$ = \$8630, the cost.

Or, $7\frac{1}{2}\% = \frac{3}{40}$ of the cost = \$647.25, the commission; $\frac{1}{40}$ = \$215.75, and $\frac{40}{40}$ = \$8630, the cost, or investment.

RULE.—*To find the base, divide the commission by the rate, or the remittance by 100% plus the rate, or the net proceeds by 100% minus the rate.*

Formula: $B = C \div R$, *or Rem.* $\div (100\% + R)$, or $NP \div (100\% - R)$.

3. Mr. Jenkins sent his agent \$5125 to be invested in flour, after deducting his commission at $2\frac{1}{2}\%$; what was the amount of the investment? *Ans.* \$5000.

4. An agent received \$2543.75 to be invested in cloth, after deducting his commission of $1\frac{3}{4}\%$; what was the value of the cloth? *Ans.* \$2500.

5. A cotton broker, charging 6%, received \$200.10 for selling cotton; what was the amount of the sale?

6. A lawyer received \$34.37 for collecting a bill at $2\frac{1}{2}\%$; what was the amount of the bill? *Ans.* \$1374.80.

7. A salesman has an income of \$4500 a year, by selling goods at a commission of 4%; what are his yearly sales? *Ans.* \$112500.

8. A collector remits \$13059.11 after retaining 3% on the amount collected; what was the amount collected? *Ans.* \$13463.

9. An agent received $4500 to be invested in flour at $9 a barrel, after deducting his commission at 25%; how many barrels did he buy? *Ans.* 400 bbl.

10. I sent my agent $12630 to buy 80 horses; if he paid $160 for transportation, $50 for their keeping, and charged 3½% commission, what did the horses cost him a piece? *Ans.* $150.

11. Mr. White sold grain on commission at 8%, through an agent, who charged him 5%. If Mr. White's commission, after paying his agent, was $189, what amount did he send his employers? *Ans.* $5796.

12. The net proceeds of a shipment of 250 tons of hay, after deducting a commission of 4%, and $630 for freight, was $3210. What was the selling price per ton? *Ans.* $16.

STOCKS.

27. A ***Company*** is an association of persons for the transaction of business.

28. A ***Corporation*** is a company authorized by law to act as a single individual. When a company is not incorporated it is called a partnership or firm.

The nature, powers, and obligations of a corporation are defined by an act of the Legislature, called a *charter*.

29. The ***Stock*** of a company is the capital invested in the business.

30. A ***Share*** is one of the equal parts into which the capital is divided. The value of a share is $100, unless otherwise stated.

31. A ***Bond*** is an interest-bearing note of a government or a corporation.

32. Bonds are generally named according to their rate of interest and date of maturity. Thus, U. S. 4½%'s, '91, signifies United States Bonds bearing 4½% interest, and payable in 1891.

33. ***Stocks*** are the shares of companies and the bonds of governments.

34. ***Certificates*** of stock are the papers issued by a corporation, and indicating the number of shares belonging to each of its stockholders.

35. An ***Installment*** is a sum required of the stockholders as a payment on their subscription.

36. An ***Assessment*** is a sum required of the stockholders to cover the losses or expenses of the business.

37. A ***Dividend*** is a sum paid to the stockholders from the gains of the business.

38. The ***Par Value*** of stock is the value marked on its face.

39. The ***Market Value*** of stock is the price for which it sells. It is also called *real value.*

40. Stock is *at par* when the selling price and the par value are equal. When it sells for more than its par value, it is said to be *above par,* or at a *premium;* and when it sells for less than its par value, it is said to be *below par,* or at a *discount.*

41. ***Stock-jobbing*** is the business of buying and selling stocks in order to realize profit from their rise and fall in the market.

42. A ***Stock Broker*** is a person who buys and sells stocks, either for himself or others.

43. ***Brokerage*** is the percentage or fee charged by a broker. It is estimated upon the par value of the stocks, and usually at $\frac{1}{4}$%.

CASE I.

44. To find the dividend, when the stock and the rate of dividend are given.

1. If I own $9500 worth of railroad stock which produces a dividend of $7\frac{1}{2}$%, what is my dividend ?

OPERATION.

$$\begin{array}{r} \$9500 \\ .07\frac{1}{2} \\ \hline \$712.50 \end{array}$$

SOLUTION.—If the stock produces a dividend of $7\frac{1}{2}$%, and I own $9500 worth, my dividend is $.07\frac{1}{2}$ times $9500 = $712.50.

RULE.—*To find the dividend, multiply the par value of the stock by the rate.*

WRITTEN EXERCISES.

2. Mr. Nash bought 36 shares in an insurance company, which declares a dividend of 8%; what is his dividend ? *Ans.* $288.

3. A company whose stock is $431000, declares a quarterly dividend of 4%; what is the yearly dividend on 85 shares ? *Ans.* $1360.

4. A banking company, on account of losses, requires an assessment of $2\frac{1}{4}$%; what amount is paid by a gentleman who owns 69 shares ? *Ans.* $155.25.

5. The Ocean Telegraph Company declares a dividend of 6%, payable in stock; how many shares increase does my stock of $25600 receive ?

Ans. 15 shares and $36 over.

6. A owned 105 shares in the Electric Light Company for two years. During the first year he was obliged to pay 2 assessments of 5% each, and the last year he received a dividend of $12\frac{1}{2}\%$; what was his gain?

Ans. $262.50.

CASE II.

45. To find the stock, when the rate and the dividend are given.

1. Mr. Bartlett received $359.70 as his share of a dividend at 5%; what amount of stock did he own?

OPERATION.

$359.70 ÷ .05 = $7194.

Or,

$5\% = \frac{1}{20} = \$359.70$

$\frac{20}{20} = \$7194$

SOLUTION.—Since 5% of the stock equals $359.70, the stock equals $359.70 ÷ .05 = $7194.

RULE.—*To find the stock, divide the dividend by the rate.*

WRITTEN EXERCISES.

2. A man received $4752 on an 8% dividend; how many shares at $50 each did he own?

Ans. 1188 shares.

3. I received $4500 payable in stock as my share of a $4\frac{1}{2}\%$ dividend; how many shares did I own?

Ans. 1000 shares.

4. If A received 63 shares on a 9% dividend, how many shares does he now own? *Ans.* 763 shares.

5. After receiving a stock dividend of $12\frac{1}{2}\%$, Mr. Blaine had 36 shares worth $50 each; how many shares had he at first? *Ans.* 32 shares.

CASE III.

46. To find the rate, when the stock and the dividend are given.

1. What rate of dividend can a company declare if it clears $11088 on a stock of $158400 ?

OPERATION.

$11088 ÷ $158400 = 7%.

SOLUTION.—As the dividend is always some per cent. of the stock, to find the rate we divide the dividend by the stock.

RULE.—*To find the rate, divide the dividend by the stock.*

WRITTEN EXERCISES.

2. If I draw a dividend of $400 on 60 shares of railroad stock, what is the rate ? *Ans.* 6⅔%.

3. At what rate will Mr. Clare receive $4221 dividend on 804 shares of bank stock ? *Ans.* 5¼%.

4. A lady owns 90 shares in a gas company whose stock is $650000, and her dividend is $315 ; what is the rate of dividend ? *Ans.* 3½%.

5. Mr. Buckley holds 520 shares in an insurance company, and is assessed twice, to the amount of 130 shares and 52 shares, respectively ; what are the rates ?

Ans. 25%, 10%.

6. Mr. Grant received a dividend of $225 ; what was the rate, if he owned 75 shares ($50) in the stock ?

Ans. 6%.

CASE IV.

47. To find the brokerage, when the par value, the market value, and the rate are given.

1. A broker sold 74 shares Erie R. R. Stock, and

charged $\frac{1}{4}$% brokerage; what amount of brokerage did he receive?

OPERATION.

74 × \$100 = \$7400.
\$7400 × $\frac{1}{4}$% = \$18.50.

SOLUTION.—The par value is equal to 74 times \$100 = \$7400; and the brokerage is .00$\frac{1}{4}$ times \$7400 = \$18.50.

RULE.—*To find the brokerage, multiply the par value by the rate.*

WRITTEN EXERCISES.

2. How much does a broker receive for purchasing 121 shares Northern Central R. R. stock, at $\frac{1}{4}$%? *Ans.* \$30.25.

3. What is the brokerage on 432 shares Baltimore & Ohio R. R. stock, at $\frac{1}{8}$%? *Ans.* \$54.

4. Mr. Jackson bought, through his agent, 36 shares Erie R. R. stock at 112, and sold them at 120; how much was the brokerage, at $\frac{1}{4}$% in both cases?

5. I sold 376 shares of stock, at $\frac{1}{8}$% brokerage, and 292 shares of another kind, at $\frac{1}{4}$%. Upon which kind was the brokerage the greater, and how much?

6. I sold, through my agent, 89 U. S. bonds at 106; how much brokerage did I pay?

NOTE.—Agents generally charge $\frac{1}{4}$% brokerage.

7. What is the brokerage on the sale of 149 shares Suffolk Bank stock, if sold through an agent?

8. Mr. Roach purchased 160 shares (\$25) of bank stock at 2$\frac{1}{4}$% premium; what was the brokerage at $\frac{1}{4}$%?

9. At $\frac{1}{2}$% brokerage, what will a broker receive for selling 35 shares Harlem R. R. stock?

10. Mr. Wilson bought 425 shares (\$50) of stock, and paid $\frac{1}{4}$% brokerage. He afterwards sold 600 shares (\$25), and paid $\frac{1}{8}$% brokerage. Upon which was the brokerage the greater, and how much?

CASE V.

48. To find the real value of stock, when the par value and the rate of premium or discount are given.

1. What is the real value of 67 shares of stock, $50 each, at 4% discount?

OPERATION.

$50 × 67 = $3350, par value.
1.00 — .04 = .96.
$3350 × .96 = $3216, real value.

SOLUTION. — As each share is worth $50, the par value = $50 × 67 = $3350. If this stock is at 4% discount, the real value is 96% of $3350 = $3216.

RULE.—*To find the real value, multiply the par value by 100% plus the rate of premium, or by 100% minus the rate of discount.*

WRITTEN EXERCISES.

2. What will be the cost of 85 shares of stock at 9% premium? *Ans.* $9265.

3. A man bought $25140 stock in the Pacific Railroad, at par, and sold it at 15% discount; what was his loss?

4. What must I pay for 320 United States $1000 bonds, at 4½% premium, and ¼% brokerage? *Ans.* $335200.

5. Mr. Bennett bought 140 railroad bonds, at 6¼% discount, and sold the same at 8¾% advance; what did he gain or lose, brokerage being ¼%? *Ans.* $2030 gain.

6. What is the market value of 560 shares of bank stock at 1¾% advance? *Ans.* $56980.

7. What is the cost of 256 shares of stock in Adams Express Company, at 5% discount, brokerage ⅛%? *Ans.* $24352.

8. A banker exchanged 126 shares of railroad stock at 4% premium, for 130 shares of bank stock at 3% discount; what did he gain or lose? *Ans.* Lost $494.

9. What is the cost of 21 shares in a gold company's stock, at $6\frac{3}{8}$% above par, the original value being $300 a share, and brokerage $\frac{1}{8}$%? *Ans.* $6709.50.

10. I bought stock at 105 and sold it at 96, and paid brokerage at $\frac{1}{4}$% in each transaction; what did I lose on 54 shares? *Ans.* $513.

11. My broker buys me 270 shares in N. Y. Central R. R. stock, at $60, and charges $\frac{1}{4}$% brokerage; how much does the stock cost me? *Ans.* $16267.50.

12. A man bought $910 in gold, at $12\frac{3}{4}$% premium, and paid $\frac{1}{4}$% brokerage; how much currency did he pay?

NOTE.—Gold is said to be at a *premium* when the currency of a country is depreciated, or at a *discount*. It thus becomes an object of investment like stocks.

CASE VI.

49. To find the par value, when the rate and the real value are given.

1. Mr. Hancock paid $15412.50 for government bonds, at $2\frac{1}{2}$% premium; what was the par value, brokerage being $\frac{1}{4}$%?

OPERATION.

$$1.00 + .02\tfrac{1}{2} = 1.02\tfrac{1}{2}.$$
$$1.02\tfrac{1}{2} + .00\tfrac{1}{4} = 1.02\tfrac{3}{4}.$$
$$\$15412.50 \div 1.02\tfrac{3}{4} = \$15000.$$

SOLUTION.—At a premium of $2\frac{1}{2}$%, the real value is $1.02\frac{1}{2}$; which together with the brokerage at $\frac{1}{4}$%, gives $1.02\frac{3}{4}$, the true cost. Hence the par value equals the real value, or cost, divided by $1.02\frac{3}{4}$ = $15000.

RULE.—*To find the par value, divide the real value by 100% plus the rate of premium, or by 100% minus the rate of discount.*

WRITTEN EXERCISES.

2. I sold 36 shares of stock in the Fulton Ferry Company, at 12% discount, and received $1584; what was the par value of 1 share? *Ans.* $50.

3. Mr. Burns paid $5175 for a draft, at $3\frac{1}{2}$% premium; what was the face of the draft? *Ans.* $5000.

4. A lady sold stock bought at par, at a premium of 5%, and gained $630; how many shares, of $25 each, did she sell? *Ans.* 504 shares.

5. How much stock can I purchase in the W. U. Telegraph Company for $20396.50, at 10% discount, and pay brokerage at $\frac{1}{4}$%? *Ans.* $22600.

6. Mr. Davis paid $7920.50 in currency, for gold at $8\frac{1}{2}$% advance; what amount did he purchase? *Ans.* $7300.

7. The discount on a certain amount of stock, at $3\frac{1}{3}$%, was $786; what was its par value? *Ans.* $23580.

8. Mr. Tilden gave a draft worth 5% premium, for 62 shares of the First National Bank stock, $25 each, at 4% discount; what was the face of the draft? *Ans.* $1417.14.

9. A man exchanged 97 shares of stock at 115, for 446 shares of another kind and $5 cash; what was the value of the latter per share? *Ans.* $25.

10. Mr. Conklin bought 31 shares of stock at 7% discount, brokerage $\frac{1}{4}$%, for $2890.75; what was the par value of a share? *Ans.* $100.

11. When gold is $106\frac{3}{4}$, how much can I purchase for $952.21 currency? *Ans.* $892.

12. When railroad stock is quoted at $85\frac{1}{3}$, how many shares can be bought for $12040, after paying $\frac{2}{3}$% brokerage? *Ans.* 140 shares.

13. A speculator invested $1191 in the Hudson River R. R. stock, at a discount of $50\frac{3}{8}\%$; how many shares did he buy? *Ans.* 24 shares.

14. Mr. Sullivan received $2063.25 to be invested in stocks at 98, after deducting his brokerage at $\frac{1}{4}\%$; what amount of stock did he purchase? *Ans.* 21 shares.

15. A man paid his broker $27675 for an investment in Ohio and Miss. stock, at 92, including brokerage at $\frac{1}{4}\%$. What was the par value of the stock?

Ans. $30000.

16. If a broker charged $10.75 for purchasing bonds, at $\frac{1}{8}\%$, how many shares did he buy? *Ans.* 86.

17. How much money must be invested in stocks at 5% discount, in order to gain $960 by selling at an advance of 7%? *Ans.* $8000.

18. How many pounds of butter at 20¢, can I buy for the proceeds of 40 shares of bank stock at 10% premium, brokerage being $\frac{1}{8}\%$ for selling, and 2% for buying?

Ans. $21544\frac{1}{10}$ lb.

19. I sent my agent $60630 to be invested in the Illinois State bonds, at $6\frac{1}{4}\%$ discount; how many can he buy, allowing $\frac{1}{4}\%$ brokerage? *Ans.* 645 bonds.

20. If I purchase 113 shares of stock at a discount of $5\frac{3}{4}\%$, and sell the same at a gain of $127, what is the selling price? *Ans.* $10777.25.

21. If my agent charges me $23739 for purchasing 193 shares New Jersey stock at 120, what is his rate of brokerage? *Ans.* 3%.

22. How many $500 U. S. bonds shall I receive for $6510, invested in U. S. $4\frac{1}{2}$'s at $108\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

Ans. 12 bonds.

23. How many shares Missouri Pacific can be bought for $9342.50 at $92\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

STOCK INVESTMENTS.

50. ***U. S. Bonds*** are divided into two classes; viz., the *Registered* and the *Coupon* Bonds.

51. ***Registered Bonds*** are those which are entered upon the books of the Treasury Department at Washington and are made transferable by indorsement.

52. ***Coupon Bonds*** are those which are payable to bearer, and are transferable without indorsement. They have coupons of interest attached to them, which may be cut off and presented for payment when due.

53. The principal United States Securities are:

Sixes continued at 3½.	U. S. cur. 6's, 1896.
Fives continued at 3½.	U. S. cur. 6's, 1897.
U. S. 4½'s, '91 reg.	U. S. cur. 6's, 1898.
U. S. 4½'s, '91 c.	U. S. cur. 6's, 1899.
U. S. 4's, 1907 reg.	Dist. of Col. 3–6 5's.
U. S. 4's, 1907 c.	U. S. P. cur. 6's, 1895.
U. S. cur. 6's, 1895.	U. S. P. cur. 6's, 1896.

54. A ***Mortgage*** is a conveyance of real estate as security for the payment of a debt.

55. ***Ground Rent*** is the rent paid for the ground occupied by certain buildings.

CASE I.

56. To find the income, when the investment, the market value, and the rate of interest are given.

1. What income will I obtain by investing $2406.40 in stock bearing 6% and purchased at 94%?

OPERATION.

$2406.40 ÷ .94 = $2560.
$2560 × .06 = $153.60.

SOLUTION.—If I can buy $1.00 worth of stock for 94 cents, for $2406.40 I can buy as many dollars worth as 94 cents are contained times in $2406.40, which are $2560. The interest or income, equals .06 times $2560 = $153.60.

RULE.—*Having found the par value, multiply it by the rate of interest, and the product will be the income.*

WRITTEN EXERCISES.

2. If Mr. Harper invest $8523.12 in U. S. 4½'s at 102, what is his annual income? *Ans.* $376.02.

3. What will be my income, if I invest $10904.25 in U. S. 4's at 117¼? *Ans.* $372.

4. Mr. Brady invested $22057.50 in U. S. cur. 6's, '96, at 108⅛. What is his annual income in gold, the latter being 110? *Ans.* $1112.73.

5. If Mr. Doyle invest $13608 in U. S. 4's at 107¾, brokerage ¼%, what will be his yearly income? *Ans.* $504.

6. Which will be the more profitable investment, $4850 in U. S. 3½'s at 97, or $3367.50 in U. S. 6's at 112¼? *Ans.* The latter by $5.

7. My present income from a house worth $35000, is $1050. Will I gain or lose by selling it at cost and investing the proceeds in U. S. 6's at 104¾, brokerage being ¼%? *Ans.* Gain $950.

8. How much can be realized from investing $54900 in U. S. cur. 6's, '95, at 114⅜? *Ans.* $2880.

9. A gentleman sold a lot on ground-rent, and redeemable on payment of $2300; what is the ground-rent at 7%? *Ans.* $161.

CASE II.

57. To find the investment, when the income, the market value, and the rate of interest are given.

1. How much must I invest in U. S. Pacific R. R. 6's at $105\frac{1}{4}$, brokerage $\frac{1}{4}\%$, to obtain an annual income of $1380?

OPERATION.

$1380 ÷ .06 = $23000
$23000 × $1.05\frac{1}{2}$ = $24265.

SOLUTION.—As every dollar of stock produces an income of .06, to produce an income of $1380, it will require as many dollars as .06 are contained times in $1380, which are $23000. And the cost of $23000 at $1.05\frac{1}{2}$ = $23000 × $1.05\frac{1}{2}$ = $24265.

RULE.—*Having found the par value, multiply it by the market value of 1 share, and the product will be the investment.*

WRITTEN EXERCISES.

2. A gentleman invested in U. S. $4\frac{1}{2}$'s at 112, and received an annual income of $3195; what amount did he invest? *Ans.* $79520.

3. What sum must be invested in Ohio 6's at $121\frac{1}{2}$, to obtain an income of $225? *Ans.* $4556.25.

4. If the U. S. $3\frac{1}{2}$'s are selling at $104\frac{3}{4}$, what amount must I invest in them to secure an income of $526.75 in currency, gold being $107\frac{1}{2}$, brokerage $\frac{1}{4}\%$? *Ans.* $14700.

5. Mr. Bryant sold $70000 California 7's at 112, and bought sufficient Ohio 6's at 102 to yield $4590.18 income; how much had he left, brokerage $\frac{1}{4}\%$ for buying and selling? *Ans.* $0.68.

6. How much must I invest in U. S. cur. 6's at $95\frac{3}{4}$%, brokerage $\frac{1}{4}$%, to secure an annual income of $492 ?

Ans. $7872.

7. What sum must be invested in Michigan 7's at $115\frac{1}{2}$, to give an income of $336, brokerage $\frac{1}{8}$% ?

Ans. $5550.

8. What amount should I invest in a 4% stock at 98, to secure the same income as that derived from $27000 invested in 7% stock at 90 ? *Ans.* $51450.

9. A gentleman desires to secure an income of $59.50 ; what sum must he invest in a $3\frac{1}{2}$% stock that sells at 65 ?

Ans. $1105.

10. What sum in currency must be invested in U. S. 6's at par, that the annual income in gold may be $480, if gold is 125 ? *Ans.* $10000.

11. If gold is at 116, what sum must I invest in U. S. $4\frac{1}{2}$'s at 102 to secure an income of $7308, in currency ?

Ans. $142800.

Case III.

58. To find the rate of income, when the market value and the rate of interest are given.

1. What is the rate of income upon money invested in U. S. $3\frac{1}{2}$'s at 105 ?

OPERATION.

$.03\frac{1}{2} \div 1.05 = .03\frac{1}{3}$.

Solution.—Every dollar of the investment costs $1.05 and gives $.03$\frac{1}{2}$ interest. Hence, if $1.05 gains $.035, $1 will gain as much as $1.05 are contained times in $.035, which are .03$\frac{1}{3}$ times, or $3\frac{1}{3}$%.

Rule.—*To find the rate of income, divide the rate of interest by the market value.*

WRITTEN EXERCISES.

2. If I buy a 5% bond at 85, what is my rate of income? *Ans.* $5\frac{15}{17}$%.

3. What rate of income will 6% stock yield if bought at 110? *Ans.* $5\frac{5}{11}$%.

4. What is my rate of income for a $4\frac{1}{2}$% bond bought at 96? *Ans.* $4\frac{11}{16}$%.

5. If an 8% stock be bought at 112, what is the rate of income? *Ans.* $7\frac{1}{7}$%.

6. What per cent. of his money will a man obtain by investing in 5% stock at 80? *Ans.* $6\frac{1}{4}$%.

7. What rate of income does a person receive on an investment in U. S. 3's at 125? *Ans.* $2\frac{2}{5}$%.

8. Which is the better investment, U. S. 4's at 105 or U. S. $4\frac{1}{2}$'s at 112? *Ans.* U. S. $4\frac{1}{2}$'s.

9. Gold being at 105, which is the more profitable investment, U. S. Pacific 6's at 102 or U. S. $3\frac{1}{2}$'s at 112? *Ans.* U. S. Pacific 6's.

10. A man bought a $3000 mortgage at 3% discount; if it yield 5% interest, what rate of income does he receive? *Ans.* $5\frac{15}{97}$%.

CASE IV.

59. To find the purchase price, when the rate of income and the rate of interest are given.

1. What must be paid for U. S. $4\frac{1}{2}$'s that the investment may yield 7%?

OPERATION.

$.045 \div .07 = 64\frac{2}{7}$%.

SOLUTION.—As $1 of the stock gives .045 interest, and $1 of the investment gives .07 income, the purchase *price equals* $.045 \div .07 = 64\frac{2}{7}$%.

RULE.—*To find the purchase price, divide the rate of interest by the rate of income.*

WRITTEN EXERCISES.

2. At what price did I purchase U. S. $3\frac{1}{2}$'s, if they yield me an income of 6%? *Ans.* $58\frac{1}{3}$%.

3. My income from a 10% bond is 5%; at what price did I buy it? *Ans.* 200%.

4. What rate of discount does 5% stock bear in the market, when an investment pays 8%? *Ans.* $37\frac{1}{2}$%.

5. If I desire to secure 7% on my investment, at what rate must I buy a 4% bond? *Ans.* $57\frac{1}{7}$%.

6. What must I pay for U. S. $3\frac{1}{2}$'s, in order to obtain 8% upon the investment? *Ans.* $43\frac{3}{4}$%.

7. What rate of premium does a 7% bond bear when an investment gives $5\frac{1}{2}$%? *Ans.* $27\frac{3}{11}$%.

8. At what rate of discount must 4% stock be purchased that the investment may yield 12%? *Ans.* $66\frac{2}{3}$%.

9. For what must U. S. Pacific R. R. 6's be purchased in order to realize 8% on the money invested? *Ans.* 75%.

DUTIES.

60. ***Duties*** are those taxes which the government levies upon imported goods.

61. There are two kinds of duties; viz.: *ad valorem* and *specific.*

62. An ***Ad Valorem Duty*** is a tax estimated at a certain per cent. on the cost of the goods in the country from which they are imported.

63. A ***Specific Duty*** is a certain sum computed on the weight or measure of goods without regard to their cost.

64. A ***Tariff*** is a schedule showing the rates of duties fixed by law, on all imported goods.

65. ***Tare*** is an allowance for the weight of whatever contains the goods ; as box, bag, etc.

66. ***Leakage*** is an allowance for loss of liquors contained in barrels or casks. When liquors are imported in bottles, such an allowance is called *breakage.*

67. The weight of the goods before any deductions have been made, is called the *Gross Weight;* and after all allowances have been deducted, it is termed *Net Weight.*

NOTE.—In all custom-house transactions, the long ton is used in weighing.

1. What is the duty at 30%, on an invoice of broadcloth valued at $320 ?

OPERATION.

$320 × .30 = $96.

SOLUTION.—Since the invoice value of the goods is $320, the duty is equal to 30% of $320, which is $96.

2. What is the duty, at 20%, on 35 boxes of candles each weighing 196 lb., invoiced at 10 cents per lb., tare being 5% ?

OPERATION.

196 lb. × 35 = 6860 lb.
6860 lb. × .05 = 343 lb.
6860 lb. — 343 lb. = 6517 lb.
6517 lb. × .10 = $651.70
$651.70 × .20 = $130.34

SOLUTION.—The gross weight of the 35 boxes equals 196 lb. × 35 = 6860 lb. Subtracting the tare, 343 lb., we find the net weight to be 6517 lb. The latter invoiced at 10 *cents* per pound is equal to $651.70, the net value; on which, computing the duty at 20%, we obtain $130.34, the required result.

RULE.—*I. To find ad valorem duties, multiply the invoiced cost of the goods by the given rate of duty.*

II. To find specific duties, first deduct the allowances, and then compute the duty on the remainder.

WRITTEN EXERCISES.

3. Find the duty at 60%, on 150 yd. of silk, invoiced at $2.50 per yard. *Ans.* $225.

4. What is the duty at 30%, on 40 barrels of molasses, each containing 31½ gallons, invoiced at 45¢ per gallon, allowing 1½% for leakage? *Ans.* $167.55.

5. A man bought two boxes of sugar valued, respectively, at $97.50, and $120. What was the duty at 25% ad valorem? *Ans.* $54.38.

6. An importer receives an invoice of 60 dozen bottles of porter, rated at $1.50 per dozen. If 3% of the bottles are broken, what will be the duty at 35%? *Ans.* $30.56.

7. What is the duty at 20%, on 75 boxes of bananas, at $5.25 per box, allowing 2% tare? *Ans.* $77.18.

8. The duty at 40%, on an importation of Rhine wine, was $320; what was the invoice of the wine? *Ans.* $800.

9. Mr. Price received an invoice of 800 tons of iron, costing $90 a ton. What is the duty at $6 per ton specific, and 50% ad valorem? *Ans.* $40800.

10. What is the duty at 20%, on 70 chests of tea, each containing 74 lb. invoiced at 83¢ per lb., tare being 7 lb. per chest? *Ans.* $778.54.

11. A merchant imported 300 bbl. of ale, each containing 36 gallons, invoiced at 25 cents a gallon; what was the duty at 20%, allowing 2% for leakage? *Ans.* $529.20.

12. Find the duty at 25%, on 12 dozen silver watches invoiced at $102 per dozen, 20 watches having been broken. *Ans.* $263.50.

13. How much duty must I pay at 50 cents a lb., and 35%, on an invoice of 712 lb. of woolen cloth, valued at £416 16*s.* ($4.86⅔ = £1)? *Ans.* $1065.95.

14. Find the duty at 20%, on an importation from Canada of 64960 pounds of hay, invoiced at $15 per long ton. *Ans.* $87.

15. If the duty on certain glass plates is 40 cents per square foot, what will be the duty on 420 boxes, each containing 40 plates, and each plate being 24 in. by 30 in.? *Ans.* $33600.

16. An importation of tobacco, invoiced at $2160, cost me $3039.50, after paying the duties and $15.50 for freight; what was the rate of duty? *Ans.* 40%.

17. After allowing 3% for breakage, a merchant paid $9166.50 duty at 60%, on an importation of 3500 bottles of champagne; what was the invoice price per bottle? *Ans.* $4.50.

18. Find the duty at 20¢ per yard and 30% ad valorem, on 2000 yards of carpet invoiced at 90¢ yer yard.

19. What is the duty on a shipment of goods weighing 4500 lb., and valued at $20000, at 40¢ per lb. and 30% ad valorem?

20. What is the entire duty on the following: 3000 lb. raisins at 2¢ per lb.; $350 worth of oranges at 20% ad valorem; 2000 boxes lemons at 35¢ per box; and 125 chests of tea, each containing 80 pounds invoiced at 40¢ per pound, at 30% ad valorem?

21. Find the duty on 850 pounds of tobacco valued at 80¢ per pound, at 50¢ per pound and 25% ad valorem.

22. What amount of duty must I pay on 80 hogsheads of sugar, each hogshead weighing 650 pounds, at 2¢ a pound, tare being 50 lb. per hogshead?

INSURANCE.

68. ***Insurance*** is a contract by which one party engages to indemnify another for loss or damage.

69. There are two kinds of insurance: viz., insurance on property, and insurance on life.

70. The ***Insurer*** is the party assuming the risk. The party protected is called the *Insured* or *Assured.*

71. The ***Policy*** is the written contract binding the parties concerned.

72. The ***Premium*** is the money paid for the insurance. It is reckoned at a certain rate per cent. on the amount insured. The difference between the amount insured and the premium is termed the *Sum Covered.*

CASE I.

73. To find the premium, when the amount insured and the rate are given.

1. Mr. Brownson insured his store for $8640 at $2\frac{1}{2}\%$; what amount of premium does he pay?

OPERATION.

$8640
.02½
$216.00, premium.

SOLUTION.—The premium is equal to $2\frac{1}{2}\%$ of the amount insured. Hence, .02½ times $8640 = $216.00, the premium.

RULE.—*To find the premium, multiply the amount insured by the rate.*

WRITTEN EXERCISES.

2. What is the premium for insuring a house and furniture for $9660 at $\frac{3}{4}\%$? *Ans.* $72.45.

3. What premium is to be paid on a policy of $14000 at $1\frac{1}{4}\%$? *Ans.* $175.

4. What is the charge for effecting an insurance of $15870 at $3\frac{1}{3}\%$, on a vessel and cargo, cost of policy being $1.75? *Ans.* $530.75.

5. Insured a factory valued at $16000 for $\frac{3}{4}$ of its value, at $1\frac{3}{4}\%$; what premium did I pay? *Ans.* $210.

6. Mr. Parsons insured his house for $4000, at $1\frac{1}{8}\%$, and his furniture for $300 at $\frac{1}{4}\%$; what was the premium?

7. A cargo of 32000 bushels of wheat at $1.15 per bu., has been insured for $\frac{3}{5}$ of its value, at $2\frac{1}{8}\%$; what is the premium? *Ans.* $469.20.

8. What is the premium for 3 years on a policy of $6000 at 3% each year? *Ans.* $540.

9. A merchant insured 2000 barrels of pork, worth $16.00 a barrel, for $\frac{3}{4}$ of its value, at $\frac{1}{2}\%$; what was the premium? *Ans.* $120.

10. Find the premium on a $3000 policy dated June 1, 1882, and expiring December 1, 1882, annual rate being $\frac{3}{4}\%$. *Ans.* $11.25.

Case II.

74. To find the amount insured, when the rate and the premium are given.

1. A gentleman paid a premium of $270 for having his factory insured at $3\frac{1}{3}\%$; what was the amount of the insurance?

OPERATION.

$\$270 \div .03\frac{1}{3} = \$8100$

Solution.—As the rate of premium is $3\frac{1}{3}\%$, and as $.03\frac{1}{3}$ times the amount insured equals the premium, $270, therefore $\$270 \div .03\frac{1}{3} = \$8100$, the amount insured.

Rule.—*To find the amount insured, divide the premium by the rate.*

WRITTEN EXERCISES.

2. If \$66 were paid for an insurance of merchandise, at $1\frac{1}{3}\%$, what was the amount insured? *Ans.* \$4950.

3. The premium for insuring $\frac{5}{7}$ of the value of a factory, at $2\frac{1}{2}\%$, was \$150; what was the value of the factory? *Ans.* \$8400.

4. A man paid \$750 for insuring a saw-mill, at $7\frac{1}{2}\%$; what was the value of the mill? *Ans.* \$10000.

5. What is the value of a house and furniture, for insuring which a man pays a premium of \$161, at $1\frac{3}{4}\%$? *Ans.* \$9200.

6. The cargo of a vessel consisting of 7000 bushels of corn, worth 56 cents a bushel, is insured at $3\frac{1}{2}\%$ on $\frac{3}{4}$ of its value; if the vessel be lost, how much will the owners of the corn lose? *Ans.* \$1082.90.

7. A merchant has his store valued at \$83450, insured at $2\frac{1}{4}\%$. If it were destroyed by fire, what would be the actual loss of the insurance company? *Ans.* \$81572.38.

8. An insurance company took a risk of \$25000 for a premium of \$125; what was the rate of insurance?

SOLUTION.—The rate of insurance = \$125 ÷ \$25000 = $\frac{1}{2}\%$.

9. I effected an insurance of \$3960 on my factory, paying \$68.35, including \$4.00 as the cost of the policy; what was the rate of insurance? *Ans.* $1\frac{5}{8}\%$.

10. The Manhattan Insurance Company took a risk at $3\frac{1}{2}\%$, and re-issued $\frac{2}{3}$ of it in another company at $2\frac{3}{4}\%$. The premium received exceeded the premium paid by \$40; what was the amount of the risk? *Ans.* \$2400.

11. Mr. Ryan insured his store and stock for $\frac{5}{8}$ of its value, at 2%, and paid \$380; if destroyed by fire, what will be his actual loss? *Ans.* \$11780.

12. A cargo of coal was insured at $3\frac{2}{3}\%$, to cover $\frac{4}{5}$ of its value. The premium paid was $191.84; if the cargo be lost what loss will the insurance company sustain?

13. On May 1, 1877, Mr. Dunn insured his house for $12000, for 10 years, at $1\frac{1}{2}\%$; what was the value of the unearned premium, May 1, 1884? *Ans.* $540.

14. A quantity of merchandise valued at $120000 and insured for $50000, is damaged to the amount of $40000; supposing the policy to contain the "average clause," how much of the loss does the insurance company pay? *Ans.* 16666\frac{2}{3}$.

NOTE.—A policy containing the "average clause," entitles the owner only to such a proportional part of the loss as the insurance is of the value of the property.

15. A laboratory worth $15000, and insured for $\frac{2}{3}$ of its value, at an annual premium of $2\frac{1}{2}\%$, was burned at the end of 4 years; what was the owner's loss? *Ans.* $6000.

16. For an insurance of $5000 on my house, and $960 on my barn, I paid for 3 years, a premium of $156.45. What was the annual rate?

SOLUTION.—$156.45 ÷ 3 = $52.15; and $52.15 ÷ $5960 = $\frac{7}{8}\%$.

17. What amount of insurance may I receive on my hotel, for a premium of $337.50, at $2\frac{1}{4}\%$?

18. A merchant had his store and goods insured at $2\frac{3}{8}\%$, paying $570 premium; what was the value of the policy? *Ans.* $24000.

19. J. C. Bryant pays a premium of $187.50 for insuring his house for $\frac{3}{4}$ of its value, at $3\frac{1}{8}\%$; what is the value of his house? *Ans.* $8000.

20. What amount is covered by an insurance, the premium of which is $42 for 3 years, at an annual rate of $\frac{5}{8}\%$? *Ans.* $1638.

LIFE INSURANCE.

75. ***Life Insurance*** is a contract by which a company agrees to pay a person, if he survive a specified age, or his heirs, if he die before attaining that age, a certain stated sum of money.

76. The ***Policies*** of life insurance are:

1. *Endowment Policies,* which are payable at a specified age, or at death, if it occur before that age.
2. *Term Policies,* which are payable at death, provided that event happens within a certain period.
3. *Ordinary Life Policies,* which are payable at death, without regard to time.

77. The ***Premium*** for life insurance is at fixed rates, determined by a table of mortality, the probable value of money, and a margin for expenses.

In the endowment policies, the premium is payable annually, or in one, five, or ten annual payments; in term policies, it is payable annually; and in life policies, it is payable annually during life, or in one, five, or ten annual payments.

1. What annual premium must I pay for a life policy of $10000, if I am 30 years of age?

OPERATION.

$22.70 × 10.000 = $227.

SOLUTION.—On examining the table, we find the premium for life policies, at the age of 30, to be $22.70 for $1000 insurance. Hence, for $10000 insurance the premium will be 10.000 times greater, or $22.70 × 10.000 = $227.

RULE.—*Multiply the premium for the given time and age, by the amount of the policy, and divide the product by 1000.*

TABLE.

ANNUAL PREMIUM FOR AN INSURANCE OF $1000.

LIFE POLICIES.					ENDOWMENT POLICIES.				
ANNUAL PAYMENTS.									
AGE	FOR LIFE.	10 YEARS.	15 YEARS.	20 YEARS.	AGE	10 YEARS.	15 YEARS.	20 YEARS.	AGE
25	$19 89	$42 56	$32 34	$27 39	25	$103 91	$66 02	$47 68	25
26	20 40	43 37	32 97	27 93	26	104 03	66 15	47 82	26
27	20 93	44 22	33 62	28 50	27	104 16	66 29	47 98	27
28	21 48	45 10	34 31	29 09	28	104 29	66 44	48 15	28
29	22 07	46 02	35 02	29 71	29	104 43	66 60	48 33	29
30	22 70	46 97	35 76	30 36	30	104 58	66 77	48 53	30
31	23 35	47 98	36 54	31 03	31	104 75	66 96	48 74	31
32	24 05	49 02	37 35	31 74	32	104 92	67 16	48 97	32
33	24 78	50 10	38 20	32 48	33	105 11	67 36	49 22	33
34	25 56	51 22	39 09	33 26	34	105 31	67 60	49 49	34
35	26 38	52 40	40 01	34 08	35	105 53	67 85	49 79	35
36	27 25	53 63	40 98	34 93	36	105 75	68 12	50 11	36
37	28 17	54 91	42 00	35 83	37	106 00	68 41	50 47	37
38	29 15	56 24	43 06	36 78	38	106 28	68 73	50 86	38
39	30 19	57 63	44 17	37 78	39	106 58	69 09	51 30	39
40	31 30	59 09	45 33	38 83	40	106 90	69 49	51 78	40
41	32 47	60 60	46 56	39 93	41	107 26	69 92	52 31	41
42	33 72	62 19	47 84	41 10	42	107 65	70 40	52 89	42
43	35 05	63 84	49 19	42 34	43	108 08	70 92	53 54	43
44	36 46	65 57	50 61	43 64	44	108 55	71 50	54 25	44
45	37 97	67 37	52 11	45 03	45	109 07	72 14	55 04	45
46	39 58	69 26	53 68	46 50	46	109 65	72 86	55 91	46
47	41 30	71 25	55 35	48 07	47	110 30	73 66	56 89	47
48	43 13	73 32	57 10	49 73	48	111 01	74 54	57 96	48
49	45 09	75 49	58 95	51 50	49	111 81	75 51	59 15	49
50	47 18	77 77	60 91	53 38	50	112 68	76 59	60 45	50

This table indicates the maximum rates of premium charged by the principal Insurance Companies of New York.

WRITTEN EXERCISES.

2. If a man is 29 years old, what annual premium must he pay for a life policy of $20000?

Ans. $441.40.

3. What annual premium will a man, aged 42 years, pay to receive an endowment policy for $1200, payable in 10 years? *Ans.* $129.18.

4. A lady, 28 years of age, takes out an endowment policy for $4000, payable in 15 years; how much does she receive more than she pays? *Ans.* $13.60.

5. A gentleman, aged 32, takes a life policy for $20000. He lives 20 years after insuring; how much will his heirs receive more than was paid? *Ans.* $10380.

6. What is the semi-annual premium on a 10-year endowment policy for $8000, age 50? *Ans.* $450.72.

7. A clergyman, aged 29, takes an endowment policy for $6000, payable in 15 years. If the dividend increases his policy $3000, how much more will he receive than he has paid the company? *Ans.* $3006.

8. Mr. Mackay, at the age of 37, took an endowment policy for $13000, payable in 10 years. When he died his family had a net gain of $4732; how many payments had he made? *Ans.* 6.

9. A lawyer had his life insured at the age of 35, taking a life policy, payable in 20 years. How old will he be when the amount of the premium paid shall exceed the face of the policy? *Ans.* 64 years.

TAXES.

78. A ***Tax*** is a certain sum of money assessed on persons and property in order to defray the expenses of a city, county, or State.

79. There are two general classes of taxes; viz., those levied by a State, county, or town; and those levied by the General Government.

80. A ***Property Tax*** is a sum assessed, at a certain rate, on the estimated value of real estate or personal property.

81. A ***Poll Tax*** is a sum levied on each male citizen, without regard to his property.

CASE I.

82. To find the tax, when the rate and the value of the taxable property are given.

1. The rate of taxation of a certain county was $2\frac{1}{2}$ mills, and the value of the property \$864000; what was the tax?

OPERATION.

$\$864000 \times .002\frac{1}{2} = \$2160.$

SOLUTION.—At the rate of $.002\frac{1}{2}$ on a dollar, the tax on \$864000 will be $.002\frac{1}{2}$ times \$864000 = \$2160.

RULE.—*To find the amount of tax, multiply the value of the taxable property by the rate.*

WRITTEN EXERCISES.

2. The taxable property of a certain town is valued at \$182540, and taxed at the rate of 4 mills on a dollar. If there are 140 polls, at \$1.20 each, what is the whole tax? *Ans.* \$898.16.

3. What is the tax of a person whose property is assessed at \$24800, if he pays 2% town tax, and $1\frac{1}{2}$% school tax? *Ans.* \$868.

4. The real estate of a certain town is valued at \$1400000. There are 960 polls, at 75 cents each; what is the whole tax, the rate being 5 mills on the dollar? *Ans.* \$7720.

5. In No. 4, what is B's whole tax whose property is valued at \$8000, and who pays for 3 polls? *Ans.* \$42.25.

6. A tax of \$11004 is to be assessed on a certain village; the value of the taxable property is \$1800000; and there are 340 polls to be assessed 60 cents each; how

much will be the tax of A, whose property is valued at $5240, and who pays for 5 polls? *Ans.* $34.44.

OPERATION.

.60 × 340 = $204 = amount raised by poll-tax.
$11004 — $204 = $10800 = " " on property.
$10800 ÷ $1800000 = .006 = tax on $1, or rate %.
.006 × $5240 = $31.44 = tax on A's property.
$31.44 + $3.00 = $34.44 = total amount of A's tax.

After finding the tax on $1, or the rate per cent., assessors greatly facilitate the operation by constructing a table on the following plan:

TABLE.

Property.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1 gives..	$.006	$10	$.06	$100	$ .60	$1000	$ 6.00
2 " ..	.012	20	.12	200	1.20	2000	12.00
3 " ..	.018	30	.18	300	1.80	3000	18.00
4 " ..	.024	40	.24	400	2.40	4000	24.00
5 " ..	.030	50	.30	500	3.00	5000	30.00
6 " ..	.036	60	.36	600	3.60	6000	36.00
7 " ..	.042	70	.42	700	4.20	7000	42.00
8 " ..	.048	80	.48	800	4.80	8000	48.00
9 " ..	.054	90	.54	900	5.40	9000	54.00

7. Find by the table, the tax of a gentleman whose property is $4980, and who pays for 7 polls, at the rate of $.90 each. *Ans.* $36.18.

8. What amount of tax is paid by a gentleman whose property is valued at $7240? *Ans.* $43.44.

9. Mr. Levins owns $6450 worth of real estate, assessed at $\frac{2}{3}$ of its value, and $2350 assessed personal property; what is his tax, according to the table given above? *Ans.* $39.90.

10. A certain school is supported by a rate-bill based on an attendance of 5750 days. If the teacher's wages and other expenses amount to $345, what is the rate? What does A pay for three pupils, 250 days each?

Ans. Rate, .06.

11. If Mr. Bell's property is assessed at $19800 and he pays .012 school tax, .016 state tax, $3.75 poll tax; what amount of taxes does he pay? *Ans.* $558.15.

12. The tax levied on a certain town is $25900, the value of the property is $531000, and there are 940 polls at $1.50 each. Find the rate of tax, and construct a table. *Ans.* Rate, .046+.

13. A corporation consisting of 50 persons, is taxed $6340; their property is assessed for $720000, and each poll is assessed 90 cents; what is the rate of taxation?

Ans. .0087+.

14. What is the total tax on 4187½ pounds tobacco at 16¢, 2180 gallons distilled spirits at 70¢, and 1082½ barrels beer at $1? *Ans.* $3278.50.

Case II.

83. To find the amount assessed, when the tax and the rate are given.

1. If my tax is $123.75, at the rate of 4½ mills on a dollar, what is the assessed value of my property?

OPERATION.

$123.75 ÷ .0045 = $27500

Solution.—If the rate is 4½ mills on a dollar, to produce a tax of $123.75 it will require as many dollars as .0045 are contained times in $123.75, which are $27500.

Rule.—*To find the amount assessed, divide the amount of tax by the rate.*

WRITTEN EXERCISES.

2. Mr. Randall's property is taxed at the rate of 7 mills on a dollar, and his taxes amount to $275.80; what is his property worth? *Ans.* $39400.

3. Mr. Butler pays a tax of $592.90, at the rate of 6½ mills on a dollar, including 6 polls at $1.75 each; what is the value of his property? *Ans.* $89600.

4. A stockholder is assessed $450, at the rate of 4½ cents on the dollar, to meet a loss sustained by the company. What is the value of his stock? *Ans.* $10000.

INTEREST.

84. ***Interest*** is a sum paid for the use of money, or its equivalent.

85. The ***Principal*** is the sum for the use of which interest is paid.

86. The ***Rate*** is a certain per cent. of the principal which indicates the interest on one dollar for one year.

87. The ***Time*** is the period during which the money is on interest.

88. The ***Amount*** is the sum of the principal and the interest.

89. ***Simple Interest*** is the sum charged for the use of the principal.

90. ***Exact Interest*** is that which is computed by reckoning 365 days to the year.

NOTE.—In computing interest it is customary to reckon a month as $\frac{1}{12}$ of a year, and a day as $\frac{1}{30}$ of a month. In dealing with the U. S. Government, each day is $\frac{1}{365}$ of a year.

91. The ***Legal Rate*** is the rate established by law.

In the first column of the following table may be seen the legal rate of interest for the several States and Territories; and in the second, the maximum rate allowed in contracts.

State or Territory.	Rate.		State or Territory.	Rate.	
Alabama.....	8%	8%	Mississippi.....	6%	10%
Alaska (Ter.)...	....	....	Missouri.....	6%	10%
Arkansas.....	6%	10%	Montana (Ter.)...	10%	Any
Arizona (Ter.)..	10%	Any	Nebraska.....	7%	10%
California.....	7%	Any	Nevada.....	10%	Any
Colorado.....	10%	Any	New Hampshire..	6%	6%
Connecticut....	6%	6%	New Jersey.....	6%	6%
Dakota (Ter.)..	7%	12%	New Mexico (Ter.)	6%	12%
Delaware.....	6%	6%	New York.....	6%	6%
Florida.....	8%	Any	North Carolina...	6%	8%
Georgia.....	7%	8%	Ohio.....	6%	8%
Idaho (Ter.)....	10%	18%	Oregon.....	8%	10%
Illinois.....	6%	8%	Pennsylvania.....	6%	6%
Indian (Ter.)...	6%	Any	Rhode Island....	6%	Any
Indiana.....	6%	8%	South Carolina...	7%	10%
Iowa.....	6%	10%	Tennessee.....	6%	6%
Kansas.....	7%	12%	Texas.....	8%	12%
Kentucky.....	6%	6%	Utah (Ter.).....	10%	Any
Louisiana.....	5%	8%	Vermont.....	6%	7%
Maine.....	6%	Any	Virginia.....	6%	6%
Maryland.....	6%	6%	Washington (Ter.)	10%	Any
Massachusetts..	6%	Any	West Virginia....	6%	6%
Michigan.....	7%	10%	Wisconsin.....	7%	10%
Minnesota.....	7%	10%	Wyoming (Ter.)..	12%	Any

92. The elements of simple interest are the *Principal*, the *Interest*, the *Rate*, the *Time*, and the *Amount*.

Case I.

93. To find the interest or the amount, when *the principal*, the rate, and the time are given.

MENTAL EXERCISES.

1. What is the interest of $5 at 6%, for one year?

SOLUTION.—Since the interest of $1 for 1 year is 6%, or 6 cents, the interest of $5 for the same time is 5 times as great, or 5 times 6 cents, which are 30 cents.

2. What is the interest of $6 at 6%, for one year and 6 months?

SOLUTION.—6 months equal $\frac{6}{12}$ or $\frac{1}{2}$ of a year, which with 1 year, equals $1\frac{1}{2}$ or $\frac{3}{2}$ years. According to the first solution, the interest of $6 at 6%, for 1 year, equals 36 cents. Therefore, the interest of $6, at 6%, for $\frac{3}{2}$ years equals $\frac{3}{2}$ of 36 cents = 54 cents.

3. What is the interest of $1 for 1 year, at 6%? At 5%? At 4%? At 8%? At 10%?

4. What is the interest of $7 at 6%, for 1 year? Of $8? Of $9? Of $20? Of $30? Of $50?

5. What is the interest of $10 at 6%, for 1 year? For 2 years? For 4 years? For 8 years? For 12 years?

6. What is the interest of $5 at 6%, for 2 yr.? For 2 yr. and 6 mo.? For 3 yr. 6 mo.? For 5 yr. 6 mo.? For 8 yr. 6 mo.?

7. What is the interest of $100 at 6%, for 1 year? Of $200? Of $500? Of $800? Of $1000? Of $350?

8. What is the interest of $100 at 6%, for 6 mo.? For 3 mo.? For 4 mo.? For 8 mo.? For 9 mo.?

9. What is the interest of $200 at 6%, for 1 month? For 2 mo.? For 5 mo.? For 7 mo.? For 10 mo.? For 11 mo.?

10. What is the yearly interest of $300 at 6%? At 5%? At 10%? At 3%? At 8%?

11. What is the yearly interest of $40 at 4%? Of $20? Of $80? Of $35? Of $75?

12. What is the yearly interest of \$70 at 7%? Of \$60 at 10%? Of \$125 at 6%? Of \$150 at 4%?

13. What is the interest of \$40 for 3 years at 4%? For 6 years at 5%? For 10 years at 8%? For 1 year at $6\frac{1}{2}$%?

14. What is the interest of \$20 for 8 years at $6\frac{1}{4}$%? For 5 years at $8\frac{1}{2}$%? For 3 years at $9\frac{1}{2}$%?

15. What is the interest of \$400 for 4 years at 7%? Of \$500? Of \$200? Of \$1000? Of \$700?

16. What is the interest of \$35.50 for 2 years at 10%? Of \$47.50? Of \$86.50? Of \$94.50?

17. What is the interest of \$80 at 6%, for 3 yr. 4 mo. 15 da.?

SOLUTION.—15 da. $= \frac{15}{30}$, or $\frac{1}{2}$ of a month; which with 4 months equals $4\frac{1}{2}$ or $\frac{9}{2}$ mo. $\frac{9}{2}$ mo. $= \frac{9}{24}$, or $\frac{3}{8}$ of a year. $\frac{3}{8}$ of a year and 3 years equal $3\frac{3}{8}$ or $\frac{27}{8}$ years. The interest of \$1 for 1 year at 6%, equals .06, or $\frac{6}{100}$. And the interest of \$1 for $\frac{27}{8}$ years $= \frac{6}{100} \times \frac{27}{8}$. Therefore, the interest of \$80 at 6%, for $\frac{27}{8}$ years $= \frac{6}{100} \times \frac{27}{8} \times 80 =$ \$$16\frac{1}{8}$.

18. What is the interest of \$50 at 10%, for 5 yr. 3 mo. and 10 da.? For 2 yr. 3 mo. and 15 da.?

19. What is the interest of \$100 at 12%, for 1 yr. 9 mo. and 12 da.? For 2 yr. 3 mo. and 20 da.?

20. What is the interest of \$300 at 8%, for 5 yr. 10 mo. and 25 da.? For 10 yr. 10 mo. and 10 da.?

21. What is the interest of \$60 at 5%, for 9 mo. and 3 da.? For 11 months? For 25 days?

22. What is the interest of \$500 for 15 da., at 12%? For 18 da.? For 21 da.? For 9 da.?

23. What is the interest of \$10 at 9%, for one day? For one month? For $12\frac{1}{2}$ years? For 3 months? For 15 days?

24. What is the amount of \$25 at 6%, for three years?

SOLUTION.—The interest of \$25 at 6%, for 3 years equals $\$25 \times \frac{6}{100} \times 3 = \$4\frac{1}{2}$. To find the amount, add the interest to the principal. Thus, $\$25 + \$4\frac{1}{2} = \$29\frac{1}{2}$.

25. What is the amount of \$200 at 5%, for 6 yr.? For 9 yr.? For 8 yr. and 9 mo.?

26. What is the amount of \$120 for 11 mo., at 7%? For 10 mo., at 9%? For 2 yr. and 6 mo., at 4%?

27. What is the amount of \$440 for 1 yr. and 10 mo., at 8%? For 3 yr., at 10%? For 5 yr., at 5%?

28. What is the amount of \$150 for 6 months, at $3\frac{1}{3}$%? For 9 months, at 15%? For 4 years, at $7\frac{1}{2}$%?

29. What is the amount of \$1000 for 10 years, 2 months, and 12 days, at 4%? For 5 years, 1 month, and 6 days?

30. What is the amount of \$80 for 5 years and 5 months, at 5%? For 4 years and 4 months, at 4%?

31. What is the amount of \$500 for 8 years and 6 months, at 6%? For 6 years and 3 months, at 10%?

WRITTEN EXERCISES.

1. What is the interest of \$5000 for 9 years, 3 months, 9 days, at 8%? Amount?

OPERATION.

```
  $5000
    .08
 $400.00 Int. for 1 yr.
   9 11/40
  11000
 360000
$3710.00 Int. for 9 yr. 3 mo. 9 da.
$5000.
$8710.00 Amount.
```

SOLUTION.—Reducing the 3 mo. and 9 da. to a fraction of a year, we obtain $\frac{11}{40}$. The interest of \$5000, at 8%, for 1 year, equals .08 times \$5000, or \$400. Hence, the interest of \$5000 for $9\frac{11}{40}$ years, equals $9\frac{11}{40}$ times \$400 = \$3710. Adding the principal, \$5000, to the interest for the given time, we get \$8710 for the amount.

RULE.—*I. Multiply the principal by the rate, and that product by the time expressed in years. The result will be the required interest.*

II. Add the interest to the principal to find the amount.

Formulas: (1) $I = P \times R \times T$. (2) $A = P + I$.

Find the interest of: Amount of:

2.	$240 for 2 yr. 6 mo., at 5%.	*Ans.* $30.
3.	$520 for 1 yr. 4 mo., at 6%.	*Ans.* $41.60.
4.	$850 for 7 yr. 8 mo., at 5%.	*Ans.* $325.83.
5.	$740 for 6 yr. 10 mo., at 7%.	*Ans.* $353.97.
6.	$920 for 3 yr. 6 mo., at 6%.	*Ans.* $193.20.
7.	$1225, 1 yr. 8 mo. 15 da., 4%.	*Ans.* $83.71.
8.	$5040, 2 yr. 4 mo. 9 da., 8%.	*Ans.* $950.88.
9.	$8429, 5 yr. 9 mo. 20 da., 6%.	*Ans.* $2936.10.
10.	$9510, 4 yr. 7 mo. 6 da., 8%.	*Ans.* $3499.68.
11.	$3562, 3 yr. 2 mo. 5 da., 7%.	*Ans.* $793.04.
12.	$4845, 7 yr. 8 mo. 12 da., 6%.	*Ans.* $2238.39.
13.	$2060, 9 yr. 6 mo. 10 da., 5%.	*Ans.* $981.36.
14.	$1865, 2 yr. 10 mo. 25 da., 6%.	*Ans.* $324.82.
15.	$1340, 1 yr. 5 mo. 6 da., 9%.	*Ans.* $172.86.

SIX PER CENT. METHOD.

1. What is the interest of $180 for 6 yr. 10 mo. 12 da., at 6%?

OPERATION.

Int. of $1 for 6 yr. = .06 × 6 = $0.36
" " " " 10 mo. = .00⅓ × 10 = .05
" " " " 12 da. = .000⅙ × 12 = .002
" " " " 6 yr. 10 mo. 12 da. = $0.412
" " $180 for 6 yr. 10 mo. 12 da. = .412 × 180
.412 × 180 = $74.16

SOLUTION.—The interest on \$1 for 1 yr., at 6%, is 6¢. For 6 yr. it is 6 times 6¢, or 36¢. For 1 month, which is $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of 6¢, or $\frac{1}{2}$ of a cent. Hence for 10 mo. it is 10 times $\frac{1}{2}$¢, or .05. Finally, since the interest on \$1 for 1 mo. is $\frac{1}{2}$¢, or 5 mills, for 1 day it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill; and, therefore, for 12 days it is 12 times $\frac{1}{6}$ of a mill, or .002. Thus we find the interest of \$1 for 6 yr. 10 mo. 12 da., to be \$0.412. Multiplying the interest on \$1 for the given time, by \$180, the principal, we obtain \$74.16, the required interest.

RULE.—*I. Multiply the rate, 6%, by the number of years, take $\frac{1}{2}$ of the number of months as cents, and $\frac{1}{6}$ of the number of days as mills.*

II. Add these partial interests for the interest of \$1 for the given time at 6%.

III. Multiply the sum thus obtained by the principal, and the product will be the required interest.

IV. For any other rate, first find the interest at 6%, and then take as many sixths of it as that rate is of six.

WRITTEN EXERCISES.

What is the interest of:

2. \$648 for 4 yr. 10 mo., at 6%? *Ans.* \$187.92.
3. \$2980, 11 yr. 6 mo. 5 da., 6%? *Ans.* \$2058.68.
4. \$430, 5 yr. 9 mo., 5%? *Ans.* \$123.63.
5. \$870, 1 yr. 6 mo. 12 da., 7%? *Ans.* \$93.38.
6. \$740, 7 yr. 6 mo., 7%? *Ans.* \$388.50.
7. \$62.75, 1 yr. 11 mo. 29 da., 4%? *Ans.* \$5.01.
8. \$8040, 9 yr. 10 mo. 24 da., 5%? *Ans.* \$3979.80.
9. \$9000, 5 yr. 5 mo. 9 da., 7%? *Ans.* \$3428.25.
10. \$27.86, 9 yr. 1 mo. 3 da., $4\frac{1}{2}$%? *Ans.* \$11.40.
11. \$1000, 1 yr. 4 mo., 6%? *Ans.* \$80.
12. \$420, 2 yr. 8 mo., 5%? *Ans.* \$56.
13. \$6000, 3 yr. 20 da., 6%? *Ans.* \$1100.

METHOD OF ALIQUOT PARTS.

1. What is the interest of $350 for 4 yr. 5 mo. 10 da., at 6%?

OPERATION.

$350 × .06 =	$21.00	= Int.	1 yr.
$21.00 × 4 =	$84.00	= "	4 yr.
4 mo. = $\frac{1}{3}$ yr.	7.00	= "	4 mo.
1 mo. = $\frac{1}{4}$ of $\frac{1}{3}$ yr.	1.75	= "	1 mo.
10 da. = $\frac{1}{3}$ mo.	.58$\frac{1}{3}$	= "	10 da.
	93.33\frac{1}{3}$		

SOLUTION.—By the usual method we find the interest for 1 year, and then for 4 years. 5 months are equal to 4 mo. + 1 mo. As 4 months = $\frac{1}{3}$ of a year, we take $\frac{1}{3}$ of $21, the interest for 1 year, which is $7. The interest for 1 month is equal to $\frac{1}{12}$ of $21 = $1.75. And the interest for 10 da., or $\frac{1}{3}$ of a month, equals $\frac{1}{3}$ of $1.75 = .58$\frac{1}{3}$. The entire interest, therefore, is the sum of these partial interests, or 93.33\frac{1}{3}$.

RULE.—*I. Find the interest for the given number of years by the first method.*

II. Take convenient fractional parts of one year's interest as the interest for the given number of months.

III. Take fractional parts of one month's interest as the interest for the given number of days.

IV. Add these results for the entire interest.

WRITTEN EXERCISES.

Find the interest of:

2. $640 for 7 years 6 mo., at 6%. *Ans.* $288.

3. $840 for 5 years 8 mo., at 5%. *Ans.* $238.

4. $1240 for 3 years 9 mo., at 7%. *Ans.* $325.50.

5. $25.50 for 8 years 3 mo., at 6%. *Ans.* $12.62.

6. $16.40 for 10 years 4 mo. 15 da., at 6%.

7. $56.25 for 2 years 6 mo. 20 da., at 8%.
8. $0.25 for 50 years 6 mo. 6 da., at 10%.
9. $2000 for 8 years 8 mo. 8 da., at 8%.
10. $1 for 20 years 9 mo. 18 da., at 5%.
11. $7500 for 10 yr. 4 mo. 6 da., at 6%.

METHOD OF EXACT INTEREST.

1. What is the exact interest of $670 from March 10, 1885, to June 1, 1887, at 8%?

OPERATION.

$670 × .08 = $53.60 Int. 1 yr.
$53.60 × 2 = $107.20 " 2 "
$53.60 × $\frac{83}{365}$ = $12.18 + " 83 da.
$119.38 + *Ans.*

SOLUTION.—The interest of $670 for 1 year, or 365 days, at 8%, is $53.60, and the interest for 2 years is $107.20. From March 10, to June 1, there are 83 days. And if the interest of $670 for one year is $53.60, for 83 days, it is $\frac{83}{365}$ of $53.60, which is $12.18 +. Hence the entire interest equals $107.20 + $12.18 + = $119.38 +.

RULE.—*Having found the interest for the integral number of years, multiply the interest for one year by the exact number of days, and divide by 365. The sum of the two results will be the required interest.*

WRITTEN EXERCISES.

2. Find the interest, at 5%, of $236.80, from May 12, 1873, to July 25, 1876. *Ans.* $37.92.

3. What is the interest of $460.20, at 6%, from Aug. 15, 1864, to Oct. 20, 1869? *Ans.* $143.05.

4. What is the interest of $1500, at 7%, from Jan. 10, 1875, to May 13, 1879? *Ans.* $455.38.

5. Mr. Larkins gave his note, Oct. 2, 1876, for $512, interest at 6%. He paid the note and interest Dec. 16, 1881. What amount did he pay? *Ans.* $671.91.

6. What interest must S. S. Cox pay on $900, at 5%, from April 26, 1865, to Aug. 30, 1869? *Ans.* $195.53.

7. Find the amount of $2180.20, at 8%, from Sept. 3, 1880, to November 28, 1884. *Ans.* $2918.96.

8. What fractional part of common interest will give exact interest? *Ans.* $\frac{72}{73}$.

9. What is the difference between the exact and the common interest of $600, at 5%, from Feb. 5, 1872, to July 7, 1875? *Ans.* $0.17.

10. How much should I receive for a note of $1000, with interest at 7%, from Jan. 10, 1877, to May 25, 1879? *Ans.* $1165.89.

Case II.

94. To find the principal, when the time, the rate, and the interest, or the amount are given.

MENTAL EXERCISES.

1. What principal, in 5 yr. 3 mo., at 5%, will give $85.05?

Solution.—The interest of $1 for 1 yr., at 5%, is $\frac{5}{100}$, or $\frac{1}{20}$ of the principal; and the interest for $5\frac{1}{4}$ yr., is $\frac{21}{4}$ times $\frac{1}{20} = \frac{21}{80}$ of the principal. Hence, $\frac{21}{80}$ of the principal is equal to $85.05; $\frac{1}{80} = \$4.05$; and $\frac{80}{80} = \$4.05 \times 80 = \$324.00$, the principal.

Note.—As the following are exercises in simple or ordinary interest, take 30 days to the month.

What principal will produce:

2. $42 in 2 yr. 4 mo., at 9%? $21? $84?

3. $1200 in 6 yr. 8 mo., at 6%? $600? $300?

4. $390 in 7 yr. 6 mo., at 5%? $130? $260?
5. $212 in 8 yr. 10 mo., at 6%? $636? $1060?
6. $900 in 7 mo. 15 da., at 10%? $180? $450?
7. $760 in 15 mo. 25 da., at 6%? $380? $570?
8. $520 in 10 yr. 10 mo., at 6%? $130? $390?
9. $270 in 7 mo. 6 da., at 4½%? $360? $450?
10. $840 in 1 yr. 6 mo., at 4%? $210? $120?
11. $460 in 5 yr. 9 mo., at 10%? $230? $690?
12. $310 in 2 yr. 7 mo., at 5%? $155? $620?
13. $600 in 3 yr. 9 mo., at 4%? $150? $1200?
14. $1 in 2 yr. 6 mo., at 8%? $100? $200?
15. $70 in 2 yr. 1 mo., at 7%? $140? $280?
16. $10.50 in 1 yr. 2 mo., at 6%? $210? $420?
17. $20.60 in 6 yr. 8 mo., at 5%? $412? $10.30?
18. $12.40 in 8 mo. 10 da., at 12%? $62? $124?
19. $18.25 in 3 yr. 4 mo., at 10%? $73? $365?

What principal will amount to:

20. $198 in 10 yr. 10 mo., at 6%? $660? $231?
21. $320 in 6 yr. 8 mo., at 9%? $560? $640?
22. $660 in 5 yr. 10 mo., at 8%? $1320? $2640?
23. $700 in 1 yr. 8 mo., at 10%? $420? $630?
24. $177 in 6 yr. 5 mo., at 12%? $354? $885?
25. $705 in 6 yr. 10 mo., at 6%? $141? $282?
26. $333 in 9 yr. 6 mo., at 7%? $666? $3330?
27. $217 in 2 mo. 5 da., at 4%? $434? $651?
28. $214 in 6 yr. 9 mo., at 5%? $270? $540?
29. $443 in 4 yr. 4 mo., at 11%? $886? $1329?
30. $174 in 9 yr. 3 mo., at 8%? $348? $522?
31. $602 in 8 yr. 5 mo., at 6%? $1204? $2408?
32. $860 in 10 yr. 10 mo., at 4%? $430? $4300?
33. $201 in 5 yr. 8 mo., at 6%? $402? $1206?
34. $340 in 4 yr. 6 mo., at 8%? $680? $1700?

WRITTEN EXERCISES.

1. What principal will produce $609.12 interest, in 5 yr. 10 mo. 15 da., at 8%?

OPERATION.

$\$1.00 \times .08 \times 5\frac{7}{8} = \$0.47$
$\$609.12 \div .47 = \$1296.$

SOLUTION.—The interest of $1 for 5 yr. 10 mo. 15 da., at 8%, is $0.47. If, therefore, $1 gives an interest of $0.47, to produce an interest of $609.12, it will require as many dollars as $0.47 are contained times in $609.12, which are $1296.

2. What principal will amount to $1617, in 3 yr. 6 mo. 15 da., at 8%?

OPERATION.

$\$1.28\frac{1}{3}$) $1617.00
3
$3.85) $4851.00 ($1260

SOLUTION.—The amount of $1 for 3 yr. 6 mo. 15 da., at 8%, is $1.28⅓. If, therefore, $1 amounts to $1.28⅓, it will require as many dollars to amount to $1617, as $1.28⅓ are contained times in $1617, which are $1260.

RULE.—*I. Divide the given interest by the interest of $1 for the given time and rate.* Or,

II. Divide the given amount by the amount of $1 for the given time and rate.

Formulas: (1) $P = I \div (R \times T)$. (2) $P = A \div [1 + (R \times T)]$.

What principal will produce:

3. $538.80 in 10 mo. 24 da., at 5%? *Ans.* $11973.33.
4. $1000 in 1 yr. 18 da., at 3%? *Ans.* $31746.03.
5. $64.26 in 12 yr. 9 mo., at 7%? *Ans.* $72.
6. $53.505 in 4 mo. 25 da., at 6%? *Ans.* $2214.
7. $306.72 in 1 yr. 11 mo. 20 da., at 12%?
8. $456.16⅔ in 7 yr. 7 mo. 7 da., at 6%?
9. $1208.48 in 5 yr. 6 mo. 12 da., at 6%?

10. $29300 in 9 mo. 23 da., at 9% ? *Ans.* $400000.
11. $458.50 in 6 yr. 6 mo. 18 da., at 7% ? *Ans.* $1000.
12. $5304 in 3 yr. 8 mo. 6 da., at 8% ? *Ans.* $18000.
13. $86.28 in 2 yr. 4 mo. 24 da., at 5% ? *Ans.* $719.
14. $323.75 in 2 yr. 6 mo. 10 da., at 7% ?
15. $636.50 in 1 yr. 8 mo. 3 da., at $6\frac{2}{3}$% ?

What principal will amount to:

16. $847 in 7 yr. 6 mo., at 10% ? *Ans.* $484.
17. $3142.50 in 9 mo. 15 da., at 6% ? *Ans.* $3000.
18. $18000 in 2 yr. 6 mo., at 5% ? *Ans.* $16000.
19. $3000.041, 9 yr. 8 mo., 9% ? *Ans.* $1604.30.
20. $24492, 6 yr. 4 mo. 24 da., 4% ? *Ans.* $19500.
21. $967.15, 1 yr. 10 mo. 10 da., 6% ? *Ans.* $870.
22. $1762.87, 1 yr. 2 mo. 21 da., 6% ?
23. $1047.20, 4 yr. 6 mo., 12% ? *Ans.* $680.
24. $3047.40, 15 yr. 4 mo. 24 da., $4\frac{1}{2}$% ? *Ans.* $1800.
25. $1403.872, 2 yr. 8 mo. 12 da., 8% ?
26. 8964.41\frac{7}{8}$, 7 yr. 4 mo. 3 da., 6% ?
27. $7288.75, 4 yr. 10 mo. 15 da., 10% ? *Ans.* $4900.
28. $5400.4365, 6 yr. 3 mo. 12 da., 5% ?
29. $14780.50, 1 yr. 8 mo. 18 da., 7% ?

Case III.

95. To find the time, when the principal, the rate, and the interest or the amount are given.

MENTAL EXERCISES.

1. In what time will $300, at 4%, give $84 interest ?

Solution.—Since $300 in 1 yr., at 4%, will gain $12 interest, for $300 to gain $84 interest, it will require as many years as $12 are contained times in $84, which are 7 times. Hence it will require 7 years for $300 to gain $84 interest.

In what time will:

2. $120 at 5%, give $54 int.? $400 at 7%, $840 int.?
3. $300 at 6%, give $36 int.? $320 at 6¼%, $400 int.?
4. $100 at 4%, give $40 int.? $500 at 7%, $350 int.?
5. $160 at 5%, give $56 int.? $600 at 5%, $756 int.?
6. $200 at 8%, give $64 int.? $700 at 3%, $847 int.?
7. $150 at 6%, give $90 int.? $300 at 4%, $721 int.?
8. $130 at 4%, give $52 int.? $100 at 9%, $183 int.?
9. $250 at 8%, give $100 int.? $200 at 3½%, $140 int.?
10. $140 at 5%, give $280 int.? $200 at 6%, $66 int.?
11. $175 at 8%, amount to $245? At 4%? At 2%?
12. $400 at 5%, amount to $520? At 2½%? At 10%?
13. $300 at 4%, amount to $396? At 8%? At 2%?
14. $100 at 7%, amount to $135? At 14%? At 21%?
15. $60 at 3%, amount to $69? At 6%? At 12%?
16. $60 at 8%, amount to $72? At 4%? At 16%?
17. $140 at 6%, amount to $182? At 1%? At 3%?
18. $30 at 5%, amount to $33? At 10%? At 20%?
19. $50 at 4%, amount to $62? At 8%? At 2%?
20. $200 at 6%, amount to $242? At 3%? At 12%?
21. $25 at 8%, amount to $35? At 4%? At 24%?
22. $160 at 5%, amount to $194? At 10%? At 20%?
23. $80 at 7%, amount to $108? At 14%? At 1%?
24. $300 at 4%, amount to $412? At 16%? At 24%?
25. $100 at 6%, amount to $124? At 3%? At 12%?
26. $500 at 10%, amount to $800? At 5%? At 30%?

27. In what time will any given principal double itself at 5%?

SOLUTION.—To double itself any sum of money must gain 100% of itself. At 1 per cent., it will require 100 years; and at 5% it will require as many years as 5% is contained times in 100%, which are 20 times. Therefore, it will require 20 years for any principal to *double itself* at 5%.

28. In what time will a given principal double itself at 7%? At 8%? At 10%? At 14%?

29. In what time will $300 double itself at 12½%? At 6%? At 4%? At 5%?

30. In what time will a given principal treble itself at 4%? At 10%? At 7%? At 6%?

31. In what time will $500 treble itself at 8%? At 16%? At 9%? At 12½%? At 20%? At 8½%?

32. In what time will $800 double itself at 4%?

33. In what time will $3000 treble itself at 20%? At 15%? At 25%? At 8%? At 33⅓%? At 66⅔%?

WRITTEN EXERCISES.

1. In what time will $1565.19 produce $834.81, at 7%?

OPERATION.

$1565.19 × .07 = $109.5633 Int. 1 yr.

		yr. mo. da.
$109.5633)	$834.8100	(7, 7, 12.9
	766.9431	
	67.8669	
	12	
	814.4028	
	766.9431	
	47.4597	
	30	
	1423.7910	
	1314.7596	
	109.0314	

SOLUTION. — The interest of $1565.19 for 1 year, at 7%, is $109.5633. If, therefore, in 1 year, the principal produces $109.5633, to produce $834.81, it will require as many years as $109.5633 are contained times in $1565.19, which are 7 + times. Hence it will require 7 yr. 7 mo. 12.9 da. for $1565.19 to produce $834.81, at 7%.

RULE.—*Divide the given interest by the interest of the principal at the given rate for 1 year.*

Formula: $T = I \div (P \times R)$.

In what time will:

2. \$325, at 6%, produce \$741 interest? *Ans.* 38 yr.
3. \$620, at 7%, produce \$150?
4. \$250, at 5%, produce \$400? *Ans.* 32 yr.
5. \$3040, at 6%, produce \$608? *Ans.* 3 yr. 4 mo.
6. \$1645.50, at 5%, produce \$850.175? *Ans.* $10\frac{1}{3}$ yr.
7. \$18.28, at 5%, produce \$22.393? *Ans.* $24\frac{1}{2}$ yr.
8. \$87.50, at 7%, produce \$49? *Ans.* 8 yr.
9. \$7856, at 5%, produce \$1276.60? *Ans.* $3\frac{1}{4}$ yr.
10. \$8340, at 6%, produce \$492.06? *Ans.* $11\frac{4}{5}$ mo.
11. \$12000, at $3\frac{3}{4}$%, produce \$2130? *Ans.* $4\frac{11}{15}$ yr.
12. \$64950, at 7%, produce \$6668.20? *Ans.* $1\frac{7}{15}$ yr.
13. \$4008.75, at 8%, produce \$1924.20? *Ans.* 6 yr.
14. \$30000, at $12\frac{1}{2}$%, produce \$30000? *Ans.* 8 yr.
15. \$14564.30, at $8\frac{1}{2}$%, produce \$29711.172?
16. \$3030, at 14%, amount to \$4613.68?
Ans. 3 yr. 8 mo. 24 da.
17. \$6400, at 9%, amount to \$16848?
Ans. 18 yr. 1 mo. 20 da.
18. \$20.10, at 8%, amount to \$32.361?
Ans. 7 yr. 7 mo. 15 da.
19. \$20850, at 7%, amount to \$21531.10?
Ans. 5 mo. 18 da.
20. \$53.68, at 5%, amount to \$57.035?
Ans. 1 yr. 3 mo.
21. \$4562, at 3%, amount to \$5006.79$\frac{1}{2}$?
Ans. 3 yr. 3 mo.
22. \$625.50, at 9%, amount to \$920.96?
Ans. 5 yr. 2 mo. 29 da.
23. \$7630.36, at 6%, amount to \$9054.20?
Ans. 3 yr. 1 mo. 9 da.
24. \$1890.60, at 8%, amount to \$2237.21?
Ans. 2 yr. 3 mo. 15 da.

CASE IV.

96. To find the rate, when the principal, the time, and the interest or the amount are given.

MENTAL EXERCISES.

1. At what rate will $50 in 6 years, produce $18 interest?

SOLUTION.—The interest of $50 for 6 years at 1% is $3. If, therefore, $50 in 6 years, at 1 per cent., produce $3 interest, to produce $18 interest, it will require as many per cent. as $3 are contained times in $18, which are 6 times. Hence it will require the rate of 6 per cent. for $50, in 6 years, to produce $18.

At what rate will:

2. $20 in 4 years, produce $24 interest? $12? 25 %
3. $35 in 6 years, produce $42 interest? $21?
4. $48 in 2 years, produce $96 interest? $48?
5. $70 in 5 years, produce $35 interest? $70?
6. $80 in 3 years, produce $72 interest? $48?
7. $100 in 8 years, produce $48 interest? $24?
8. $150 in 10 years, produce $75 interest? $150?
9. $250 in 6 years, produce $225 interest? $300?
10. $300 in 9 years, produce $540 interest? $270?
11. $600 in 6 years, produce $216 interest? $144?
12. $500 in 8 years, produce $240 interest? $400?
13. $125 in 6 years, produce $150 interest? $300?
14. $400 in 5 years, amount to $600? In 10 years?
15. $300 in 2 years, amount to $660? In 4 years?
16. $100 in 7 years, amount to $250? In 1 year?
17. $210 in 1 year, amount to $420? In 2 years?
18. $400 in 2½ years, amount to $455? In 10 years?
19. $500 in 10 years, amount to $550? In 5 years?
20. At what per cent. will a given principal double itself in 4 years?

SOLUTION.—As a given principal must gain 100 per cent. to double itself in 1 year, to double itself in 4 years will require $\frac{1}{4}$ of 100 per cent., or 25 per cent.

21. At what per cent. will $300 double itself in 12 years? In 6 years? In 5 years?

22. At what rate will $400 double itself in 10 years? In 20 years? In 7 years? In 30 years? In 50 years?

23. At what per cent. will $100 treble itself in 12 years? In 40 years? In 60 years? In 25 years?

24. At what per cent. will $150 treble itself in 25 years? In 80 years? In 30 years? In 70 years? In 40 years?

25. At what per cent. will $50 gain 5 times itself in 10 years? In 20 years? In 50 years?

WRITTEN EXERCISES.

1. At what rate will $1461.75, in 4 yr. 9 mo. 15 da., produce $420.25?

OPERATION.

$\$1461.75 \times .01 = \$14.6175$
$\$14.6175 \times 4\frac{19}{24} = \$70.0421\frac{7}{8}$
$\$420.25 \div \$70.0421\frac{7}{8} = 6\%.$

SOLUTION.—The interest of the principal for 4 yr. 9 mo. 15 da., at 1%, is $70.04+. If, therefore, $1461.75 for the given time, at 1%, produce $70.04+, it will require as many per cent. to produce $420.25, as $70.04+ is contained times in $420.25, which are 6 times. Hence it will require 6% for the given principal, in the given time, to produce $420.25.

RULE.—*Divide the given interest by the interest of the principal for the given time at one per cent.*

Formula: $R = I \div (P \times 1\% \times T)$.

At what rate will:

2. $320 in 4 yr. 6 mo., produce $72? *Ans.* 5%.

3. $960 in 2 yr. 8 mo., produce $179.20? *Ans.* 7%.

4. $325 in 6 yr., produce $58.50 ? *Ans.* 3%.

5. $2500 in 1 yr. 3 mo. 10 da., produce $191.67 ?

6. $127.60 in 1 yr. 11 mo. 3 da., produce $24.563 ?

7. $5040 in 3 yr. 6 mo. 15 da., produce $1428 ?

8. $1825.41 in 5 mo. 18 da., produce $42.60 ?

9. $930.84 in 3 yr. 7 mo., produce $166.7755 ?
Ans. 5%.

10. $175.25 in 9 mo. 12 da., produce 8.236\frac{3}{4}$?
Ans. 6%.

11. $3400 in 3 yr. 4 mo. 12 da., produce 801.26\frac{2}{3}$?

12. $2501 in 2 yr. 11 mo., produce $437.675 ?

13. $201.60 in 3 yr. 2 mo. 20 da., amount to $253.568?
Ans. 8%.

14. $304.08 in 4 yr. 4 mo., amount to $383.1408 ?
Ans. 6%.

15. $1800 in 12 yr. 6 mo., amount to $3375 ?
Ans. 7%.

PROMISSORY NOTES.

97. A ***Promissory Note*** is a written promise made by one party to pay another a specified sum at a specified time, or on demand.

98. The ***Face*** of a note is the sum promised to be paid.

99. The ***Maker*** of a note is the party who signs it, and who thus becomes responsible for its payment.

100. The ***Payee*** is the person to whom, or to whose order it is made payable.

101. The ***Holder*** is the one who owns it.

102. A ***Negotiable Note*** is a note that can be transferred from one party to another. To be negotiable

it must be made payable to "bearer," or to the "order" of the payee.

103. The ***Indorser*** of a note is the person who writes his name on the back as security for its payment.

104. The ***Maturity*** of a note is the legal date on which it becomes due.

105. ***Days of Grace*** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

WRITTEN EXERCISES.

Calculate the interest upon each of the following notes:

1.—TIME NOTE, INTEREST-BEARING.

$795\frac{22}{100}$. *Cincinnati, Ohio, July 20, '83.*

Three months after date, I promise to pay G. C. Taylor, or bearer, Seven Hundred Ninety-five and $\frac{22}{100}$ *Dollars, with interest at 7 per cent.*

Value received. *Thomas Renwood.*

2.—JOINT AND SEVERAL NOTE.

$3000\frac{00}{100}$. *Philadelphia, March 12, 1882.*

Six months after date, we jointly and severally promise to pay William Hancock, or order, Three Thousand Dollars, with interest from date, for value received, without defalcation. *Richard Dana.*
Samuel Tilden.

NOTE.—I. A note is on interest from the day it becomes due, even though no interest be mentioned.

II. When no rate of interest is given, take the legal rate of the place mentioned in the note; if no particular place be mentioned, take 6%.

3.—PRINCIPAL AND SURETY NOTE.

1245\frac{00}{100}$. *San Francisco, Cal., Nov. 8, 1881.*

Four months after date I promise to pay Peter Cooper, or bearer, One Thousand Two Hundred and Forty-five Dollars, with interest, for value received.

Joseph Lynch.

Surety, Peter Conrad.

4.—JOINT NOTE.

936\frac{37}{100}$. *St. Louis, Mo., May 19, 1882.*

On demand, for value received, we promise to pay Philip Sheils, or order, Nine Hundred Thirty-six and $\frac{37}{100}$ Dollars, with interest, negotiable and payable without defalcation or discount.

Matthew Butler.
Bernard Crome.

NOTE.—This note was presented for payment Nov. 19, '82.

5. Write out a 90-day note for $600, bearing interest from date, and calculate the interest due in 145 days.

6. Write a 60-day joint and several note for $250, without interest; and find the amount due if paid in 125 days.

ANNUAL INTEREST.

106. ***Annual Interest*** is the simple interest of the principal, and of each year's interest from the time of its accruing until payment.

1. What amount is due on a note of $400, at 5%, for 3 yr. 6 mo., interest payable annually?

OPERATION.

$400 × .05	= $20	= Interest for 1 year.
$20 × 3½	= $70	= " " 3½ years.
$20 × .05 × 2½	= $2.50	= Interest on 1st year's interest.
$20 × .05 × 1½	= 1.50	= " " 2d " "
$20 × .05 × ½	= .50	= " " 3d " "
	400.00	= Principal.
	$474.50	

WRITTEN EXERCISES.

2. At 6%, interest payable annually, how much would be due Oct. 1, 1883, on a note of $5000, dated Oct. 1, 1879, no payments having been made? *Ans.* $6308.

3. What interest is due on a note for $2400, dated February 10, 1860, interest payable annually at 8%; if no payments have been made till May 10, 1864?

Ans. $923.52.

4. Find the amount of $3200, at 6%, interest payable annually from April 8, 1881, to Oct. 8, 1884, no interest having been paid except for the first year.

Ans. $3703.04.

5. What must be paid July 25, 1888, in settlement of a note for $1200, dated March 10, 1883, at 6%, said note promising interest annually, and no interest having been paid? *Ans.* $1638.30.

6. What interest was due on the following note, Dec. 1, 1887, if no payments had been made?

$1800. *Charleston, S. C., June 1, 1884.*

On demand, I promise to pay James Murphy, or order, Eighteen Hundred Dollars, for value received, with interest annually at 7%. *Thomas Walsh.*

Ans. $480.69.

PARTIAL PAYMENTS.

107. ***Partial Payments*** are payments in part of notes, mortgages, etc., made at different times.

108. ***Indorsements*** are acknowledgments of payments, written on the back of the note, mortgage, etc., stating the amount and time of the payment.

UNITED STATES RULE.—*I. Find the amount of the given principal to the time of the first payment; and if this payment exceed the interest then due, subtract it from the amount obtained, and treat the remainder as a new principal.*

II. But if the interest be greater than any payment, compute the interest on the same principal to a time when the sum of the payments shall equal or exceed the interest due, and subtract the sum of the payments from the amount of the principal.

III. Proceed in the same manner with the remaining payments until the time of settlement.

1.

$2000. *New York, May 1, 1881.*

One year after date I promise to pay William Johnson & Co., or order, Two Thousand Dollars, for value received, with interest from date at 6%. John A. Spice.

On this note are the following indorsements:

April 21, 1882,	received	$400
Dec. 1, 1882,	"	50
Aug. 1, 1883,	"	200
July 7, 1884,	"	800

How much remained due, Jan. 19, 1885?

OPERATION.

Principal, or face of note, from May 1, 1881,		$2000.00
Interest from May 1, 1881, to April 21, 1882 (11 mo. 20 da.),		116.67
Amount, April 21, 1882,		$2116.67
First payment, April 21, 1882,		400.00
New principal from April 21, 1882,		$1716.67
Int. of $1716.67 from April 21, 1882, to Dec. 1, 1882 (7 mo. 10 da.),	$62.94	
(Interest exceeding payment, a new principal is not found.)		
Int. of $1716.67 from Dec. 1, 1882, to Aug. 1, 1883 (8 mo.),	68.67	$131.61
[Payments ($50 + $200), now greater than the interest due ($131.61)].		
Amount, Aug. 1, 1883,		$1848.28
Second and third payments, $50 + $200,		250.00
New principal from Aug. 1, 1883,		$1598.28
Int. of $1598.28 from Aug. 1 to July 7, 1884 (11 mo. 6 da.)		89.50
Amount July 7, 1884,		$1687.78
Fourth payment, July 7, 1884,		800.00
New principal from July 7, 1884,		$887.78
Int. of $887.78 from July 7, 1884, to Jan. 19, 1885 (6 mo. 12 da.),		28.41
Amount due Jan. 19, 1885, the final day of settlement,		$916.19

WRITTEN EXERCISES.

2.

$3000. *Buffalo, N. Y., May 10, 1876.*

Four years after date, for value received, I promise to pay Thomas Kane, or order, Three Thousand Dollars, with interest from date. *Felix Kent.*

Indorsements: May 10, 1877, $200; May 25, 1878,

$120; June 20, 1879, $500. How much was due May 10, 1880? *Ans.* $2883.47.

3.

$500. *Philadelphia, Jan. 12, 1879.*

For value received, I promise to pay G. W. Farrell, Five Hundred Dollars, on demand, with interest at 8%.

L. H. Rucke.

Indorsements: May 12, 1880, $200; July 15, 1881, $50; Jan. 1, 1882, $100. What was due April 1, 1882?

4. A note of $1000 was given Sept. 1, 1880. Indorsements: Dec. 20, 1880, $200; Mar. 15, 1881, $25; Nov. 10, 1881, $300. What was due Sept. 1, 1882, interest 5%?

5. A note of $6000 was given Oct. 2, 1883. Indorsements: Feb. 2, 1884, $500; July 17, 1884, $1000; Dec. 25, 1884, $2000. What was due Jan. 4, 1885, interest 7%?

The following rule is generally employed by business men in settling notes and interest accounts payable within a year:

MERCHANTS' RULE.—*I. Find the amount of the principal, and also the amount of each payment, to the time of settlement.*

II. Subtract the amount of the payments from the amount of the principal, and the remainder will be the balance due.

WRITTEN EXERCISES.

1.

$2550. *New York, Jan. 1, 1877.*

Thirty days after date, for value received, I promise to pay John Brown & Co., or order, Two Thousand Five Hundred and Fifty Dollars. *Joseph Wakely.*

Indorsements: May 20, $300; July 30, $100; Oct. 12, $200. What is due Dec. 1, 1877? *Ans.* $2064.48.

2. A note of $1680 was given Sept. 8, 1876. Indorsements: Nov. 8, $500; Jan. 1, 1877, $150; June 1, 1877, $400. What was due Sept. 8, 1877, at 6%?

Ans. $693.16.

3. A note of $3600 was given Mar. 20, 1860. Indorsements: Apr. 10, $1000; Jan. 1, 1861, $500. What was due Feb. 1, 1861, at 5%? *Ans.* $2213.

4. A note of $5400 was given June 2, 1870. Indorsements: Aug. 2, $3000; Nov. 20, $2000; Feb. 1, 1871, $100. What was due June 2, 1871, at 7%?

DISCOUNT.

109. ***Discount*** is an allowance made for the payment of a debt before it is due.

110. The ***Present Worth*** of a debt payable at a future time without interest, is such a sum as being on interest for the time at a certain rate, will amount to the debt.

111. The ***True Discount*** is the difference between the present worth and the debt.

1. What is the present worth of $650, due 8 years hence without interest, money being worth 6%?

OPERATION.

$0.06 × 8 = $0.48, Int. $1 for 8 yr.
$1.00 + $0.48 = $1.48, Amt. " "
$650 ÷ $1.48 = $439.18+, *Ans.*

SOLUTION.—The amount of $1.00 for 8 years at 6%, is $1.48; and the present worth of $1.48, is $1.00. Hence the present worth of $650 for 8 yr. at 6%, is as many dollars as $1.48 is contained times in $650, which *are $439.18+.*

RULE.—*I. Divide the given sum by the amount of $1 for the given time and rate. The quotient will be the present worth of the debt.*

II. Subtract the present worth from the debt and the remainder will be the discount.

NOTE.—When several payments are made without interest, find the present worth of each separately, and take their sum.

WRITTEN EXERCISES.

2. What is the present worth of $51.50, due in 2½ years, at 9%? *Ans.* $42.04.

3. What is the discount of $7000 due in 18 mo., at 4%? *Ans.* $396.23.

4. What is the present worth of $3200.90 due in 9 mo., at 4½%? *Ans.* $3096.40.

5. Which is the more profitable, $1000 due in 20 months, at 6%, or $850 cash?

Ans. The former, $59.09 more.

6. What is the discount on $4301.20, due in 60 days, at 5½%? *Ans.* $39.07.

7. Find the present worth of $10250, due in 5 years, at 5%. *Ans.* $8200.

8. A owes $5700, payable in 6 yr. 9 mo. without interest, but wishing to pay it immediately, what should he in equity pay, money being worth 8%? *Ans.* $3701.30.

9. I can sell my house for $4800 cash, or $5000 in 3 yr. 6 mo. How much will I gain or lose by choosing the former, money being worth 6%? *Ans.* Gain $667.77.

10. Mr. Wilson owes $300, of which one-third is to be paid in one year, and the remainder in two years. What is the present value, money being worth 6 per cent.?

Ans. $272.91.

11. How much do I gain by borrowing money at 5%, to pay a debt of $5000, due in two years, discounted at 6%? *Ans.* $89.29.

12. Mr. Lynch gave his note for $760 due in 3 yr. 6 mo. without interest. At the end of 18 mo. he wishes to pay it. Money being worth 6%, what should the holder of the note receive? *Ans.* $678.57.

13. What is the present worth of $3500, one-half due in 1 yr., one-third in 6 mo., and the remainder in 2 yr. 4 mo., money being worth 5%?

14. What is the discount of $1870, due in 4 yr. 3 mo., at 4%?

15. Find the difference between the interest and the discount of $950, due in 3 yr. 4 mo. 12 da., at 6%.

16. If I receive 4% less than the face of a bill for $16000, due in 8 months, what do I gain, money being worth 7%?

BANK DISCOUNT.

112. A ***Bank*** is a chartered institution which receives and loans money, or furnishes a paper circulation.

113. ***Bank Notes*** are notes issued by banks to circulate as money. They are payable in specie at the banks.

114. ***Bank Discount*** is the simple interest on a note for the number of days it has to run. It is paid in advance, and may be computed by any of the methods of simple interest.

115. ***The Proceeds*** of a note is the amount received by the holder when the note is discounted. It is equal *to the face* minus the discount.

CASE I.

116. To find the discount and the proceeds of a note, when the face is given.

1. What are the proceeds of a note for $400, due in 27 days, discounted at a bank at 6%?

OPERATION.

27 da. + 3 da. = 30 da.
30 da. × .000⅙ = .005
$400 × .005 = $2.00
$400 − $2 = $398

SOLUTION.—The interest of $400 for 27 da. + 3 da. of grace, or 30 da., is $2.00, which is the bank discount. Subtracting this discount from the face, $400, we find the proceeds, $398.

RULE.—*I. Find the interest on the face of the note for three days more than the specified time; the result will be the discount.*

II. Subtract the discount from the face; the remainder will be the present worth.

NOTES.—1. In taking the discount of notes bearing interest, compute it on the amount of the note at its maturity.

2. When the time of a note is given in months, calendar months are reckoned; and the maturity is 3 days later than the date corresponding to that of the note.

WRITTEN EXERCISES.

2. What are the proceeds of a note of $500 for 60 days, discounted at 7%? *Ans.* $493.88.

3. Find the proceeds of a note of $800 for 20 days, discounted at 6%. *Ans.* $796.93.

4. Required the discount of a note for $525, due in 30 days, discounted at a bank at 6%. *Ans.* $2.89.

5. Find the proceeds of a note for $1800, with interest at 6%, dated Feb. 8, at 2 mo., discounted March 8, at 8%. *Ans.* $1805.56.

6. A note for $420.50, drawn at 60 days, is discounted at the Bank of New York. Find the proceeds.

Ans. $416.08.

7. Wishing to borrow $1080 of a Southern bank which is discounting paper at 8%, I gave my note for $960, payable in 30 days. How much do I need to make up the required amount?

8. A note for $235.60, drawn at 30 days, is discounted at the Vermont Bank. Find the proceeds.

9. What are the proceeds of a note for $30000, at 2 mo., discounted at the Philadelphia Bank?

NOTE.—In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, both the day of *discount* and the day of *payment* are counted. Hence, in these States, it is necessary to add 4 days instead of 3, to the given time.

10. Bought a horse for $250, and sold him for a note, at 60 days, for $325, which I had discounted at 6%. What was my gain? *Ans.* $71.59.

11. Discounted a note of $800 for 30 days, at 6%, and invested the proceeds in flour at $9.36 per bbl. How many bbl. did I purchase? *Ans.* 85 bbl.

12. What are the proceeds of a note of $600, due in 30 days, bearing interest at 9%, which I discounted at a bank at the same rate?

13. What is the difference between the true and the bank discount of $212 for 2 yr., at 8%?

NOTE.—The true discount is equal to $212 minus the present worth. The bank discount is equal to $212 minus the proceeds of $212 for 2 yr. and 3 da., at 8%.

14. What is the difference between the true and the *bank* discount of $400 for 90 days, at 7%?

15. Find the difference between the true and the bank discount of $897.50 for 30 days, at 6%. *Ans.* $0.47.

16. Find the difference between the true and the bank discount of $490, due in 3 yr. 4 mo., omitting days of grace, money being worth 10%.

Find the maturity, the term of discount, and the proceeds of each of the following notes:

17.

$3000. *Brooklyn, N. Y., May 12, 1882.*

Sixty days from date, I promise to pay G. H. Penman, or order, Three Thousand Dollars, value received.

Thomas Giles.

Discounted, June 15, 1882, at 6%.

18.

$8000. *New York, Jan. 15, 1883.*

Three months after date, I promise to pay to the order of Smith & Son, Eight Thousand Dollars, at the First National Bank, value received.

John Edison.

Discounted, Jan. 20, 1883.

19.

230\frac{50}{100}$. *Boston, May 4, 1884.*

Two months after date, I promise to pay S. T. Wilson, or order, Two Hundred Thirty and $\frac{50}{100}$ *Dollars, value received.*

J. F. Thomas.

Discounted, June 1, 1884, at 6%.

20.

$840. *Washington, Oct. 12, 1882.*

Eight months from date, for value received, I promise to pay P. O'Shea, or order, Eight Hundred Forty Dollars, with interest, at 6%.

Henry Jaden.

Discounted, Dec. 12, 1882, at 6%.

21.

537\frac{16}{100}$. *St. Louis, Aug. 21, 1884.*

Ninety days after date, for value received, I promise to pay to the order of W. Jackson, Five Hundred Thirty-seven and $\frac{16}{100}$ Dollars, with interest, at 8 per cent.

Joseph Cream.

Discounted, Aug. 31, 1884, at 6%.

CASE II.

117. To find the face of a note, when the proceeds are given.

1. If I wish to borrow $600 from a bank, for what amount must I give my note at 30 days, discounting at 6%?

OPERATION.

$1.0000
Interest of $1 = .0055
Proceeds of $1 = .9945
$600 ÷ .9945 = $603.31 +

SOLUTION.—To find the proceeds of $1, subtract the interest of $1 for the given time and rate, from a dollar. Then, if the proceeds of $1 are $0.9945, to give $600 proceeds, it will require as many dollars as $0.9945 are contained times in $600, which are $603.31 +.

RULE.—*Divide the given proceeds by the proceeds of $1 for the given time and rate, and the quotient will be the face of the note.*

WRITTEN EXERCISES.

2. What is the face of a note at 60 days, the proceeds of which, discounted at 6%, are $4000 ?

Ans. $4042.45.

3. A broker buys a 30-day note for $50 less than the face ; what is the face, discounted at 7% ?

Ans. $7792.21.

4. For what sum must a note be drawn at 90 days to net $6500, when discounted at 6% ? *Ans.* $6602.34.

5. A gentleman wishes to borrow $750 from a bank at 60 days ; for what sum must he give his note, discounting at 6% ? *Ans.* $757.96.

6. Mr. Snelling presented a note for 60 days, at a New York bank for discount ; the proceeds being $3958, what was the face of the note ? *Ans.* $4000.

7. The proceeds of a note having two months to run, discounted at a bank at 6%, were $6431.75 ; what was the face of the note ? *Ans.* $6500.

8. Find the face of a 9 mo. note, the proceeds of which, discounted at 2% a month, are $652.

9. For what sum must a note be drawn at 60 days, to net $1000 when discounted at 8% ? *Ans.* $1014.20.

10. What would be the face of a note for 6 months in payment of a debt of $530, if discounted at 1½% a month?

11. Mr. Jenkins buys a bill of merchandise in New York at cash price, to the amount of $3567.80, and gives in payment his note at 3 months at 4½% ; what must be the face of the note ?

12. What must be the face of a note at 60 days, discounted at 6%, to realize $62.75 ? *Ans.* $63.42.

13. What must be the face of a note payable in 4 months, that I may realize $1925 when discounted at 8%?

14. How much more or less will it cost me to borrow $7500 at 90 days, at bank discount, than at true discount, the rate being 10% in both cases? *Ans.* $11.39 more.

15. If the proceeds of a 30-day note, discounted at 8%, are $900, what is the face?

16. For what sum must a 60-day note be drawn, so that when discounted at 7% it will yield $2312?

Ans. $2340.67.

COMPOUND INTEREST.

118. ***Compound Interest*** is interest on both principal and interest, when the interest is not paid when due.

1. What is the compound interest of $800 for 3 years, at 6%?

OPERATION.

$800,	Principal.
.06	
$48.00,	Interest 1st year.
800	
$848.00,	Amount 1st year.
.06	
$50.8800,	Interest 2d year.
848.	
$898.88,	Amount 2d year.
.06	
$53.9328,	Interest 3d year.
898.88	
$952.8128,	Amount 3d year.
800. ,	Original Principal.
$152.8128,	C. Interest for 3 years.

SOLUTION.—The interest for 1 year at 6%, on $800 is $48. Adding the principal, $800, to this, we find $848 for the amount for the first year, which is also the principal for the second year. The interest on $848 is $50.88; which, added to $848, gives $898.88, the amount for the second year, or the principal for the third year. The interest on $898.88 is $53.9328, which plus $898.88, equals $952.8128, the amount for the third year. Subtracting from this the original principal, we get $152.8128 as the compound interest.

RULE.—*I. Find the amount of the given principal for the first period of time, and make it the principal for the second.*

II. Find the amount of the second principal for the second period of time, and make it the principal for the third.

III. Continue thus for the whole time; the last amount, minus the given principal, will be the compound interest.

NOTE.—1. Semi-annual and quarterly interest are reckoned in the same manner as annual, by counting the half years and quarters as periods.

2. When the time cannot be divided into exact periods, find the amount of the principal to the end of the last period. Then find the interest on this amount for the remaining time, and add it to the last amount. The sum will give the required amount, from which subtract the given principal, as before.

WRITTEN EXERCISES.

2. What is the compound interest of $750 for 2 years, at 6%? *Ans.* $92.70.

3. What is the compound interest of $200 for 3 years, at 7%? *Ans.* $45.

4. What is the compound interest of $300 for 1 yr. 6 mo., at 5%, payable semi-annually? *Ans.* $23.07.

5. What is the compound interest of $437.82 for 3 yr. 3 mo., at 6%, payable quarterly?

6. What is the amount of $630.25 for 2 yr. 6 mo. 10 da., at 8%, payable semi-annually, compound interest?

7. Find the amount, by using table, of $763, for 7 yr., at 5%, compound interest.

SOLUTION.—In the column of the table marked 5%, and opposite 7 yr., we find the amount of $1 to be $1.4071. Hence the amount of $763 is 763 times as much.

8. Find the amount of $1200 for 12 yr., at 6%, compound interest. *Ans.* $2414.64.

The following table is intended to facilitate the calculation of compound interest, by showing the amount of $1, at 2%, 3%, to 10%, for any number of years from 1 to 30.

Yrs.	2%	2½%	3%	3½%	4%	4½%	Yrs.
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	2
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	3
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.1925	4
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	5
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	6
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	7
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	8
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.4861	9
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	10
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	11
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	12
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	13
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	14
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	15
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	16
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	17
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	18
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	19
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	20
21	1.5157	1.6796	1.8603	2.0594	2.2788	2.5202	21
22	1.5460	1.7216	1.9161	2.1315	2.3699	2.6337	22
23	1.5769	1.7646	1.9736	2.2061	2.4647	2.7522	23
24	1.6084	1.8087	2.0328	2.2833	2.5633	2.8760	24
25	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	25
26	1.6734	1.9003	2.1566	2.4460	2.7725	3.1407	26
27	1.7069	1.9478	2.2213	2.5316	2.8834	3.2820	27
28	1.7410	1.9965	2.2879	2.6202	2.9987	3.4297	28
29	1.7758	2.0464	2.3566	2.7119	3.1187	3.5840	29
30	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	30

9. What is the difference between the simple and the compound interest of $700 for 15 yr., at 3%?

10. What is the amount of $300 for 4 yr. 5 mo. 20 da., at 4%, compound interest? *Ans.* $357.60.

11. What is the compound interest of $840 for 15 yr. 10 mo., at 9%?

Yrs.	5%.	6%.	7%.	8%.	9%.	10%.	Yrs.
1	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1
2	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	2
3	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	3
4	1.2155	1.2625	1.3108	1.3604	1.4115	1.4641	4
5	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	5
6	1.3401	1.4185	1.5007	1.5868	1.6771	1.7715	6
7	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	7
8	1.4775	1.5938	1.7182	1.8509	1.9925	2.1435	8
9	1.5513	1.6895	1.8385	1.9990	2.1718	2.3579	9
10	1.6289	1.7908	1.9672	2.1589	2.3673	2.5937	10
11	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	11
12	1.7956	2.0122	2.2522	2.5181	2.8126	3.1384	12
13	1.8856	2.1329	2.4098	2.7196	3.0658	3.4522	13
14	1.9799	2.2609	2.5785	2.9371	3.3417	3.7974	14
15	2.0789	2.3966	2.7590	3.1721	3.6424	4.1772	15
16	2.1829	2.5404	2.9522	3.4259	3.9703	4.5949	16
17	2.2920	2.6928	3.1588	3.7000	4.3276	5.0544	17
18	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	18
19	2.5270	3.0256	3.6165	4.3157	5.1416	6.1159	19
20	2.6533	3.2071	3.8697	4.6609	5.6044	6.7275	20
21	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	21
22	2.9253	3.6035	4.4304	5.4365	6.6586	8.1402	22
23	3.0715	3.8197	4.7405	5.8714	7.2578	8.9543	23
24	3.2251	4.0489	5.0724	6.3411	7.9110	9.8497	24
25	3.3864	4.2919	5.4274	6.8484	8.6230	10.8347	25
26	3.5557	4.5494	5.8074	7.3963	9.3991	11.9181	26
27	3.7335	4.8223	6.2139	7.9880	10.2450	13.1099	27
28	3,9201	5.1117	6.6488	8.6271	11.1671	14.4209	28
29	4.1161	5.4184	7.1143	9.3172	12.1721	15.8630	29
30	4.3219	5.7435	7.6123	10.0626	13.2676	17.4494	30

EQUATION OF PAYMENTS.

119. ***Equation of Payments*** is the process of finding the mean or equitable time for paying, at once, and without loss to either debtor or creditor, several sums of money due at different times.

120. The ***Equated Time*** is the date at which the several debts may be paid in one sum.

121. The ***Term of Credit*** is the time allowed for the payment of a debt.

122. The ***Average Term of Credit*** is the equated time at which the several debts may be paid together.

123. The ***Focal Date*** is the date from which we reckon in arranging an account.

CASE I.

124. To find the average term of credit, when all the terms of credit begin at the same time.

1. A merchant owes $320 due in 8 months, and $500 due in 2 months; what is the average term of credit?

OPERATION.

$$\$320 \times 8 = 2560$$
$$500 \times 2 = 1000$$
$$\$820 \qquad 3560$$
$$3560 \div \$820 = 4\tfrac{14}{41} \text{ mo.}$$
$$\tfrac{14}{41} \times 30 = 10\tfrac{1}{4} \text{ da.}$$
$$4\tfrac{14}{41} \text{ mo.} = 4 \text{ mo. } 10 \text{ da.}$$

SOLUTION.—The credit, or interest of $320 for 8 months equals the interest of $1 for 2560 months; and the interest of $500 for 2 months equals that of $1 for 1000 months. By adding we obtain the credit of the several sums equal to the credit of $1 for 3560 months. Hence, if $1 has a credit of 3560 months, *$820 will* have a credit of $\frac{1}{820}$ of 3560 months, which is $4\frac{14}{41}$ months.

RULE.—*I. Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient will be the average term of credit.*

II. Add the average term of credit to the date at which the credits begin, and the result will be the equated time.

WRITTEN EXERCISES.

2. Bought merchandise, June 1, as follows: $150 on 4 mo., $348 on 6 mo., $200 on 4 mo., and $720 on 8 mo.; what is the average term of credit? *Ans.* 6 mo. 16 da.

3. I agreed to pay $\frac{1}{2}$ of a debt in 6 mo., $\frac{1}{3}$ of it in 8 mo., and the remainder in 2 years; in what time can I cancel the obligation by a single payment?

Ans. 9 mo. 20 da.

4. Find the average term of credit of $360 due in 5 mo., $1200 in 2 mo., and $200 in 4 mo.

5. A broker sold a house for $20000, as follows: $10000 cash, $5000 in 4 mo., and the remainder in 1 year; for what time should a note be drawn for the whole amount? *Ans.* 4 mo.

6. A merchant bought a bill of goods, May 20, 1881, amounting to $4325, as follows: $3000 in 2 mo., and the remainder in 6 mo.; what is the equated time of payment? *Ans.* Aug. 27, 1881.

7. Mr. Harman bought merchandise, Feb. 12, 1883, as follows: $1200 for cash, $600 on 3 mo., $900 on 4 mo., $1000 on 1 mo.; what is the equated time of payment? *Ans.* April 4, 1883.

8. Thomas Lalor owes Henry Brant $8000, $\frac{1}{2}$ due in 3 mo.; $\frac{1}{4}$ in 6 mo.; and the remainder in 8 mo.; what is the average term of credit? *Ans.* 5 mo.

9. Henry Gray owes $3500, $1200 of which is due now, $1420 in 4 mo., and the remainder in 6 mo.; what is the average time of payment? *Ans.* 3 mo. 4 da.

CASE II.

125. To find the equated time, when the terms of credit begin at different dates.

1.

J. J. HILLMAN, NEW YORK, Mar. 10, 1884.

To EDWARD KANE, *Dr.*

1883, June 5..	To Cash		$300.
" July 20..	" Mdse. on 3 mo		400.
" Aug. 10..	" Mdse. on 2 mo		200.
" Oct. 1..	" Cash		500.

Find the equated time of settlement.

OPERATION.

June 5, $300 × 0 = 00000.
Oct. 20, 400 × 137 = 54800.
Oct. 10, 200 × 127 = 25400.
Oct. 1, 500 × 118 = 59000.
$1400 139200.
139200 ÷ $1400 = 99$\frac{3}{7}$ da.
June 5, 1883, + 99 da. = Sept. 12, 1883.

SOLUTION.—Taking for convenience, June 5, the earliest date, as the focal date, the first item, $300, has no credit. The second being payable in 3 mo. from July 20, becomes due Oct. 20, which is 137 da. from the focal date. In like manner the third has a credit of 127 da. and the fourth 118 da. Then by averaging, as in Case I., we find the term of credit to be 99 da. +. Hence June 5, 1883, + 99 da. = Sept. 12, 1883.

RULE.—*I. Select the earliest date as the focal date.*

II. Find the term of credit from the focal date to the date when each item becomes due.

III. Find the equated time, as in Case I.

WRITTEN EXERCISES.

2. Mr. Warren sold goods as follows:

Feb. 2, 1870,	to amount of		$150	on	2 mo.	credit.
Mar. 10, "	"	"	340	"	3 mo.	"
April 21, "	"	"	600	"	4 mo.	"

Find the equated time of payment. *Ans.* July 10.

3. What is the equated time of payment of the following account?

1871, July 1	Mdse. at 4 mo	$380.
" July 16	Cash	800.
" Aug. 8	Mdse. at 3 mo	600.
" Sept. 29	Cash	700.

Ans. Sept. 20.

4. What is the equated time for the payment of the following bill?

JOHN WALSH,

1870. To JAMES FRENCH, *Dr.*

May 25,	to 320 yd.	Carpet @ $1.75,	at 2 mo.	credit.
May 30,	" 500 "	Oil Cloth @ .60,	at 5 mo.	"
June 14,	" 720 "	Sheeting @ .10,	at 6 mo.	"

5. On Sept. 9, 1880, I bought $400 worth of goods on 4 months' credit; Oct. 10, $1000 worth for cash; Oct. 14, $600 worth on 3 months' credit. At what equated time can I pay the whole amount? *Ans.* Nov. 26.

CASE III.

126. When a debt due at some future time, has received partial payments, to find when the remainder should be paid.

1. I gave a note for $2000 to be paid in 10 mo.; but in 3 months I paid $1200, and in 2 months after, $400 more. When, in equity, should I pay the balance?

OPERATION.

$1200 × 7 = 8400
400 × 5 = 2000
$1600 10400
$2000 − $1600 = $400
10400 ÷ $400 = 26 mo.
10 mo. + 26 mo. = 36 mo.

SOLUTION.—By paying $1200 in 3 mo., or 7 mo. before it is due, I become entitled to a credit of $1 for 8400 mo. In like manner by paying $400 5 months before it is due, I obtain a credit of $1 for 2000 months. Adding both sums, we get a credit of $1 for 10400 months. Hence the balance, $400, should have an additional credit of $\frac{1}{400}$ of 10400 mo. = 26 mo. And 10 mo. + 26 mo. = 36 mo., the equated time for paying the balance.

RULE.—*I. Multiply each payment by the time it was paid before it was due.*

II. Find the term of credit for the balance of the debt, and add it to the given time.

WRITTEN EXERCISES.

2. On a loan of $1500, at 6 months, made Nov. 1, 1872, there was paid Nov. 30, $600; and Dec. 15, $400. What is the equated time for paying the balance?

Ans. Feb. 17, 1874.

3. A merchant lent $500 for 4 mo., and $700 for 5 mo. At the end of 3 mo. he received $1000. At what time, in equity, should the remainder be paid?

Ans. $12\frac{1}{2}$ mo. after borrowing.

4. A gentleman borrowed $800 for 7 months. In 2 months he paid $\frac{1}{2}$ of it, and in 3 months $\frac{1}{4}$ of the remainder. How long, in equity, may the balance remain unpaid? *Ans.* 8 mo. after due.

5. I bought a bill of $600 for 3 mo., and $900 for 6

mo. At the end of 1 month, I paid $1200. When, in equity, should I pay the remainder? *Ans.* 20 mo.

CASE IV.

127. To find the equated time for the payment of the balance of an account.

1. At what equated time will the balance of my account with Lawrence Hogan, become due?

Dr. LAWRENCE HOGAN. *Cr.*

1882.				1882.			
Jan. 12	To Mdse.	100	00	Mar. 10	By Cash	150	00
Mar. 12	" "	300	00	June 26	" "	100	00
July 20	" "	200	00	Oct. 15	" "	100	00

OPERATION.

Due	*Dr.*		Due	*Cr.*	
Jan. 12,	$100 × 0	= 00000	Mar. 10,	$150 × 57	= 8550
Mar. 12,	300 × 59	= 17700	June 26,	100 × 165	= 16500
July 20,	200 × 189	= 37800	Oct. 15,	100 × 276	= 27600
	$600	55500		$350	52650
	350	52650			
	$250	2850			

$$2850 \div \$250 = 11\tfrac{2}{5} \text{ da.}$$
$$\text{Jan. } 12 + 11 \text{ da.} = \text{Jan. } 23.$$

SOLUTION.—The date of the first item due is Jan. 12. Taking this as the focal date, and performing the operation as in the foregoing cases, we find on the Dr. side a total credit of $1 for 55500 days, and on the Cr. side a total credit of $1 for 52650 days. Comparing the two sums, we find that the Dr. side must suffer a discount of $1 for 2850 days more than the Cr. The balance, $250, therefore, must suffer a discount of $\frac{1}{250}$ of 2850 days = $11\frac{2}{5}$ days.

2. Find the equated time of payment of the following account, and the cash balance Dec. 1, 1881.

Dr. SAMUEL BLAKEMAN. *Cr.*

1881.				1881.			
May 4	To Mdse.	500	00	July 9	By Cash	400	00
June 16	" "	800	00	Sept. 11	" Mdse.	600	00
July 21	" "	300	00	Oct. 29	" Cash	200	00

OPERATION.

Due	*Dr.*		Due	*Cr.*	
May 4,	$500 × 0 =	00000	July 9,	$400 × 66 =	26400
June 16,	800 × 43 =	34400	Sept. 11,	600 × 130 =	78000
July 21,	300 × 78 =	23400	Oct. 29,	200 × 178 =	35600
	$1600	57800		$1200	140000
	1200				57800
	$400				82200
	.069$\frac{1}{2}$				
	$27.80 Int. for 417 da.				
	400				
	$427.80				

82200 ÷ $400 = 205$\frac{1}{2}$ da.
May 4, 1881 − 206 = Oct. 10, 1880.
Dec. 1, 1881 − Oct. 10, 1880 = 417 da.
417 × .000$\frac{1}{6}$ = $.069$\frac{1}{2}$.

Equated time, Oct. 10, 1880.
Cash balance, $427.80.

SOLUTION.—Having found the balance by the same operation as in the last exercise, we observe that although there is a balance of $400 on the Dr. side, the credit contains a greater term of credit by 82200 days. Hence, as S. Blakeman has had the use of $1 for 82200 days more than his creditor, he should pay the balance, $400, soon enough to make the terms of credit equal. $\frac{1}{400}$ of 82200 days = 205$\frac{1}{2}$ da. And 206 da. before May 4, 1881 = Oct. 10, 1880, the equated time for paying the balance. If the balance, $400, becomes due Oct. 10, 1880, and is paid only Dec. 1, 1881, Mr. Blakeman must pay the interest for the intervening time. The interest of $400 for 417 days, at 6% = $27.80; which, added to the $400, gives *a total* balance of $427.80.

RULE.—*I. Find the term of credit for each item and the products as in the last case.*

II. Divide the balance of the sum of the products by the balance of the accounts and the quotient will be the average term of credit.

III. When the balances are on the same side of the account reckon the equated time FORWARD *from the focal date; but when they are on opposite sides reckon it* BACKWARD.

IV. If the balance becomes due before the time of settlement, find the cash balance by adding the interest for the interval; if after, deduct the bank discount.

INTEREST METHOD.

3. At what equated time will the balance of the following account become due?

Dr. WILLIAM F. GAYNOR. *Cr.*

1879.				1879.			
Mar. 10	To Mdse.	240	00	Apr. 17	By Cash	120	00
" 27	" "	500	00	May 27	" "	400	00
May 18	" "	350	00	June 10	" "	200	00

OPERATION.

Dr. Due		da.	Int.	Due		da.	*Cr.* Int.
Mar. 10,	$240	0	$0.00	Apr. 17,	$120	38	$0.76
" 27,	500	17	1.42	May 27,	400	78	5.20
May 18,	350	69	4.03	June 10,	200	92	3.07
	$1090		$5.45		$720		$9.03
	720						5.45
	$370						$3.58

$3.58 ÷ .061⅔ = 58 days before Mar. 10, or Jan. 11, 1879.

SOLUTION.—In this method, instead of the products, we find the interest on the several items for their respective terms of credit, at 6%. The balance of interest is divided by the interest of the balance of the account for one day; which gives an average term of credit for 58 days. As the balances are found on opposite sides of the account, the equated time is reckoned backward. Hence the balance of the account becomes due Jan. 11, 1879.

RULE.—*I. Find the interest on each item for its corresponding term of credit.*

II. Divide the balance of interest by the interest of the balance of the account for one day; and the quotient will give the average term of credit.

NOTE.—In doing the following exercises by the Interest Method, reckon the exact number of days on a basis of 360 to the year.

WRITTEN EXERCISES.

4. When is the balance of the following account due?

Dr. MICHAEL BANIM. *Cr.*

1864.				1864.			
Feb. 5	To Mdse.	500	00	Feb. 28	By Cash...	400	00
April 2	" "	800	00	May 15	" "	600	00
May 25	" "	600	00	June 20	" Sundries	150	00

Ans. Mar. 5.

5. At what date may the balance of the following account be paid without loss to either party?

Dr. HENRY F. SPICE. *Cr.*

1880.				1880.			
Jan. 6	To Mdse.	720	00	Feb. 10	By Cash	600	00
Mar. 10	" "	340	00	Apr. 15	" "	400	00
" 30	" "	800	00	May 12	" "	500	00

Ans. Sept. 29, 1879.

6. What will be the equated time and the cash balance of the following account, Dec. 12, 1885?

Dr. JOHN. F. CREAL. *Cr.*

1884.				1884.			
Jan. 21	To Mdse.	300	00	Feb. 2	By Cash	200	00
May 28	" "	900	00	June 20	" "	750	00
July 12	" "	1000	00	Aug. 15	" "	800	00

Ans. Feb. 5, 1884; Cash balance, $500.70.

7. Average the following account:

Dr. PHILIP MANGAN. *Cr.*

1881.				1881.			
June 1	To Mdse.	625	00	June 30	By Cash	525	00
July 25	" "	400	00	Aug. 10	" "	450	00

8. Find the equated time of payment, and the cash balance of the following account Feb. 1, 1883, interest at 7%.

Dr. CLARK IN ACC'T WITH SMITH. *Cr.*

1882.				1882.			
July 25	To Mdse.	500	00	July 30	By Cash	300	00
Oct. 8	" "	2500	00	Oct. 15	" "	2000	00
Nov. 10	" "	5000	00	Nov. 28	" "	4700	00

Ans. June 25; Cash balance, $1042.97.

9. Average the following account:

Dr. GEORGE GRAVES. *Cr.*

1886.				1886.			
Jan. 15	To Mdse.	200	00	Jan. 27	By Cash	150	00
May 3	" "	6500	00	June 1	" "	4000	00

EXCHANGE.

128. ***Exchange*** is the method of making payments at distant places by means of written orders called *Drafts*, or *Bills of Exchange.*

129. ***Domestic Exchange*** is that which is made between different places in the same country.

130. ***Foreign Exchange*** is that which takes place between different countries.

131. A ***Bill of Exchange*** is a written order requesting one person to pay a certain sum to another, or his order, at a specified time. In domestic exchange, such bills are called *Drafts.*

132. A ***Sight Draft*** is one requiring payment to be made "at sight," or as soon as presented to the person ordered to pay. Bills made payable a certain number of days "after sight," are called *Time Bills.*

133. The ***Drawer,*** or ***Maker,*** is the person who signs the bill.

134. The ***Drawee*** is the person to whom the order is addressed.

135. The ***Payee*** is the person to whom the money is ordered to be paid.

136. The ***Buyer,*** or ***Remitter,*** is the person who purchases the bill.

137. The ***Holder*** is the person who has possession, or who owns the bill.

138. The ***Indorsement*** of a bill is the writing upon its back, by which the payee transfers the payment to another.

139. The ***Acceptance*** is the promise of the *drawee* to pay the bill at maturity. In accepting a bill the drawee writes the word "Accepted," with his name and the date, across the face.

140. The ***Par of Exchange*** is the estimated value of the coins of one country as compared with those of another.

141. The ***Rate of Exchange*** is the rate per cent. which is reckoned upon a draft.

142. The ***Course of Exchange*** is the current price paid in one place for bills of exchange on another place.

The following are examples of the usual forms of drafts:

FIRST NATIONAL BANK.

$6000. *New York, May 20, 1875.*

At sight, pay to the order of Joseph Petrie, Six Thousand Dollars. *John Larkins, Cashier.*

To the Merchants National Bank,
Philadelphia, Pa.

$4500. *St. Louis, Mo., July 10, 1880.*

At thirty days' sight, pay to the order of G. P. Lawrence, Four Thousand Five Hundred Dollars, and charge the same to account of

Thomas Cleary, Cashier.

To the Fifth National Bank,
New York.

The first of these drafts illustrates the method of remitting money from New York to Philadelphia. It supposes that Joseph Petrie of New York wishes to pay $6000 to Henry Jackson of Philadelphia. He goes to a bank in New York and gets the draft. Having written on the back of it, "Pay to the order of Henry Jackson," and signed his name, he forwards it to Henry Jackson in Philadelphia. The latter takes it to the Merchants National Bank, and having written his name on the back, receives the money.

The second draft differs from the first inasmuch as it is a time draft. The "thirty days' sight" means after acceptance. It is presented to the bank as soon as received, when the cashier writes upon it "accepted," with the date, and his name as cashier, and then returns it to the payee. This is an agreement on the part of the bank to pay it after thirty days.

Case I.

143. To find the cost of a domestic bill of exchange.

1. What must I give for a sight draft of $700 on New Orleans, at 3½% discount?

OPERATION.

$1.000 — .035 = .965, Rate of Ex.
.965 × 700 = $675.50, Cost.

Solution. — Every dollar of the draft is purchased at 3½% discount; therefore the whole draft will cost .965 per dollar, or 700 times .965, = $675.50.

2. What must be paid for a draft of $5000, on Boston, at 60 days, interest 6%, when exchange is at 2% premium?

OPERATION.

$1.00 + .02 = $1.02, C. of Ex.
$1 × .0105 = Disc. $1, 63 da.
$1.02 — $.0105 = $1.0095
$1.0095 × 5000 = $5047.50, Cost.

Solution.—The course of exchange being $1.02, and the bank discount of $1 for 60 da. (+3 da. of grace) being .0105, the cost of $1 of the draft is equal to $1.02 — .0105 = $1.0095. Hence, the entire cost *of the draft* = $1.0095 × $5000 = $5047.50.

RULE.—*Multiply the cost of $1 of exchange by the face of the draft.*

NOTE.—As seen above, the cost of time drafts is affected by both the course of exchange and the bank discount for the specified time.

WRITTEN EXERCISES.

3. What must I pay for a sight draft of $850 on St. Louis, at $3\frac{1}{2}$% premium? *Ans.* $879.75.

4. Find the cost of a draft of $4150, payable at 30 days after sight, at 6%, exchange being $1\frac{1}{2}$% premium? *Ans.* $4189.43.

5. What must a New York firm pay for a 15-day draft of $10500 on Baltimore, at 7%, exchange being $2\frac{3}{4}$% discount? *Ans.* $10174.50.

6. What must be paid in Philadelphia for a draft of $5300 on St. Paul, drawn at 60 days, exchange being $1\frac{3}{8}$% premium? *Ans.* $5317.23.

7. What is the cost of a draft on New York for $6750 at $\frac{3}{8}$% premium?

Find the cost of the following drafts:

8. $3000, 2% premium. *Ans.* $3060.
9. $5260, 4% discount. *Ans.* $5049.60.
10. $6783, $\frac{3}{8}$% discount. *Ans.* $6757.56.
11. $1200, $1\frac{1}{4}$% premium. *Ans.* $1215.
12. $4500, $2\frac{1}{2}$% discount. *Ans.* $4387.50.
13. $9150, $\frac{1}{4}$% discount. *Ans.* $9127.13.
14. $1560, $\frac{1}{8}$% premium. *Ans.* $1561.95.
15. $6416, $\frac{1}{2}$% discount. *Ans.* $6383.92.
16. $8712, $1\frac{3}{4}$% premium. *Ans.* $8864.46.
17. $4736, $3\frac{1}{8}$% discount. *Ans.* $4588.
18. $3500, 3% discount, 30 da., 6%. *Ans.* $3375.75.
19. $8420, $1\frac{1}{4}$% premium, 10 da., 7%. *Ans.* $8503.97.

20. \$2700, $1\frac{1}{4}$% discount, 20 da., 5%. *Ans.* \$2657.63.
21. \$3800, $3\frac{1}{2}$% premium, 60 da., 6%. *Ans.* \$3893.10.
22. \$5700, $2\frac{1}{8}$% premium, 90 da., 8%. *Ans.* \$5703.33.
23. \$4990, 5% discount, 30 da., 5%. *Ans.* \$4717.63.
24. \$7600, 9% discount, 20 da., 4%. *Ans.* \$6896.58.
25. \$3200, 2% premium, 27 da., 4%.
26. \$3550, 10% discount, 12 da., 7%.
27. \$1625, $2\frac{1}{2}$% premium, 42 da., 6%.
28. \$8500, 5% discount, 30 da., 8%.

Case II.

144. To find the face of a draft, when the cost is given.

1. What is the face of a draft on Buffalo, at 30 days, which can be bought for \$1623.20, interest being 6%, and exchange 2% premium?

OPERATION.

\$1.00
.02
\$1.02 = Course of Exchange.
.0055 = Disc. for 33 da.
\$1.0145 = Cost of \$1 of draft.
\$1623.20 ÷ \$1.0145 = \$1600.

Solution.—Following the rule for Case I., we find the cost of \$1 of the draft to be \$1.0145. Hence, if \$1 cost \$1.0145, for \$1623.20, we can get as many dollars as \$1.0145 is contained times in \$1623.20, which are \$1600.

Rule.—*Divide the given cost by the cost of \$1 of exchange.*

WRITTEN EXERCISES.

2. How large a 60-day draft can I buy for \$866.88$\frac{7}{20}$, interest at 6%, and exchange $1\frac{1}{2}$% premium? *Ans.* \$863.

3. Find the face of a 30 days' draft at $\frac{1}{4}$% discount, which can be bought for \$8027.775, money being worth 7%. *Ans.* \$8100.

4. What is the face of a sight draft purchased for $1101.24, exchange being at a premium of $3\frac{1}{2}$% ?

Ans. $1064.

5. A merchant in Albany bought a sight draft on Richmond at $\frac{5}{8}$% discount, for 447.18\frac{3}{4}$. What was the face of the draft ? *Ans.* $450.

Find the face of the following drafts :

6. $7150, 5% discount. *Ans.* $7526.32.
7. $9264, $3\frac{1}{2}$% discount. *Ans.* $9600.
8. $6045, $\frac{3}{4}$% premium. *Ans.* $6000.
9. $2005, $\frac{1}{4}$% premium. *Ans.* $2000.
10. $4700, $\frac{1}{8}$% premium. *Ans.* $4694.13.
11. $5200, $1\frac{1}{4}$% discount. *Ans.* $5265.82.
12. $3000, $\frac{1}{2}$% premium, 30 days, 5%. *Ans.* $2998.75.
13. $1500, 6% discount, 20 days, $4\frac{1}{2}$%. *Ans.* $1600.64.
14. $2850, 8% premium, 60 days, $5\frac{1}{4}$%. *Ans.* $2661.53.
15. $5000, $\frac{3}{8}$% discount, 90 days, 7%. *Ans.* $5111.60.
16. $6500, $1\frac{1}{4}$% premium, 30 da., 6%. *Ans.* $6454.82.
17. $1199.34, 1% discount, 10 da., 8%. *Ans.* $1215.
18. $2524.024, $1\frac{1}{2}$% premium, 20 da., 4%. *Ans.* $2493.
19. $3526.65, 4% premium, 30 da., 3%. *Ans.* $3400.
20. $5345.20, 3% discount, 90 da., 6%. *Ans.* $5600.
21. $3428.05, $1\frac{7}{8}$% premium, 60 da., 6%.
22. $1645.56, $1\frac{1}{2}$% discount, 30 da., 6%.
23. $6089, 2% premium, 30 da., 7%.
24. $8100, 3% premium, 25 da., 4%.
25. $5300, $3\frac{1}{2}$% discount, 60 da., 3%.

FOREIGN EXCHANGE.

145. In computing foreign exchange it is necessary to reduce the currency of one nation to that of the other.

146. ***Money of Account*** comprises the divisions of money of any particular country in which accounts are kept.

ESTIMATE OF VALUES OF FOREIGN COINS, JAN. 1, 1885.

Country.	Monetary Unit.	Standard.	Value in U.S. Money.
Argentine Republic	Peso	Gold and silver	.96,5
Austria	Florin	Silver	.39,3
Belgium	Franc	Gold and silver	.19,3
Bolivia	Boliviano	Silver	.79,5
Brazil	Milreis of 1000 reis	Gold	.54,6
British Possessions in N.A.	Dollar	Gold	$1.00
Chili	Peso	Gold and silver	.91,2
Cuba	Peso	Gold and silver	.92,2
Denmark	Crown	Gold	.26,8
Ecuador	Peso	Silver	.79,5
Egypt	Piaster	Gold	.04,9
France	Franc	Gold and silver	.19,3
German Empire	Mark	Gold	.28,8
Great Britain	Pound sterling	Gold	4.86,6½
Greece	Drachma	Gold and silver	.19,3
Hayti	Gourde	Gold and silver	.96,5
India	Rupee of 16 annas	Silver	.37,8
Italy	Lira	Gold and silver	.19,3
Japan	Yen	Silver	.85,8
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.86,4
Netherlands	Florin	Gold and silver	.40,2
Norway	Crown	Gold	.26,8
Peru	Sol	Silver	.79,5
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks	Silver	.63,6
Spain	Peseta of 100 centimes	Gold and silver	.19,3
Sweden	Crown	Gold	.26,8
Switzerland	Franc	Gold and silver	.19,3
Tripoli	Mahbub of 20 piasters	Silver	.71,7
Turkey	Piaster	Gold	.04,4
United States of Colombia	Peso	Silver	.79,5
Venezuela	Bolivar	Gold and silver	.19,3

Remittances to and from different places in Europe are frequently made through the following leading commercial centres, as quoted May 29, 1885 :

	60 Days.			3 Days.		
Prime banker's sterling bills	4.86	@	4.86½	4.88	@	4.88½
Good banker's sterling bills	4.85½	@	4.86	4.87½	@	4.88
Prime com. sterling bills	4.85½	@	4.86	4.87½	@	4.88
Paris (franc)	5.18¾	@	5.18⅛	5.16¼	@	5.15⅝
Antwerp (franc)	5.21¼	@	5.20⅝	5.18¾	@	5.17⅛
Swiss (franc)	5.18⅛	@	5.17½	5.15⅝	@	5.15
Amsterdam (guilder)	.40⅜	@	.40½	.40½	@	.40⅝
Hamburg (reichmark)	.95	@	.95⅛	.95⅝	@	.95¾
Frankfort (reichmark)	.95	@	.95⅛	.95⅝	@	.95¾
Bremen (reichmark)	.95	@	.95⅛	.95⅝	@	.95¾
Berlin (reichmark)	.95	@	.95⅛	.95⅝	@	.95¾

The words "prime," "good," and "commercial," denote the different degrees of credit in which such bills are held.

The unit, in exchange on Paris, Antwerp, and Switzerland, is the *franc*, and the quotation expresses the number of francs and centimes to the dollar. In exchange on Amsterdam, the unit is the guilder expressed by its value in cents. And the quotation on Hamburg, Frankfort, Bremen, and Berlin gives the number of cents in 4 reichmarks.

In the first column are found the highest and lowest prices of 60-day bills, and in the second those of 3 days.

147. A ***Set of Exchange*** consists of three copies of the same bill. Each copy is remitted in a different manner, and when one has been paid the others are void.

1.

Exchange for £2000. *Chicago, February 12, 1883.*

Sixty days after sight of this FIRST OF EXCHANGE, *Second and Third unpaid, pay to the order of Walter, Campbell & Co., Two Thousand Pounds Sterling, value received, and charge the same to account of*

No. 3145. *Snelling, Ross & Co.*

To Hamilton, Hampton & Co.,
Liverpool, England.

2.

Exchange for £2000.

Chicago, February 12, 1883.

Sixty days after sight of this SECOND OF EXCHANGE, *First and Third unpaid, pay to the order of Walter, Campbell & Co., Two Thousand Pounds Sterling, value received, and charge the same to account of*

No. 3145. *Snelling, Ross & Co.*

To Hamilton, Hampton & Co.,
Liverpool, England.

3.

Exchange for £2000.

Chicago, February 12, 1883.

Sixty days after sight of this THIRD OF EXCHANGE, *First and Second unpaid, pay to the order of Walter, Campbell & Co., Two Thousand Pounds Sterling, value received, and charge the same to account of*

No. 3145. *Snelling, Ross & Co.*

To Hamilton, Hampton & Co.,
Liverpool, England.

1. What must be paid in Philadelphia for a bill of exchange on London for £940, the course of exchange being $4.87½ ?

OPERATION.

Course of Ex., value of £1 = $4.875
$4.875 × 940 = $4582.50

SOLUTION.—Since by the course of exchange £1 = $4.875, £940 = 940 times $4.875, which are $4582.50.

2. What is the face of a bill on Liverpool which may be purchased in New Orleans for $1555.88, exchange *being 4.85 ?*

OPERATION.

£1 = $4.85
$1555.88 ÷ $4.85 = £320.8
£320.8 = £320 16*s*.

SOLUTION.—If £1 costs $4.85, as many pounds can be bought for $1555.88 as $4.85 are contained times in $1555.88, which are £320.8 = £320 16*s*.

RULE.—*I. To find the cost of a bill of exchange, multiply the cost of a unit of the currency in which the bill is given by the face.*

II. To find the face, divide the cost of the bill by the cost of a unit of the currency in which the bill is given.

WRITTEN EXERCISES.

What is the value of:

3.	£300 at 4.83¼ ?	*Ans.* $1449.75.
4.	£675 at 4.85 ?	*Ans.* $3273.75.
5.	£900 at 4.83⅞ ?	*Ans.* $4354.88.
6.	£90 15*s*. at 4.86¼ ?	*Ans.* $441.27.
7.	£25 10*s*. 8*d*. at 4.85⅞ ?	*Ans.* $124.06.
8.	£612 17*s*. 9*d*. at 4.86¾ ?	*Ans.* $2983.23.
9.	£325 16*s*. 6*d*. at 4.88⅛ ?	*Ans.* $1590.43.
10.	4004 francs at 5.20 ?	*Ans.* $770.
11.	6000 francs at 5.18¼ ?	*Ans.* $1157.74.
12.	9420 francs at 5.21¾ ?	*Ans.* $1805.46.
13.	7540 francs at 5.23¼ ?	*Ans.* $1440.99.
14.	2000 guilders at .38⅞ ?	*Ans.* $777.50.
15.	4500 guilders at .39¼ ?	*Ans.* $1766.25.
16.	1600 guilders at .36⅝ ?	*Ans.* $586.
17.	7200 reichmarks at .93⅜ ?	*Ans.* $1680.75.
18.	8480 reichmarks at .94⅜ ?	*Ans.* $2000.75.
19.	3160 reichmarks at .93⅜ ?	*Ans.* $737.66.
20.	5400 reichmarks at .94⅜ ?	*Ans.* $1274.06.
21.	6100 reichmarks at .93⅜ ?	*Ans.* $1423.97.

22. 4960 reichmarks at .94⅜? *Ans.* $1170.25.

23. What was the cost of a set of exchange on England for £660 5*s.* at 4.87½? *Ans.* $3218.72.

24. When exchange on Paris is 5.20, how many francs of exchange will $1065 buy? *Ans.* 5538 francs.

25. What will be the cost in Hamburg of a bill on Buffalo for $2660, the course of exchange being 95? *Ans.* 11200 reichmarks.

26. When exchange on Berlin is 95½, what will be the face of a bill of exchange that $7162.50 will purchase? *Ans.* 30000 reichmarks.

27. When exchange is quoted at 482 cts. and gold is at par, what will be the amount of a bill in Liverpool that $6000 will buy? *Ans.* £1244 16*s.* 3*d.*

28. If Paris exchange is quoted at 522 centimes, what must I give for a bill of exchange of 5000 francs, gold being at 108? *Ans.* $1034.48.

29. By the second quotation in the table on page 115, what amount of exchange on Frankfort, at 3 days' sight, will $2298 in gold buy? *Ans.* 9600 reichmarks.

30. I wish to remit 2435 reichmarks from Bremen to New York; what will be the face of a bill of exchange at 60 days, at the first quotation? *Ans.* $578.31.

31. When gold is at 112½, and exchange on Paris at 5.16, what will be the face of a bill on Paris that can be bought for $4000 in currency? *Ans.* 18346.67 francs.

32. Find the face of a bill of exchange on Dublin that can be bought for $270.48, when sterling exchange is quoted at 4.84. *Ans.* £55 17*s.* 8*d.*

33. What will be the face of a draft on Geneva that can be bought for $7686, exchange being 5.22½? *Ans.* 40159.35 francs.

34. What was the face of a bill of exchange on Am-

sterdam, for which an exporter received $945.43, exchange selling at $41\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

Ans. 2271.30 guilders.

35. What will be the cost of a bill of exchange on Antwerp, at 3 days' sight, for 19280 francs, at $5.19\frac{3}{8}$?

Ans. $3712.15.

36. What amount was received for a bill of exchange on Hamburg for 26400 reichmarks, at $94\frac{1}{8}$, brokerage $\frac{1}{8}\%$?

Ans. $6220.50.

37. What is the cost of a draft on Paris for 6210 francs, at $5.19\frac{3}{4}$, brokerage $\frac{1}{8}\%$? *Ans.* $1194.52.

38. An exporter received $4666.959 for a bill of exchange that was sold through a banker at $4.86\frac{3}{4}$; what was the face of the bill, commission being $\frac{1}{8}\%$?

ARBITRATION OF EXCHANGE.

148. ***Arbitration of Exchange*** is the method of solving the exchange between two places by means of one or more intermediate exchanges.

149. The object of arbitration is to enable a person to ascertain whether it will be more profitable to remit directly from one place to another, or by means of intermediate exchanges. As the rates are constantly changing, the latter course is frequently found to be the more advantageous.

150. When there is only one intermediate exchange, the method is called *Simple Arbitration;* when there are two or more, it is called *Compound Arbitration.*

1. A man in New York wishes to remit $1500 to New Orleans. Exchange on St. Louis is $\frac{1}{2}\%$ premium; and between St. Louis and New Orleans 2% discount. What will the remittance cost, if sent by St. Louis?

OPERATION.

$\$1.005$ N. Y. $= \$1.00$ St. L.
$\$1.00$ " $= \$\frac{1}{1.005}$ "
$\$0.98$ St. L. $= \$1.00$ N. O.
$\$1.00$ " $= \$\frac{1}{.98}$ "
$\$\frac{1}{1.005}$ " $= \$\frac{1\times1}{.98\times1.005}$ N. O.
$\$1.00$ N. Y. $= \$\frac{1\times1}{.98\times1.005}$ "
$\$1500$ " $= \$\frac{1\times1\times1500}{.98\times1.005}$ "
$\$\frac{1\times1\times1500}{.98\times1.005} = \$1522.99+$

SOLUTION.—Exchange in New York on St. Louis, being $\frac{1}{2}\%$ premium, $1.005 in New York = $1.00 in St. Louis. Therefore, $1.00 in New York = $\frac{1}{1.005}$ in St. Louis. Secondly, if exchange in St. Louis on New Orleans is 2% discount, $.98 in St. Louis = $1.00 in New Orleans; and $1.00 in St. Louis = $\frac{1}{.98}$ in New Orleans. Hence, $1.00 in New York, which is = $\frac{1}{1.005}$ in St. Louis, is = $\frac{1\times1}{.98\times1.005}$ in New Orleans; and $1500 in New York = $\frac{1\times1\times1500}{.98\times1.005} = \$1522.99+$, or $1523, N. O.

2. What will it cost in New York to pay a debt in Hamburg of 2560 marks by remitting through Paris, exchange between New York and Paris being 5 francs 16 centimes per dollar, and between Paris and Hamburg, 1.23 fr. per mark?

OPERATION.

$$5.16 \text{ fr.} = \$1$$
$$1 \text{ fr.} = \frac{\$1}{5.16}$$
$$1 \text{ mark} = 1.23 \text{ fr.} = \frac{\$1 \times 1.23}{5.16}$$
$$2560 \text{ marks} = \frac{\$1 \times 1.23 \times 2560}{5.16}$$
$$\frac{\$1 \times 1.23 \times 2560}{5.16} = \$610.23$$

SOLUTION.—Since 5.16 fr. = $1, 1 fr. = $1 divided by 5.16; and 1.23 fr., or 1 mark, = 1.23 times the value of 1 fr., or ($1 × 1.23) ÷ 5.16. Then if 1 mark = ($1 × 1.23) ÷ 5.16, 2560 marks = [($1 × 1.23) ÷ 5.16] × 2560 = $610.23. Hence, to pay a debt of 2560 marks in Hamburg, will cost $610.23 in New York.

NOTE.—Analysis is the simplest, as well as the most satisfactory method of treating Arbitration of Exchange, Compound Proportion, and, in general, all those cases in which it is required to find the value of a given quantity in terms of another quantity by means of *some* intermediate quantity.

WRITTEN EXERCISES.

3. What must I pay in Buffalo for a remittance of $8034 from Baltimore, through Chicago, exchange between Baltimore and Chicago being $2\frac{1}{2}$% discount, and between Chicago and Buffalo 3% premium? *Ans.* $8000.

4. A merchant in New York directs his agent in Boston to draw on Cincinnati for $5000, at $2\frac{1}{2}$% premium. After allowing $\frac{1}{4}$% commission, he draws upon his agent in Boston at $1\frac{1}{2}$% discount. What amount does he realize? *Ans.* $5035.50.

5. If at Boston, exchange on Liverpool is $4.83\frac{3}{8}$, and at Liverpool, on Paris 21 francs $85\frac{1}{4}$ centimes per £1, what is the arbitrated course of exchange between Boston and Paris through Liverpool?

6. What must be paid in Paris for a remittance of $2300, through London, to Philadelphia, if exchange between London and Philadelphia is £9 to $46, and between Paris and London 54 francs to £2? *Ans.* 12150 fr.

7. What will it cost a man in Sacramento to pay a debt of $1400 in St. Louis, if the direct exchange is $1\frac{1}{2}$% in favor of St. Louis? *Ans.* $1421.

8. Would it be more profitable to pay the above debt through Philadelphia, if exchange on Philadelphia were 2% in favor of Philadelphia, and between Philadelphia and St. Louis $1\frac{1}{4}$% in favor of St. Louis?

9. A lawyer in Pittsburgh wishes to pay a debt of $2500 in Amsterdam. Exchange between Pittsburgh and Amsterdam is $.415 per guilder; between Amsterdam and Liverpool 18*d.* per guilder; and between Pittsburgh and Liverpool $\$5\frac{1}{6}$ per £. What is the difference between the direct and the circular exchange?

Ans. 497.64 guilders.

RATIO AND PROPORTION.

151. ***Ratio*** is the relation of two similar quantities as indicated by the quotient of the second divided by the first.

152. Ratio is usually expressed by the colon (:); thus, 6 : 3 signifies the ratio of 6 to 3, and is equal to $\frac{3}{6} = \frac{1}{2}$.

153. The ***Terms*** of a ratio are the two numbers compared. They form a couplet, the first of which is called the *antecedent,* and the second, the *consequent.*

NOTE.—The numbers compared must be of the same kind. Fractions are compared either by direct division, or by reducing them to a common denominator, when they will be to each other as their numerators.

154. A ***Simple Ratio*** is the ratio of two numbers; as, 12 : 6.

155. A ***Compound Ratio*** is the product of two or more simple ratios; as $(2 : 3) \times (4 : 5)$, or $\frac{3}{2} \times \frac{5}{4}$.

156. ***Inverse Ratio*** is obtained by dividing the antecedent by the consequent.

ORAL EXERCISES.

1. What part of 8 is 2?

SOLUTION.—Since 2 is contained 4 times in 8, it is equal to $\frac{1}{4}$ of 8. This relation is expressed thus: 2 has the same ratio to 8 as 1 *has to* 4.

2. What part of 6 is 3? Is 1? Is 2? Is 5? Is 4?

3. What part of 10 is 5? Is 4? Is 8? Is 9? Is 3?

4. What is the ratio of 30 to 6? Of 20 to 4? Of 60 to 10?

5. If the antecedent is 10 and the ratio 2, what is the consequent?

6. If the consequent is 50 and the ratio 5, what is the antecedent?

7. What is the antecedent when the ratio is 4, and the consequent 48?

8. What is the consequent, if the antecedent is 13, and the ratio 9?

WRITTEN EXERCISES.

1. What is the ratio of 5 to 7? Of 9 to 4? Of 8 to 13?

2. What is the ratio of 15 to 37? Of 14 to 28? Of 3 to 18? Of 21 to 84? Of 15 to 45? Of 12 to 36?

3. What is the ratio of 7 to 8? Of 8 to 5? Of 12 to 7? Of 1 to 20? Of 9 to 12? Of 14 to 6?

4. What is the ratio of 15 to 11? Of 20 to 30? Of 10 to 25? Of 30 to 45? Of 50 to 35?

5. What is the ratio of \$4 to \$10? Of \$12 to \$5? Of \$16 to \$8? Of \$25 to \$60? Of \$13 to \$39?

6. What is the ratio of $\frac{2}{5}$ to $\frac{3}{5}$? Of $\frac{1}{4}$ to $\frac{3}{4}$? Of $\frac{2}{3}$ to $\frac{3}{4}$?

7. What is the ratio of $\frac{7}{10}$ to $\frac{9}{10}$? Of $\frac{5}{6}$ to $\frac{2}{3}$? Of $\frac{5}{12}$ to $\frac{8}{9}$? Of $\frac{1}{6}$ to $\frac{2}{3}$? Of $\frac{4}{5}$ to $\frac{9}{10}$?

8. What is the ratio of $2\frac{1}{2}$ to $\frac{6}{7}$? Of $3\frac{1}{2}$ to $\frac{8}{15}$? Of $\frac{4}{5}$ to $5\frac{1}{3}$? Of $\frac{9}{10}$ to $4\frac{1}{2}$? Of $\frac{1}{8}$ to $1\frac{3}{5}$?

9. What is the ratio of $3\frac{1}{4}$ to $4\frac{1}{2}$? Of $5\frac{1}{4}$ to $2\frac{1}{3}$? Of $10\frac{1}{2}$ to $7\frac{2}{3}$? Of $2\frac{1}{5}$ to $7\frac{1}{2}$? Of $\frac{3}{4}$ to $\frac{1}{7}$?

10. What is the inverse ratio of 4 to 3? Of 7 to 8? Of 10 to 16? Of 17 to 9? Of 16 to 3?

11. What is the inverse ratio of 20 to 30? Of 15 to 35? Of 40 to 60? Of 120 to 15?

12. What is the inverse ratio of 30 to 60? Of 10 to 100? Of 80 to 20? Of 160 to 180?

13. What is the inverse ratio of $\frac{4}{7}$ to $\frac{5}{9}$? Of $\frac{3}{4}$ to $\frac{5}{7}$? Of $\frac{4}{5}$ to $\frac{7}{10}$? Of $\frac{15}{16}$ to $\frac{3}{8}$? Of $\frac{1}{2}$ to $\frac{8}{15}$?

14. Which is the greater ratio, 7 : 8, or 4 : 5?

15. What is the difference between the ratios 6 : 7 and 10 : 8? 9 : 10 and 3 : 5?

16. How much greater than the ratio 10 : 12 is 2 : 3?

What is the value of each of the following compound ratios?

17.	*18.*	*19.*	*20.*	*21.*
3 : 4 4 : 7	6 : 8 9 : 12	5 : 6 2 : 9	8 : 15 6 : 13	12 : 15 10 : 5
8 : 1 3 : 2	5 : 7 4 : 6	$4\frac{1}{2}$: 5 2 : $3\frac{1}{3}$	$1\frac{1}{4}$: 5 7 : $6\frac{1}{7}$	8 : $2\frac{3}{4}$ 9 : 3

Reduce the following couplets to couplets of lower numbers having the same ratio:

Examples.	*22.*	*23.*	*24.*	*25.*
6 : 12 = 1 : 2	10 : 45	26 : 52	34 : 46	24 : 120
8 : 14 = 4 : 7	20 : 60	84 : 36	57 : 90	16 : 128
10 : 6 = 5 : 3	25 : 65	81 : 54	50 : 85	35 : 105
9 : 15 = 3 : 5	32 : 40	21 : 56	14 : 80	60 : 300
24 : 8 = 3 : 1	75 : 10	33 : 77	91 : 49	75 : 225

SIMPLE PROPORTION.

157. ***Proportion*** is an equality of ratios.

The ratio of 6 to 12 is 2, and the ratio of 5 to 10 is 2. Hence, from these two equal ratios the following proportion can be formed: 6 : 12 = 5 : 10. Instead of the sign of equality, a double colon (::) is generally used. And the proportion is read either 6 is to 12 as 5 *is to* 10, or, the ratio of 6 to 12 equals the ratio of 5 to 10.

158. The ***Terms*** of a proportion are the four numbers used in the comparison. The first and fourth are called the *extremes;* and the second and third, the *means.*

159. The ***Couplets*** are the two ratios compared. The first and second terms constitute the *first couplet;* and the third and fourth, the *second couplet.*

160. ***Simple Proportion*** is an equality of simple ratios.

161. In every proportion, the product of the extremes is equal to the product of the means. Thus, in the proportion $7 : 5 :: 21 : 15$, if the ratios be expressed as $\frac{5}{7} = \frac{15}{21}$, and both multiplied by the product of the denominators (7×21), the result will be $\frac{5 \times 7 \times 21}{7} = \frac{15 \times 7 \times 21}{21}$ $= 5 \times 21 = 15 \times 7$.

Find the value of x in each of the proportions:

1. $18 : 15 :: 42 : x$.
2. $9 : 20 :: x : 12$.

OPERATION.

(1)

$$18 : 15 :: 42 : x$$
$$18 \times x = 15 \times 42$$
$$18\,x = 15 \times 42$$
$$x = \frac{15 \times 42}{18} = 35$$

SOLUTION.—Placing the product of the extremes equal to the product of the means, we have $18 \times x = 15 \times 42$, from which we find x to be equal to 35.

(2)

$$9 : 20 :: x : 12$$
$$20 \times x = 12 \times 9$$
$$20\,x = 12 \times 9$$
$$x = \frac{12 \times 9}{20} = 5\tfrac{2}{5}$$

In the second proportion the unknown term is a mean. We, therefore, place the product of the means equal to the product of the extremes, and find x to be equal to $5\frac{2}{5}$. Hence, we conclude that either extreme is equal to the product of the means divided by the other extreme; and either mean is equal to the product of the extremes divided by the other mean.

WRITTEN EXERCISES.

Find the unknown term in each of the following proportions:

1.	$x : 5 :: 4 : 20 = 20x = 4 \times 5 = x = 1.$	*Ans.* 1.
2.	$3 : x :: 9 : 18.$	*Ans.* 6.
3.	$10 : 30 :: 8 : x.$	*Ans.* 24.
4.	$15 : x :: 2 : 6.$	*Ans.* 45.
5.	$16 : 5 :: x : 10.$	*Ans.* 32.
6.	$40 : 8 :: 25 : x.$	*Ans.* 5.
7.	$36 : x :: 18 : 3.$	*Ans.* 6.
8.	$45 : 90 :: x : 8.$	*Ans.* 4.
9.	$22 : 66 :: 11 : x.$	*Ans.* 33.
10.	$x : \frac{1}{4} :: \frac{3}{5} : 6.$	*Ans.* $\frac{1}{40}$.
11.	$\frac{5}{8} : x :: 9 : \frac{6}{7}.$	*Ans.* $\frac{5}{84}$.
12.	$\frac{3}{4} : \frac{7}{8} :: x : \frac{3}{10}.$	*Ans.* $\frac{9}{35}$.
13.	$120 : 24 :: 225 : x.$	*Ans.* 45.
14.	$500 : 800 :: x : 80.$	*Ans.* 50.
15.	$72 : x :: 108 : 324.$	*Ans.* 216.
16.	$x : 45 :: 80 : 400.$	*Ans.* 9.
17.	$575 : x :: 324 : 1620.$	*Ans.* 2875.
18.	$216 : 72 :: x : 192.$	*Ans.* 576.
19.	$500 : 750 :: 200 : x.$	*Ans.* 300.
20.	$350 : 700 :: 70 : x.$	*Ans.* 140.
21.	$120 : 600 :: x : 90.$	*Ans.* 18.
22.	$x : 1050 :: 200 : 600.$	*Ans.* 350.
23.	$250 : x :: 72 : 288.$	*Ans.* 1000.

APPLICATION.

162. The method of simple proportion is applied in solving problems whose quantities are so related that *three* being given, the fourth may be found by the

equality of ratios. Of the three given terms, two must always be like numbers, giving the complete ratio. The third must be of the same name or kind as the required term.

163. The solution of a problem in proportion may also be considered as the comparison of two *causes* and two *effects*. As like causes produce like effects, every problem in proportion must give either of the following statements:

1st cause : 2d cause :: 1st effect : 2d effect.
1st effect : 2d effect :: 1st cause : 2d cause.
1st cause : 1st effect :: 2d cause : 2d effect.

But long experience has shown that it is far more advantageous for the student to solve the problems usually given under the rules of Proportion, by the method of Analysis. For an illustration of this method, see the fourth solution given below.

1. If five bushels of wheat cost \$7, what will 8 bushels cost?

STATEMENT.

bu. bu. \$ \$
5 : 8 :: 7 : (?)
1st cause : 2d cause :: 1st effect : 2d effect.

OPERATION.

$$5 \times (?) = 8 \times 7$$

$$(?) = \frac{8 \times 7}{5} = \$11\tfrac{1}{5}$$

SOLUTION. — In this question the required term is an effect. The statement of the proportion is 5 bu. is to 8 bu. as \$7 : (?), or how many dollars. Taking the product of the means, and dividing by the known extreme, we have $\frac{8 \times 7}{5} = \$11\frac{1}{5}$, the required term.

2. How many yards of cloth will \$36 buy, if 7 yd. cost \$28?

STATEMENT.

x : 7 yd. : : \$36 : \$28

OPERATION.

$x \times \$28 = 7 \text{ yd.} \times \$36$

$x = \frac{\cancel{7} \times \cancel{\$36}^{9}}{\cancel{28}_{\cancel{4}}} = 9 \text{ yd.}$

SOLUTION.—The relation of the unknown term to 7 yd. is the same as the relation of the cost of the unknown number of yards is to the cost of 7 yd. Hence, from these equal relations, we derive the proportion, x, the required number of yd. is to 7 yd. as \$36 is to \$28; from which we obtain the number of yards that can be bought for \$36.

3. If 5 men can dig a ditch in 16 days, how long will it take 8 men to do the same work ?

STATEMENT.

men		da.
5	=	16
8	=	x

men	men		da.	da.
8 :	5	: :	16 :	x

OPERATION.

$8 \times x = 5 \times 16$

$x = \frac{5 \times \cancel{16}^{2}}{\cancel{8}} = 10 \text{ da.}$

SOLUTION.—In this problem the like terms are 8 men and 5 men ; and 16 da. is of the same kind as the unknown time. But since 8 men will require less time to do the work than 5 men, the required term must be less than 16 da. The ratio of both couplets, therefore, must be less than 1. Hence, we obtain the proportion, 8 men : 5 men : : 16 da. : x ; from which we derive the required term, 10 da.

4. If 24 men can dig a ditch in 14 days, how long will it take 21 men to dig it ?

OPERATION.

If 24 men can dig a ditch in 14 days,

1 man " " $14 \times 24 = 336$ days.

21 men " " $\frac{14 \times 24}{21} = 16$ days, *ans.*

SOLUTION.—Arrange t..e statement so that what is required may come last. Thus, if 24 men can dig a ditch in 14 days, 1 man will require 24 times as many days, or 24 times 14 days = 336 days. And if 1 man requires 336 days to dig a ditch, 21 men will require *21 times* less, or $\frac{336}{21} = 16$ days.

5. If $\frac{8}{7}$ bushels of oats cost $\$\frac{6}{5}$, what will $\frac{1}{5}$ of a bushel cost?

OPERATION.

$\frac{8}{7}$ bu.	cost $\$\frac{6}{5}$			
$\frac{1}{7}$ "	will cost	$\frac{5}{5\times8}$	$=$	$\$\frac{3}{20}$
$\frac{7}{7}$ "	"	$\frac{6\times7}{5\times8}$	$=$	$\frac{21}{20}$
$\frac{6}{5}$ "	"	$\frac{6\times7}{5\times8}$	$=$	$\frac{21}{20}$
$\frac{1}{5}$ "	"	$\frac{6\times7}{5\times8\times5}$	$=$	$\frac{21}{100}$
$\frac{4}{5}$ "	"	$\frac{6\times7\times4}{5\times8\times5}$	$=$	$\$\frac{21}{25}$

SOLUTION.—As dollars are required, we place the given $\$\frac{6}{5}$ last. Then, if $\frac{8}{7}$ bu. of oats cost $\$\frac{6}{5}$, $\frac{1}{7}$ bu. will cost 8 times less, or $\frac{6}{5\times8} = \$\frac{3}{20}$; and $\frac{7}{7}$, or 1 bu., will cost 7 times as much as $\frac{1}{7}$ bu., or $\frac{3}{20} \times 7 = \$\frac{21}{20}$. Again, if 1 bu., or $\frac{6}{5}$ bu., cost $\$\frac{21}{20}$, $\frac{1}{5}$ bu. will cost 5 times less, or $\$\frac{21}{100}$; and $\frac{4}{5}$ bu. will cost 4 times as much as $\frac{1}{5}$ bu., or $\frac{21}{100} \times 4 = \$\frac{84}{100} = \$\frac{21}{25}$.

RULE 1.—*I. With the two given numbers which are of the same name or kind, form a ratio greater or less than 1, according as the answer is to be greater or less than the third given number.*

II. Find the fourth term by dividing the product of the means by the known extreme.

RULE 2.—*So arrange the terms that the causes shall form one couplet, and the effects the other. Then find the unknown cause or effect as indicated above.*

WRITTEN EXERCISES.

6. When 12 barrels of apples cost $37.50, what will 165 barrels cost? *Ans.* $515.63.

7. What is the cost of 36 hhd. of molasses, if 9 hhd. cost $180? *Ans.* $720.

8. If 16 bushels of wheat make 3 barrels of flour, how many barrels of flour will 120 bushels make?

Ans. $22\frac{1}{2}$ bbl.

9. How many men will it take to do a piece of work in 15 days, if 30 men require 10 days to do it?

Ans. 20 men.

10. If 45 oxen eat 72 acres of grass in a month, how many oxen will 288 acres keep for the same time?

Ans. 180 oxen.

11. If $200 gain $10 in 18 months, how much will $320 gain in the same time? *Ans.* $16.

12. If 15 gallons of wine cost $35, what will 75 gallons cost? *Ans.* $175.

13. Two numbers are to each other as 5 : 40, and the less is 136.25; what is the greater? *Ans.* 1090.

14. If 39 men earn $312 in two weeks, how many men can earn $504 in the same time? *Ans.* 63 men.

15. If 63 cords of wood cost $236.25, what will 25 cords cost? *Ans.* $93.75.

16. If 23 horses consume 46 bundles of hay in 2 weeks, how many bundles will 32 horses consume in the same time? *Ans.* 64 bundles.

17. How much will 35 cwt. 25 lb. of sugar cost, at the rate of 12 cwt. 75 lb. for $68? *Ans.* $188.

18. If a pole 18 feet high cast a shadow 22 feet long, what must be the length of a pole to cast a shadow 110 feet long? *Ans.* 90 ft.

19. In what time will a train run from New York to Washington, 236 miles, at the rate of 6 miles in 12 min. 15 sec.? *Ans.* 8 hr. 1 mi. 50 sec.

20. In what time can a man pump 60 barrels of water, if he pumps 35 barrels in 1 hr. 45 mi.? *Ans.* 3 hr.

21. At the rate of 10 yards for $28.50, how many yards of cloth can be bought for $179.55? *Ans.* 63 yd.

22. What quantity of sugar can be bought for $256.25, *if 5 lb. cost* 62½ cents? *Ans.* 2050 lb.

23. If a piece of land 10 rods wide, contain $\frac{9}{10}$ of an acre, how much would there be if it were $16\frac{2}{3}$ rods wide?
Ans. $1\frac{1}{2}$ A.

24. If a cistern containing 2500 gallons, leak 2 gal. 3 qt. a minute, how long will it take to empty it?
Ans. 15 hr. 9 + min.

25. If a certain quantity of provisions will supply a garrison of 520 men 9 months, how long will it last 3640 men? *Ans.* $1\frac{2}{7}$ mo.

26. If $630 purchase 420 hats, how many hats can be purchased for $75? *Ans.* 50 hats.

27. If it requires 75 men, working 8 hours a day, to do a piece of work, how many men working 12 hours a day, will it require to do the same work? *Ans.* 50 men.

28. If $\frac{5}{7}$ of a yard of cloth cost $\$\frac{23}{28}$, what will $12\frac{5}{6}$ yd. cost? *Ans.* $12.30.

29. What will 162 bushels of wheat cost, if 80 bushels cost $97.60? *Ans.* $197.64.

30. How long will it take 60 men to dig a well, if 10 men can do it in 32 days? *Ans.* $5\frac{1}{3}$ days.

COMPOUND PROPORTION.

164. ***Compound Proportion*** is that proportion in which one or both ratios are compound. It embraces all those problems in which the unknown quantity depends upon the ratios of two or more pairs of similar quantities.

WRITTEN EXERCISES.

Find the unknown term in each of the following compound proportions:

1. $10 : x :: \left\{\begin{matrix} 3 : 5 \\ 4 : 6 \end{matrix}\right\} = 10 : x :: 12 : 30.$

2. $x : 20 :: \left\{\begin{matrix} 2 : 1 \\ 3 : 4 \end{matrix}\right\}$ *Ans.* 30.

3. $\left\{\begin{matrix} 10 : 5 \\ 4 : 8 \end{matrix}\right\} :: x : 3$ *Ans.* 3.

4. $\left\{\begin{matrix} 7 : 3 \\ x : 5 \end{matrix}\right\} :: \left\{\begin{matrix} 24 : 21 \\ 3 : 9 \end{matrix}\right\}$ *Ans.* $\frac{40}{45}$.

5. $\left\{\begin{matrix} 14 : 6 \\ 15 : 10 \end{matrix}\right\} :: \left\{\begin{matrix} 9 : 5 \\ x : 7 \end{matrix}\right\}$ *Ans.* $13\frac{11}{18}$.

6. $\left\{\begin{matrix} 22 : 44 \\ 11 : x \end{matrix}\right\} :: \left\{\begin{matrix} 8 : 35 \\ 1 : 15 \end{matrix}\right\}$ *Ans.* $360\frac{15}{16}$.

7. $\left\{\begin{matrix} \frac{2}{3} : \frac{3}{4} \\ \frac{3}{4} : \frac{1}{2} \end{matrix}\right\} :: \left\{\begin{matrix} \frac{2}{5} : x \\ \frac{6}{8} : \frac{3}{7} \end{matrix}\right\}$ *Ans.* $\frac{21}{40}$.

8. $\left\{\begin{matrix} \frac{9}{10} : \frac{3}{8} \\ \frac{5}{6} : \frac{7}{12} \end{matrix}\right\} :: \left\{\begin{matrix} \frac{6}{15} : \frac{7}{8} \\ \frac{5}{16} : x \end{matrix}\right\}$ *Ans.* $\frac{1}{24}$.

APPLICATION.

1. If 6 men can earn \$36 in 9 days, how much can 15 men earn in 8 days?

STATEMENT.

$$\left\{\begin{matrix} \text{men} & \text{men} \\ 6 : & 15 \\ \text{da.} & \text{da.} \\ 9 : & 8 \end{matrix}\right\} :: \$36 : \$x.$$

OPERATION.

$$\$x = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}}} \times \frac{8}{\cancel{9}} \times \overset{24}{\cancel{36}} = \$80.$$

SOLUTION.—The required term is the number of dollars that 15 men can earn in 8 days.

If the number of men be increased, the number of dollars will be increased in direct proportion. And if the number of days be diminished, the number of dollars will also be diminished. Hence the ratio of the given sum to the required sum is in proportion to the combined ratios of the given number of men and days. From these relations, expressed as a compound proportion, we obtain by *the operation* in the margin, the required sum, \$80.

2. How many horses will be required to plow 66 acres in 36 days, if 24 horses plow 22 acres in 10 days?

STATEMENT.

$$\left\{\begin{array}{l}22 \text{ acres} : 66 \text{ acres}\\ 36 \text{ days} : 10 \text{ days}\end{array}\right\} :: \begin{array}{cc}\text{h.} & \text{h.}\\ 24 & x\end{array}$$

OPERATION.

$$x = \frac{\cancel{66}^{\,3}}{\cancel{22}} \times \frac{10}{\cancel{36}_{\,12}} \times \cancel{24}^{\,2} = 20 \text{ horses.}$$

SOLUTION.—If it take 24 horses to plow 22 acres in 10 days, to plow 66 acres will require a proportionally greater number of horses, that is, the number of acres is directly proportional to the number of horses.

Secondly, if it take 24 horses 10 days to plow 22 acres, to plow 66 acres in 36 days, will require fewer horses, that is, the number of days is inversely proportional to the number of horses. These relations may be expressed in the form of a compound proportion, from which we will obtain the unknown term, x, = 20 horses.

3. If 12 men working 10 hours a day for 5 days, earn \$50, in how many days of 9 hours each, can 5 men earn \$180?

STATEMENT.

men	hr.	\$	da.
12	10	50	5
5	9	180	x

OPERATION.

12 men working 10 hours a day, earn \$50 in 5 days.

1	"	"	10	"	"	can	"	"	$5 \times 12 = 60$	days.
1	"	"	1	"	"	"	"	"	$5 \times 12 \times 10 = 600$	"
1	"	"	1	"	"	"	"	\$1 "	$\frac{5 \times 12 \times 10}{50} = 12$	"
5	"	"	1	"	"	"	"	\$1 "	$\frac{5 \times 12 \times 10}{50 \times 5} = \frac{12}{5}$	"
5	"	"	9	"	"	"	"	\$1 "	$\frac{5 \times 12 \times 10}{50 \times 5 \times 9} = \frac{4}{15}$	"
5	"	"	9	"	"	"	"	\$180 "	$\frac{5 \times 12 \times 10 \times 180}{50 \times 5 \times 9} = 48$	"

4. If 12 men working 10 hours a day for 5 days, earn \$50, how many hours a day must 5 men work to earn \$180 in 48 days?

STATEMENT.

men	\$	da.	hr.
12	50	5	10
5	180	48	x

SOLUTION.

12 men earn \$50 in 5 days, by working 10 hr. a day

1 man can earn \$50 in 5 days, by working $10 \times 12 = 120$ hr.

1 man can earn \$1 in 5 days, by working $\frac{10 \times 12}{50} = \frac{12}{5}$ hr.

1 man can earn \$1 in 1 day, by working $\frac{10 \times 12 \times 5}{50} = 12$ hr.

5 men can earn \$1 in 1 day, by working $\frac{10 \times 12 \times 5}{50 \times 5} = \frac{12}{5}$ hr.

5 men can earn \$180 in 1 day, by working $\frac{10 \times 12 \times 5 \times 180}{50 \times 5} = 432$ hr.

5 men can earn \$180 in 48 days, by working $\frac{10 \times 12 \times 5 \times 180}{50 \times 5 \times 48} = 9$ hr.

NOTE.—When the pupil has become familiar with the method of reasoning employed in the last two solutions, he may be permitted to abbreviate by omitting the results of the intermediate steps. But in no case should he be allowed to advance until he has given a satisfactory reason for each operation.

RULE.—*I. With the required term and its like known quantity form ratios with each pair of the other known quantities.*

II. Express these ratios in the form of a compound proportion.

III. Find the required term from the equal products of the extremes and the means.

NOTE.—As shown in the third and the fourth solution given above, every question in Compound Proportion may be solved by the Analytic, or Unity Method.

WRITTEN EXERCISES.

5. In how many days can 9 machines reap 540 acres, if 21 machines can reap 2520 acres in 24 days?

Ans. 12 days?

6. If I gain $230 on $3600 in 5 months, how much will I gain on $10800 in 1 year 8 months? *Ans.* $2760.

7. If 100 bushels of corn at .75 a bushel, will discharge a certain debt, how many bushels at .96, will discharge a debt 3 times as great? *Ans.* 234⅜ bu.

8. If 12 men, in 70 days, can build a wall 280 rd. long, and 6 ft. high, how many men, in 80 days, can build a wall of the same thickness, 288 rd. long, and 5 ft. high? *Ans.* 9 men.

9. What will be the cost of 80 yd. of calico, 1⅛ yd. wide, if 64 yd. of the same kind, 1¼ yd. wide, cost $10?

Ans. $11.25.

10. If a block of granite, 16 ft. long, 11 ft. wide, and 3 ft. thick, weigh 33000 lb., what would be the weight of a block of the same kind, 40 ft. long, 16 ft. wide, and 6 ft. thick? *Ans.* 240000 lb.

11. If a pasture of 10 acres will feed 8 horses for 5 months, how many acres will feed 25 horses 15 months?

Ans. 93¾ A.

12. How far could a man travel in 30 days, at the rate of 9 hours a day, if he can travel 13 miles in 2 days, at the rate of 6 hours a day? *Ans.* 292½ mi.

13. If 20 persons are fed for 15 days on $10 worth of flour, how many dollars worth of flour will 16 persons require for 12 days? *Ans.* $6.40.

14. If 6 tailors can make 72 pairs of pants in 4 days, how many pairs can 20 tailors make in a week?

Ans. 360 pairs.

15. If 25% of a man's fortune, invested at 5%, makes $650 in 6 mo., in what time will $37\frac{1}{2}$% of his fortune, invested at 10%, make $325? *Ans.* 1 mo.

16. If 12 men, in $9\frac{1}{2}$ days of 10 hours each, can do a piece of work, in how many days of 12 hours each can 20 men do the work? *Ans.* $4\frac{3}{4}$ days.

17. If 8 lb. of thread make $26\frac{2}{3}$ yd. of linen, $\frac{7}{8}$ yd. wide, how many pounds of the same kind of thread will make 99 yd., $1\frac{1}{4}$ yd. wide? *Ans.* $42\frac{3}{7}$ lb.

18. If 25 yd. of carpet $\frac{5}{6}$ yd. wide, cost $30, what will 121 yd. $\frac{2}{3}$ yd. wide, cost at the same rate?

Ans. $154.88.

19. If 20 persons drink 40 gallons of water in 5 days, how many gallons will 9 persons drink in a year?

Ans. 1314 gal.

PARTITIVE PROPORTION.

165. ***Partitive Proportion*** is the method of dividing a number into parts having a given relation to each other.

NOTE.—All problems of this kind should be solved by the analytic method.

WRITTEN EXERCISES.

1. The sum of two numbers is 425 and their difference is 75; what are the numbers?

OPERATION.

$425 - 75 = 350$
$\frac{1}{2}$ of $350 = 175$
$175 + 75 = 250$

SOLUTION.—Since the sum of any two numbers is equal to twice the less plus the difference, the sum $425 - 75$, the difference, $= 350$ = twice the less. Hence the less $= \frac{1}{2}$ of $350 = 175$; and the greater $= 175 + 75 = 250$.

2. Divide the number 615 into two such parts that 4 times the less shall be equal to the greater.

OPERATION.

4 times the less = the greater
5 " " = sum = 615
1 " " = 123 = less
4 " 123 = 492 = greater

SOLUTION.—If 4 times the less equal the greater part, the sum of the two parts, 615, equals 5 times the less. Hence the less = $\frac{1}{5}$ of 615 = 123; and 4 times 123 = 492 = the greater.

3. The sum of two numbers is 327, and their difference is 63; what are the numbers? *Ans.* 132, 195.

4. A house and farm are valued at $90500; what is the value of each, if the farm is worth $40300 more than the house? *Ans.* $25100, $65400.

5. Messrs. Haggerty and Brennan have together $100000; but Mr. Haggerty's money is 3 times Mr. Brennan's. How much has each?

6. A and B commenced business together; A gained a certain sum, and B gained 5 times as much. If both their gains amounted to $18000, what was the gain of each? *Ans.* A's, $3000; B's, $15000.

7. The sum of 2 numbers is 1056, and 3 times the first equals 5 times the second; what are the numbers?

OPERATION.

3 times first = 5 times second
once " = $\frac{5}{3}$ " "
$\frac{3}{3} + \frac{5}{3} = \frac{8}{3}$ second = 1056
$\frac{1}{3}$ " = 132
$\frac{3}{3}$ " = 396 = second
$\frac{5}{3}$ " = $\frac{5}{3} \times 396 = 660$ = first

SOLUTION.—If 3 times the first = 5 times the second, the first = $\frac{5}{3}$ of the second. The sum of the two numbers is, therefore, equal to $\frac{5}{3} + \frac{3}{3} = \frac{8}{3}$ of the second = 1056. $\frac{1}{3}$ of the second = $\frac{1}{8}$ of 1056 = 132; and $\frac{3}{3}$, or the second, = 396. And the first = $\frac{5}{3}$ of 396 = 660.

8. The sum of two numbers is 330, and $\frac{1}{4}$ of the less equals $\frac{1}{7}$ of the greater; what are the numbers?

SOLUTION.

$\frac{1}{4}$ of the less = $\frac{1}{7}$ of the greater
$\frac{4}{4}$ " " = $\frac{4}{7}$ " "
$\therefore \frac{7}{7}$, the greater, + $\frac{4}{7}$ of the greater (= the less) = 330 = the sum
$\frac{11}{7}$ of the greater = 330
$\frac{1}{7}$ " " = 30
$\frac{7}{7}$ " " = 210 = the greater
$\frac{4}{7}$ " " = $\frac{4}{7}$ of 210 = 120 = the less

9. The sum of two numbers is 780, and $\frac{1}{2}$ of the less is equal to $\frac{1}{4}$ of the greater; what are the numbers?

Ans. 520, 260.

10. A merchant bought 240 gallons of wine and brandy; how many gallons of each did he purchase, if 10 times the quantity of brandy equals 6 times the quantity of wine? *Ans.* B., 90 gal.; W., 150 gal.

11. Two men own together 810 acres of land, and $\frac{5}{6}$ of the share of one is equal to $\frac{2}{3}$ of the share of the other; how many acres does each own? *Ans.* 360, 450.

12. The united salaries of A and B amount to $4400; what is the salary of each, if $\frac{1}{8}$ of A's equals $\frac{1}{3}$ of B's?

Ans. A, $3200; B, $1200.

13. C's and D's salaries united amount to $1900, and $\frac{1}{3}$ of C's equals $\frac{7}{15}$ of D's; what is the salary of each?

Ans. C's, $700; D's, $1200.

14. Five times the amount which I gained during my first year of business was equal to 9 times the amount I gained during the second: how much did I gain during each year, if both gains amounted to $2800?

Ans. $1800, $1000.

15. Divide $600 into parts which shall be to each other as 3, 5, and 7.

SOLUTION.

It is evident that if the number to be divided were $3 + 5 + 7 =$ \$15, the three parts would be, respectively, \$3, \$5, and \$7. Therefore,

I.

If on \$15 the first part would be \$3
" \$1 " " " $\$\frac{3}{15}$
" \$600 " " " $\frac{3}{15} \times 600 = \$120$

II.

If on \$15 the second part would be \$5
" \$1 " " " $\$\frac{5}{15}$
" \$600 " " " $\frac{5}{15} \times 600 = \$200$

III.

If on \$15 the third part would be \$7
" \$1 " " " $\$\frac{7}{15}$
" \$600 " " " $\frac{7}{15} \times 600 = \$280$

16. Divide \$840 into parts proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{12}$.

SOLUTION.—Since the value of fractions is not changed by reducing them to a common denominator, we may reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{12}$ to $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{7}{12}$. Now, it is evident that these fractions are proportional to the numbers 6, 8, and 7. Dividing \$840 into parts proportional to these last numbers, by the method shown in the last solution, we obtain 240, 320, and 280, the required parts.

17. Divide \$16750 among three persons so that the share of the first shall be to that of the second as 3 to 5, and that the share of the second shall be to that of the third as 4 to 7.

SOLUTION.—If the share of the second is 5 when that of the first is 3, when the second is 1, the first will be $\frac{3}{5}$. In like manner, when the second is 1, the third will be $\frac{7}{4}$. Hence, the parts will be proportional to the numbers $\frac{3}{5}$, 1, and $\frac{7}{4}$; and by the last solution we find the required parts to be \$3000, \$5000, and \$8750.

18. Three workmen together earned $524; the first worked 10 hours a day for 8 days, the second 8 hours a day for 9 days, and the third 11 hours a day for 10 days. How much did each receive?

SOLUTION.—Each man's part is in proportion to the number of hours he worked. Since, therefore, the first worked 80 hours, the second 72 hours, and the third 110 hours, $524 must be divided into parts proportional to these numbers.

19. A man bequeathed his two sons $5800 in the proportion of $\frac{2}{3}$ to $\frac{3}{4}$; what was the share of each?

Ans. $3000, $2800.

20. Three clerks together received $2400; the first received $10 as often as the second received $8; and the second received $4 as often as the third received $3. How much did each receive? *Ans.* $1000, $800, $600.

21. Three merchants sold 620 barrels of flour; how many barrels did each sell, if their amounts are to each other as 6, 10, and 15. *Ans.* 120, 200, and 300 bbl.

22. A, B, and C hire a pasture for $150; A puts in 2 cows for 9 months, B 4 cows for 5 months, and C 6 cows for 2 months. How much should each pay?

Ans. A, $54; B, $60; C, $36.

PARTNERSHIP.

166. ***Partnership*** is the association of two or more persons in business, who unite their capital and services, and share the gains or losses according to a stipulated agreement.

167. ***Partners*** are the persons thus associated.

168. The ***Capital*** of a company or firm is the money or other property invested by the partners.

CASE I.

169. To find each partner's share of the gain or loss when their capital is invested for equal periods of time, and they have agreed to divide the gains or losses in proportion to their respective investments.

1. A and B enter into partnership; A invests a capital of \$500, and B \$200; they gain \$665; what is each one's share of the profit?

OPERATION.

\$500, A's Capital.
200, B's "
\$700, whole stock.

$\frac{500}{700} = \frac{5}{7}$, A's stock.
$\frac{200}{700} = \frac{2}{7}$, B's "
$\$665 \times \frac{5}{7} = \$475$, A's gain.
$\$665 \times \frac{2}{7} = \$190$, B's "

SOLUTION.—The whole capital is the sum of A's and B's investments = \$700. Since A invests \$500, his share of the whole capital is $\frac{500}{700} = \frac{5}{7}$; and B's share is $\frac{2}{7}$. Now, as they have agreed to share the profits in proportion to their investments, each one's share of the gain will have the same relation to the whole gain, as his share of the capital has to the whole capital. A, therefore, will have $\frac{5}{7}$ of the whole gain, and B $\frac{2}{7}$.

SECOND METHOD.

\$700 : \$500 :: \$665 : \$475, A's gain.
\$700 : \$200 :: \$665 : \$190, B's gain.

RULE.—*Find the ratio of the whole capital to each partner's share of the capital, and multiply it by the whole gain or loss.*

Or, *The whole capital is to each one's share of the capital as the whole gain or loss is to each one's share of the gain or loss.*

WRITTEN EXERCISES.

2. Three merchants engage in trade. A advances $10000, B $25000, and C $30000. They gain $31200. What is each man's share?

Ans. A's, $4800; B's, $12000; C's, $14400.

3. Three men ship goods on joint stock. A puts in $6000, B $9000, and C $13000. What will be each man's share, if the whole gain is $8400?

Ans. A's, $1800; B's, $2700; C's, $3900.

4. Messrs. Lynch and Kelly, in trading for 6 months, made a profit of $9100. Mr. Kelly's stock was equal to $\frac{5}{8}$ of Mr. Lynch's. What was the gain of each?

Ans. Kelly's, $3500; Lynch's, $5600.

5. A, B, and C commence business with a joint capital of $66400. At closing business A has $42000, B $26250, and C $15750. What was each one's original stock?

Ans. A's, $33200; B's, $20750; C's, $12450.

6. A, B, and C enter into partnership in the lumber business for 10 years, and make a profit of $10400; A contributes $2150; C contributes $6350; and B's share of the gain is $3600. How much does B contribute, and what is A's and C's gain?

Ans. A's gain, $1720; C's gain, $5080; B's stock, $4500.

7. Four men engaged in speculation with a joint capital of $150000. At the end of one year, A's gain was $14000; B's, $16000; C's, $20000; D's, $25000. What was the stock of each?

Ans. A's, $28000; B's, $32000; C's, $40000; D's, $50000.

8. A, B, and C traded in company. A agreed to manage the business on condition that he should receive

$\frac{3}{7}$ as much of the profits as B and C together. B furnished $5600, and C $8400. They gained $7000. What was each one's share of the profits?

Ans. A's, $2100; B's, $1960; C's, $2940.

9. A gentleman bequeathed his wife $5400, his son $3600, and his daughter $3200. But his estate was found to be only $6100. How much should each receive?

Ans. Wife, $2700; son, $1800; daughter, $1600.

10. A general, captain, and 30 soldiers capture stores to the amount of $3150. The general takes 9 shares, the captain 6, and each soldier 1. How much does each receive?

Ans. General, $630; captain, $420; each soldier, $70.

11. A bank with available means amounting to $140000, failed for $300000. What will each of the creditors receive, A claiming $45900; B $22950; and C $12840?

Ans. A, $21420; B, $10710; C, $5992.

12. Two persons engaged in the coal trade. A made purchases to the amount of $16570, and paid $596.25 for expenses. B made purchases to the amount of $12776, and paid $957 for expenses. After one year A sold the entire stock for $21600. How did the partners settle?

Ans. A's loss, $5166.25; B's, $4133.

13. Messrs. Gould and Brown buy grain to the amount of $923.75, of which Mr. Gould pays $369.50, and Mr. Brown the remainder. They gain $1125. What is each one's share? *Ans.* Gould's, $450; Brown's, $675.

14. Mr. Wilson left $48000 to four heirs in the proportion of $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{16}$ to the first three respectively, and the remainder to the fourth. How much did the fourth receive? *Ans.* $3000.

15. A, B, and C purchased a store for $2500, of which A paid $1800. They sold it so as to gain a certain sum,

of which B took \$340.25, and C \$478.75. What was A's share of the gain? *Ans.* \$2106.

16. A gentleman divides his fortune worth \$38664, between his wife and son, giving his wife $\frac{1}{4}$ more than the son. What is the son's share? *Ans.* \$15036.

17. A, B, and C contribute to an undertaking, respectively, \$1240, \$6200, and \$3720; they gain \$5580; how shall this gain be divided among them?

CASE II.

170. To find each partner's share of the gain or loss when their capital is invested for different periods of time and they agree to share the gain or loss in proportion to their respective investments.

1. Two persons engage in trade and gain \$660. A puts in \$400 for 3 mo.; and B \$500 for 2 mo.; what is each man's share of the gain?

OPERATION.

\$400 × 3 = \$1200, A's capital.
\$500 × 2 = \$1000, B's "
\$2200, Entire "

$\frac{1200}{2200} = \frac{6}{11}$ = A's share.
$\frac{1000}{2200} = \frac{5}{11}$ = B's "
\$660 × $\frac{6}{11}$ = \$360, A's gain.
\$660 × $\frac{5}{11}$ = \$300, B's "

SOLUTION.—The use of \$400 for 3 months is the same as the use of \$1200 for 1 month; and \$500 for 2 months is equivalent to \$1000 for 1 month. Thus the original investments for unequal periods of time, are made equivalent to other investments for equal periods of time. The rest of the solution is the same as in Case I.

RULE.—*Multiply each partner's investment by the time it was employed, and divide the gain or loss in proportion to the respective products.*

WRITTEN EXERCISES.

2. A and B enter into partnership and loso $340. A puts in $300 for 6 mo., and B $200 for 8 mo.; what does each one lose? *Ans.* A, $180; B, $160.

3. Three gentlemen invested their joint stock in flour. A furnished $800 for 6 mo.; B $1000 for 4 mo.; and C $500 for 9 mo. They gained $2660. What was each man's share? *Ans.* A's, $960; B's, $800; C's, $900.

4. A, B, C, and D traded in company. A furnished $200 for 6 mo.; B $800 for 2 mo.; C $500 for 3 mo.; and D $900 for 1 mo. They gained $3120. What was each partner's share?

Ans. A's, $720; B's, $960; C's, $900; D's, $540.

5. A and B engaged in business with a capital of $52560. A's capital was in trade 7 years, and his share of the gain was $66465; B's capital was in 8 years, and his share of the gain was $50184. What amount of capital had each in the firm?

Ans. A, $31650; B, $20910.

6. Two persons agree to trade together for 21 months. At first A puts in $1200, and B $1600. But at the end of 3 months A draws out $200, and at the end of 12 months B invests $600 more. When about to settle they find a gain of $1212. What is each man's share of the gain? *Ans.* A's, $432; B's, $780.

7. J. Liefel and H. Kenny agree to work a farm for $\frac{1}{4}$ of the proceeds. Liefel engages 10 men for 5 months, and Kenny 12 men for 6 months. If the proceeds amount to $6832, what will each partner receive?

Ans. Liefel, $700; Kenny, $1008.

8. C and D enter into partnership for one year. C's investments are: Jan. 1, $3000; May 1, $1200; Sept. 1,

$1800; on July 1, he draws out $600. D invests as follows: Jan. 1, $5000; March 1, $1500; June 1, $1000; and withdraws, Oct. 1, $2000. How much should C invest Nov. 1, to entitle him to $\frac{1}{2}$ the gain?

Ans. $13400.

9. V. Diner and T. Eckels were partners in a cotton speculation. Diner put in $7000 for 2 years and 3 months; and Eckels put in $4000 for 3 years and 6 months. Their net losses were $8925. What was each one's share?

Ans. Diner's, $4725; Eckels's, $4200.

10. A, B, and C commenced business with a joint capital of $1600. A's money was invested 4 months, B's 6 months, and C's 8 months. They gained $688, which was so divided that $\frac{1}{6}$ of C's share was equal to $\frac{2}{15}$ of B's and to $\frac{1}{8}$ of A's. How much did each invest and what did each gain?

Ans. { Capital: A, $800; B, $500; C, $300.
Gain: A, $256; B, $240; C, $192.

11. Three contractors agree to build a mansion for $200000. A supplies 160 men for 25 months; B 140 men for 20 months; and C 40 men for 30 months. How much does each contractor receive?

Ans. A, $100000; B, $70000; C, $30000.

12. A and B become partners. A's capital is to B's as 5 to 3. At the end of 4 months A withdraws $\frac{1}{2}$ of his capital, and B $\frac{2}{3}$ of his. The gain for the year amounts to $6000. How much belongs to each?

Ans. A, $4000; B, $2000.

13. A commenced business with a capital of $4250. 3 months after he took in B with a capital of $2000, 4 months after which he took in C with a capital of $5000. At the end of the year the loss was $4700. What was the share of each? *Ans.* A's, $2550; B's, $900; C's, $1250.

14. A invests \$600 for 2 mo., and B \$100 for 6 mo.; they gain \$900; required the gain of each.

15. A, B, and C form a partnership in manufacturing silk. A contributes \$3600 for 7 months, B a certain sum for 4 months, and C \$5400 for a certain time. On settling accounts, A receives \$16200 for his share, B \$13500, and C \$13500. What was B's stock? C's time?

Ans. \$4500; 3 months.

16. Three men take an interest in a cotton speculation. A invests his capital for 6 months and claims $\frac{1}{6}$ of the gains. B's capital is in 10 months; and C invests \$400 for 8 months, and claims $\frac{5}{9}$ of the gains. How much did A and B put in? *Ans.* \$160 each.

17. A company doubled their joint stock of \$4800, at the end of the first year. B put in $\frac{3}{8}$ for $\frac{1}{3}$ of a year, C $\frac{1}{4}$ for $\frac{3}{4}$ of a year, and D the remainder for one year. How much is each one's share of the entire stock at the end of the year?

Ans. B's, \$4666.05; C's, \$2874.42; D's, \$2059.53.

Case III.

171. When each partner's share of the gains or losses is fixed, and interest is allowed for the difference between his proportion of capital and the amount he actually contributes.

1. A and B enter into partnership. A contributes \$3000 and is to receive $\frac{3}{5}$ of the gains. B contributes \$2500 and is to receive $\frac{2}{5}$ of the gains. Each partner is to receive or pay interest at 6% per annum, for any excess or deficit in his proportionate share of the capital. At the end of 6 months the profits are \$1500. What is each partner's share?

OPERATION.

$\$3000 + \$2500 = \$5500$, entire capital.
$\frac{3}{5}$ of $\$5500 = \$3300$, A's prop. share of capital.
$\frac{2}{5}$ of $\$5500 = \$2200$, B's " "
$\$3300 - \$3000 = \$300$, A's deficit.
$\$2500 - \$2200 = \$300$, B's excess.
$\$300 \times .03 = \$9.00$, Int. for 6 mo. @ 6%.
$\frac{3}{5}$ of $\$1500 = \$900$, $- \$9 = \$891$, A's gain.
$\frac{2}{5}$ of $\$1500 = \$600$, $+ \$9 = \$609$, B's "

SOLUTION.—The entire capital is $5500. A should have furnished $\frac{3}{5}$ of this capital, or $3300. His deficit, therefore, is $300, on which he must pay $9 interest. B should have contributed $\frac{2}{5}$, or $2200. His excess is, therefore, $300; on which he receives $9 interest. A gains $900 − $9 = $891. B gains $600 + $9 = $609.

RULE.—*I. From the proportional gains or losses subtract or add the interest on the excess or deficit of each partner's share of capital.*

II. Subtract the interest on the additions from the gross profits, but add that on the withdrawals.

III. To find the present value of each share, add to each partner's original stock the net gain and the additions, but subtract the withdrawals.

WRITTEN EXERCISES.

2. Two men embarked as partners in the printing business. A furnished $12000 and was to have $\frac{3}{4}$ of the profits. B furnished $8000 and was to have $\frac{1}{4}$ of the profits. Each partner was to receive or pay interest at 6% per annum on any excess or deficit in his share of the capital. At the end of two years they closed with a profit of $16000. What was the net worth of each?

Ans. A's, $23640; B's, $12360.

3. In No. 2, what would have been the net worth of each, had they closed with a profit of $24000 ?

4. Messrs. Banks and Drake formed a partnership in the wholesale hardware business. Banks invested $28000 and was to receive $\frac{5}{8}$ of the profits. Drake invested $20000 and was to receive $\frac{3}{8}$ of the profits. They agreed to receive or pay interest at 6% for their excess or deficit in capital. Their profits in three years amounted to $27600. What was each one's share ?

Ans. Banks's, $16890 ; Drake's, $10710.

5. The fourth year Banks adds $3000 capital, averaging July 1 ; and Drake draws out $1000, averaging Sept. 1. Profits $14400. Find the net worth of each.

Ans. Banks, $56794.65 ; Drake, $35205.35.

ALLIGATION.

172. ***Alligation*** treats of combining two or more quantities of different values. It is divided into *Alligation Medial* and *Alligation Alternate.*

173. ***Alligation Medial*** is the process of finding the average price or quality of the combination.

174. ***Alligation Alternate*** is the process of finding the proportional quantities to be used in any required combination.

CASE I.

175. To find the mean value, when the quantity and the value of each ingredient are given.

1. A grocer mixed 8 lb. of tea at 50 cents a pound, with 4 lb. at $.60, 1 lb. at $1.30, and 3 lb. at $.90. How much is 1 lb. of the mixture worth ?

OPERATION.

$$
\begin{array}{rl}
\$.50 \times 8 = & \$4.00 \\
.60 \times 4 = & 2.40 \\
1.30 \times 1 = & 1.30 \\
.90 \times 3 = & 2.70 \\
\hline
16\,) & \$10.40 \\
\hline
 & \$.65
\end{array}
$$

SOLUTION.—The value of 8 lb. at .50 is $4.00, of 4 lb. at .60 is $2.40, of 1 lb. at $1.30 is $1.30, and of 3 lb. at .90 is $2.70. The whole compound, therefore, is equal to $8 + 4 + 1 + 3 = 16$ lb., whose value is $\$4 + 2.40 + 1.30 + 2.70 = \$10.40$. Hence 1 lb. of the compound is worth $\$10.40 \div 16 = \$.65$.

RULE.—*Find the whole value of the ingredients and divide it by the sum of the ingredients.*

WRITTEN EXERCISES.

2. A merchant mixed 30 lb. of sugar at 12 cents a pound, 25 lb. at 10 cents, 40 lb. at 20 cents, and 10 lb. at 15 cents. What is the average price of the mixture?

Ans. $14\frac{8}{17}$¢ per lb.

3. A person mixed 20 gal. of wine at $.60, 34 gal. at $.70, 50 gal. at $2.40, with 10 gal. of water; what was the value of one gallon of the mixture? *Ans.* 1.36\frac{2}{3}$.

4. A grocer bought $10\frac{1}{2}$ dozen of eggs at 12 cents a dozen, 8 dozen at 8 cents, and 15 dozen at $9\frac{2}{3}$ cents. What was the average price per dozen? *Ans.* 10¢.

5. If the eggs mentioned above were sold so as to make 50% on the cost, how much would the grocer receive per dozen? *Ans.* 15¢.

6. A goldsmith combined 10 oz. of gold, 18 carats fine, 12 oz. 20 carats fine, 15 oz. 22 carats fine, with 13 oz. of alloy. What was the fineness of the composition?

Ans. 15 carats.

7. A drover bought 25 sheep at $3.00 a head, 40 at $2.50, and 50 at 1.67\frac{1}{2}$. At what average price per head must he sell them, to gain 10%? *Ans.* 2.47\frac{1}{2}$.

8. A miller mixes 20 bu. of wheat at $2.25 per bu.,

with 8 bu. at \$1.63¼, and 9 bu. at \$2.10. What will he lose per bushel, if he sell the mixture at \$2.00?

Ans. 8¢.

9. A grocer mixed 20 lb. of tea at 90 cts. per lb., with 12 lb. at 75 cts., and 8 lb. at \$1.00. Will he gain or lose, and how much per pound, if he sell the mixture at 80 cts.? *Ans.* Lose 7½ cts.

Case II.

176. To find the proportional quantity of each ingredient, when the mean value, and the value of each are given.

1. What proportion of sugars worth, respectively, 5, 8, 12, and 15 cents a pound, must be taken to form a mixture worth 9 cents a pound?

	OPERATION.			PROOF.	
¢	¢ lb.	¢	¢	¢ lb.	¢
9 { 5	4 × (1 + 1)	= 8	9 Gain.	5 × 2	= 10
8	1 × 1	= 1		8 × 1	= 8
12	−3 × 1	= 3	9 Loss.	12 × 1	= 12
15	−6 × 1	= 6		15 × 1	= 15
	5			5)	45¢
					9¢ Ave.

Solution.—By taking 1 lb. of sugar at 5¢ for the mixture worth 9¢, we gain 4¢; and by taking 1 lb. at 8¢, we gain 1¢.

But by taking 1 lb. at 12¢, we lose 3¢; and by taking 1 lb. at 15¢, we lose 6¢.

By using 1 lb. of each ingredient, the loss is 4¢ greater than the gain. To preserve the equality of the gain and loss, take the least additional number of pounds of either ingredient below the average price, and sufficient to gain 4¢. Thus 1 lb. of the 1st ingredient will make the gain and loss equal. Hence the mixture will contain 2 lb. at 5¢, 1 lb. at 8¢, 1 lb. at 12¢, and 1 lb. at 15¢.

RULE.—*I. Write the price or qualities of the several ingredients in a column, and the mean price or quality at the left.*

II. Find the gain or loss on one unit of each ingredient; and then take such additional units of any as will make the gains and losses equal.

WRITTEN EXERCISES.

2. What relative quantitives of tea worth 9, 11, 16, and 20 dimes a pound, must be taken to form a mixture worth 13 dimes a pound? *Ans.* 2, 1, 1, 1.

3. Having a quantity of 2, 5, 25, and 50 cent pieces, I wish to exchange them for 10 cent pieces. What is the relative number of pieces to be exchanged?

Ans. 5, 3, 1, 1.

4. What proportions of water, and wine worth $1.50 and $3.50 a gallon, must be used to form a mixture worth $2.00 a gallon? *Ans.* 2 water; 1, 3 wine.

5. What relative quantities of gold $\frac{2}{3}$ fine, $\frac{7}{8}$ fine, and $\frac{3}{10}$ fine, will make a mixture $\frac{3}{4}$ fine? *Ans.* 1, 10, 2.

6. A merchant has flour worth 6\frac{1}{2}$, $8, and 9\frac{3}{4}$ per barrel. How many barrels of each must he sell to realize an average price of $9 per barrel? *Ans.* 2, 1, 8.

7. What quantities of cloth at 12, 16, 22, and 24¢ a yard, must be sold that the price may average 20 cents?

Ans. 1, 1, 2, 2.

8. In what proportion should corn at $.70 per bushel, be mixed with oats at $.50 per bushel, to sell the mixture at $.60 per bushel? *Ans.* 1, 1.

9. What quantities of coffee worth, respectively, 25, 30, 42, and 50 cents, must be mixed to sell at 40 cents *per* pound? *Ans.* 2, 1, 15, 1.

CASE III.

177. To find the proportional quantity of each ingredient, when one or more of the quantities are required to be in a given proportion.

1. A drover bought 3 calves at \$7 each, sheep at \$6, and pigs at \$4. How many sheep and pigs must he sell, that the average price may be \$5 each?

OPERATION.

	\$	\$ A	\$	
5	7	2×3	$= 6$	\$7 Loss.
	6	1×1	$= 1$	
	4	$-1 \times (1+6)$	$= 7$	\$7 Gain.

PROOF.

$$\begin{array}{rl} \$4 \times 7 = & \$28 \\ 6 \times 1 = & 6 \\ 7 \times 3 = & 21 \\ \hline 11 \;) & \$55 \\ \hline & \$5 \end{array}$$

SOLUTION.—As in Case II., we find the gain and loss on 1 of each. But as the number of calves is limited, we must find the gain on 3 calves. Then we observe that by taking 3 calves and 1 of each of the others, the losses exceed the gains by \$6. Hence by taking an additional number of pigs, we preserve the balance and get for the required number of animals 3 calves, 1 sheep, and 7 pigs.

RULE.—*Having found the difference between the gains and losses, when using the given number of the limited quantity with one of each of the others, take such additional units as are necessary to preserve the balance.*

WRITTEN EXERCISES.

2. A man bought 30 ducks at 8 dimes each. How many more must he buy at 4 and 10 dimes each, so that the average price may be 6 dimes?

Ans. 32 at 4; 1 at 10.

3. How many yards of cloth worth 35 cents a yard, must be added to a piece of 75 yards, worth 50 cents a yard, that the average value may be 40 cents?

Ans. 150 yd.

4. A man bought 6 sheep for $30. How many must he buy at $7½, and $3, that the average price may be $4 a head? *Ans.* 2 at $7½; 13 at $3.

5. What proportions of sugars at 9 cts., 11 cts., 15 cts., and 18 cts. are necessary to form a compound worth 10 cts. per pound, using equal parts of the last three kinds?

Ans. 14, 1, 1, 1.

6. I bought 16 yards of cloth at $¾ a yard, and 12 yards of another kind for $15. How many yards must I buy at $1¾ per yard, that the average price of the whole may be $1⅛? *Ans.* 12 yd.

7. How much wheat at $1.25 a bushel, must be mixed with 80 bushels of corn at 60 cents a bushel, that the mixture may be worth $1.00 a bushel? *Ans.* 128 bu.

8. How many pounds of tea at 89¢, 62¢, and 50¢ must be mixed with 20 lb. at 80¢, that the mixture may be worth 72¢ per pound?

9. What quantities of sugars at 6¢, 8¢, and 9¢, must be mixed with 29 lb. at 4¢, that the average price may be 5¢ per pound?

Case IV.

178. To find the quantity of each ingredient when the value of each, the mean value, and the entire quantity are given.

1. A grocer mixed teas worth 20¢, 25¢, 45¢, and 50¢ a pound, making a compound of 120 lb. worth 40¢ a pound. *Find the* quantity of each kind.

OPERATION.

	¢	¢	lb.	¢	
40	20	20 ×	1 =	20	$.35 G.
	25	15 ×	1 =	15	
	45	− 5 ×	1 =	5	$.35 L.
	50	− 10 ×	(1 + 2) =	30	
			6		

$$120 \div 6 = 20$$
$$1 \times 20 = 20 \text{ lb. @ \$.20}$$
$$1 \times 20 = 20 \text{ " " \$.25}$$
$$1 \times 20 = 20 \text{ " " \$.45}$$
$$3 \times 20 = 60 \text{ " " \$.50}$$

SOLUTION.—By the last cases, we find that to make a mixture of 6 pounds, it requires 1 pound of each of the first three kinds, and 3 pounds of the last kind. Hence, to make a mixture of 120 pounds, it will take as many times these amounts as 6 is contained times in 120, which are 20 times. Multiplying by 20, we obtain 20 lb. at $.20, 20 lb. at $.25, 20 lb. at $.45, and 60 lb. at $.50.

RULE.—*Having found the sum of the proportional quantities by Case III., divide it into the required quantity, and multiply each proportional quantity by the quotient.*

WRITTEN EXERCISES.

2. How much water must be mixed with wine at $1.20 a gallon, to make 90 gallons worth 70 cents a gallon?

Ans. 37½ gal. water, 52½ gal. wine.

3. A hatter sold 30 hats comprising three different qualities worth, respectively, $5, $4, and $7 each. If the average price was $6, how many of each quality did he sell? *Ans.* 6, 6, 18.

4. How much wool at 40, 60, and 75 cents a pound, will be required to make a mixture of 85 pounds worth 65 cents a pound?

Ans. 17 lb. at 40; 17 lb. at 60; 51 lb. at 75.

5. How many gallons of alcohol of 95% and 80% strong, should be mixed with water to form a mixture of 90 gallons 85% strong?

Ans. 78 gal. at 95%; 3 gal. at 80%; 9 gal. water.

6. A druggist made a mixture of syrups of 48 gallons, at \$.50, \$.70, and \$1.00 a gallon, and sold it at \$.80 a gallon. What quantity of each did he sell?

7. A drover purchased 200 sheep for \$450, paying \$4$\frac{1}{6}$, \$7$\frac{1}{3}$, and \$1$\frac{1}{2}$ a head. How many of each kind did he buy? *Ans.* 30, 12, 158.

8. A woman sold 100 eggs for \$2, some at $\frac{1}{2}$¢, some at 1¢, some at 2$\frac{1}{2}$¢, and some at 4¢ apiece. How many did she sell of each kind? *Ans.* 25 of each.

9. How shall I mix gold that is 20 carats fine with gold 14 carats and 18 carats fine, respectively, to obtain 15 ounces of a mixture 16 carats fine? *Ans.* 3, 9, 3.

10. How much water must be mixed with cider worth 40 cents a gallon, in order that 80 gallons of the mixture shall be worth 30 cents a gallon?

Ans. 20 gal. water with 60 gal. cider.

11. How many pounds of raisins at 11, 15, and 22 cents a pound, respectively, are required to fill a cask containing 200 pounds, worth 16 cents a pound?

Ans. 66$\frac{2}{3}$ lb. of each.

12. A goldsmith has gold 15, 17, 18, and 22 carats fine, and wishes to make 40 ounces 20 carats fine; how much of each must he take? *Ans.* 5, 5, 5, 25.

13. A farmer has 25 bushels of barley worth 28 cents a bushel, which he wishes to mix with rye worth 36 cents a bushel, and oats worth 48 cents a bushel, so that he may have 100 bushels worth 40 cents a bushel; how many bushels of rye and oats must he take?

Ans. 50 bu. oats, 25 bu. rye.

INVOLUTION.

179. ***Involution*** is the process of raising a number to a given power.

180. A ***Power*** of a number is either the number itself, or the product arising from using the number a certain number of times as a factor.

The *first power* of a number is the number itself. Thus, the first power of 6, is 6.

The *second power*, or the *square* of a number, is the product arising from using the number twice as a factor. Thus, the second power of 4 is equal to $4 \times 4 = 16$.

The *third power*, or the *cube* of a number, is the product arising from using the number three times as a factor. Thus, the third power of 5 is equal to $5 \times 5 \times 5 = 125$.

NOTE.—The number of multiplications is one less than the number of factors.

181. A ***Perfect Power*** is a number which is the product of equal factors. Thus, the number 9 is a perfect power, because it is the product of the equal factors, 3 and 3.

182. An ***Imperfect Power*** is a number which is not the product of equal factors. Thus, the numbers, 10, 11, and 13, are examples of imperfect powers.

183. The ***Exponent*** of a number is a small figure placed above and to the right of the number, to show how many times it is taken as a factor; thus,

$3^1 =$ the first power of 3, or the root.

$3^2 = 3 \times 3 = 9$, the second power, or square of 3,

$3^3 = 3 \times 3 \times 3 = 27$, the third power, or cube of 3.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power of 3.

$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, the second power of $\frac{3}{4}$.

$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$, the third power of $\frac{1}{2}$.

NOTE.—As the first power of a number is the number itself, the exponent indicating the first power is generally understood.

PRINCIPLES.—*I. A number is raised to a given power by taking it as a factor as many times as there are units in its exponent. Thus,* $2^3 = 2 \times 2 \times 2 = 8$.

II. The product of any two or more powers of the same number, is equal to the power indicated by the sum of their exponents. Thus, $7^2 \times 7^3 = (7 \times 7) \times (7 \times 7 \times 7) = 7^{2+3} = 7^5$.

III. Any power of a number is raised to a given power by finding the power indicated by the product of the exponents. Thus, $(9^2)^3 = (9 \times 9) \times (9 \times 9) \times (9 \times 9) = 9^{2 \times 3} = 9^6$.

IV. A power of a number may be divided by another power of the same number, by subtracting the exponent of the divisor from the exponent of the dividend. Thus, $8^5 \div 8^3 = (8 \times 8 \times 8 \times 8 \times 8) \div (8 \times 8 \times 8) = 8^{5-3} = 8^2$.

MENTAL EXERCISES.

1. What is the third power of 4?

OPERATION.—$4^3 = 4 \times 4 \times 4 = 64$.

2. What is the third power of 3? Of 2? Of 5?

3. Raise 10 to the third power. 9 to the second power.

4. Find the fifth power of 1. The second power of 12. The third power of 6.

5. Square all the numbers from 1 to 12.

6. Find the cube of all the numbers from 1 to 12.

7. What is the square of $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$?

8. Square $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{3}{7}$, $\frac{5}{8}$, $1\frac{1}{2}$, $2\frac{1}{3}$.

9. Cube $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{4}{9}$, $2\frac{1}{2}$.

10. Square .4, .7, .9, .03, .05, .12, .08, .07, .06.

11. Cube .2, .04, .30, .1, .10, .20, .5.

12. How often is 3 used as a factor in finding the fifth power of 3? 8 in finding the second power of 8?

13. The cube of 6 equals 6 used how often as a factor? The fifth power of 6? The seventh power?

14. What power of 5 is 5^3 multiplied by 5^2? What power of 7 is 7^4 multiplied by 7^3?

15. What power of 12 is equal to $12^2 \times 12^3 \times 12^4$? What power of 4 is equal to $4^5 \times 4^2$?

16. What power of a number is the square of its square? The cube of its cube?

17. What power of 3 is the square of its cube? The cube of its square?

18. What power of 6 is equal to 6^5 divided by 6^2? What power of 8, to 8^4 divided by 8^3?

19. What number multiplied by 14 gives $\frac{7}{8}$ of the square of the number? By 10, gives $\frac{2}{5}$ of its cube?

20. What power of 4 is the cube of the cube of 4^2?

WRITTEN EXERCISES.

1. Find the fifth power of 4.

OPERATION.

$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024.$

Or,

$4^5 = 4^2 \times 4^3$

$4^2 = 16$

$4^3 = 64$

$4^5 = 16 \times 64 = 1024.$

SOLUTION.—Multiplying the number, 4, four times by itself, we get $4 \times 4 \times 4 \times 4 \times 4 = 1024$. Or, having first formed the second and the third power of 4, separately, we take their product for the fifth power.

2. What is the 8th power of 3? The 9th power of 2?
3. What is the 15th power of 4? The 10th power of $\frac{2}{3}$?
4. What is the square of 47? Of 56? Of 100?
5. What is the cube of 32? Of 26? Of 65? Of 75?
6. What is the square of 63.05? Of 10.21?
7. What is the square of $6\frac{1}{4}$? Of $9\frac{1}{2}$? Of $12\frac{1}{5}$?

Find the value of each of the following expressions:

8. $(12^2)^3$.
9. $(\frac{5}{9})^4$.
10. $(1\frac{3}{5})^2$.
11. $4.5^2 \times 6^3$.
12. $(\frac{3}{4})^2 \times \frac{4}{5} \times (10^2)^2$.
13. $(3\frac{1}{3})^5 \div (3\frac{1}{3})^3$.
14. $(8.04)^3 \div (8.04)^2$.
15. $[(2\frac{1}{4})^2 \times 6^3]^3 \div [6^3 \times (2\frac{1}{4})^2]^6$.

Second Method.

What is the square of 42?

OPERATION.

$$
\begin{array}{rcl}
a & & b \\
42 & = & 40 + 2 \\
42 & = & 40 + 2 \\
\hline
84 & = & (40 \times 2) + (2 \times 2) \\
168 & = & (40 \times 40) + (40 \times 2) \\
\hline
1764 & = & (40 \times 40) + 2\,(40 \times 2) + (2 \times 2)
\end{array}
$$

Solution.—The number 42 consists of 40 units + 2 units, or 4 tens + 2 units. Multiplying each part separately, the results under a = those under b. Or, by substituting letters, we get the result $t^2 + 2tu + u^2$. Hence,

The square of a number expressed by two figures, equals the square of the tens, plus twice the product of the tens into the units, plus the square of the units.

In like manner find the square of each of the following numbers:

1. 24; 35; 52; 81; 93; 19; 64; 75; 27; 83; 72; 91.
2. 15; 26; 39; 47; 86; 58; 95; 66; 53; 38; 65; 49.

What is the cube of 25 ?

OPERATION.

$$
\begin{array}{rrl}
25^2 = & 20^2+2\,(20\times 5)+5^2 = & 625 \\
 & 20+5 = & 25 \\
\hline
 & (20^2\times 5)+2\,(20\times 5^2)+5^3 = & 3125 \\
 & 20^3+2\,(20^2\times 5)+\ \ (20\times 5^2) = & 1250 \\
\hline
25^3 = & 20^3+3\,(20^2\times 5)+3\,(20\times 5^2)+5^3 = & 15625
\end{array}
$$

Hence, the cube of a number expressed by two figures, equals the cube of the tens, plus three times the product of the square of the tens into the units, plus three times the product of the tens into the square of the units, plus the cube of the units.

In like manner find the cube of each of the following numbers :

1. 32 ; 16 ; 24 ; 31 ; 44 ; 56 ; 72 ; 82 ; 54 ; 39 ; 26.
2. 15 ; 29 ; 37 ; 68 ; 94 ; 59 ; 45 ; 63 ; 54 ; 35 ; 47.

When the number consists of three or more figures, we complete the square or cube, according to the following formulas :

$$(h+t+u)^2 = h^2+2ht+t^2+2\,(h+t)u+u^2.$$
$$(h+t+u)^3 = h^3+3h^2t+3ht^3+t^3+3\,(h+t)^2u+ 3\,(h+t)\,u^2+u^3.$$

WRITTEN EXERCISES.

Raise to the second power :

1. 127.	*Ans.* 16129.		*5.* 2150.	*Ans.* 4622500.
2. 209.	*Ans.* 43681.		*6.* 7863.	*Ans.* 61826769.
3. 518.	*Ans.* 268324.		*7.* 1.937.	*Ans.* 3.751969.
4. 870.	*Ans.* 756900.		*8.* 40.95.	*Ans.* 1676.9025.

Raise to the third power:

9. 100.	*Ans.* 1000000.	*13.* 1000.	*Ans.* 1000000000.
10. 999.	*Ans.* 997002999.	*14.* 2.002.	*Ans.* 8.024024008.
11. 6.05.	*Ans.* 221.445125.	*15.* 1.001.	*Ans.* 1.003003001.
12. .004.	*Ans.* .000000064.	*16.* 4.4.	*Ans.* 85.184.

Raise to the powers indicated:

17. $.015^2$.	*Ans.* .000225.	*20.* $(8.25\frac{1}{2})^2$.	*Ans.* $68.1450\frac{1}{4}$.
18. 2.61^2.	*Ans.* 6.8121.	*21.* $(.33\frac{1}{3})^2$.	*Ans.* $.1111\frac{1}{9}$.
19. 210^3.	*Ans.* 9261000.	*22.* $(1.06\frac{2}{3})^2$.	*Ans.* $1.1377\frac{7}{9}$.

EVOLUTION.

184. ***Evolution*** is the process of finding the root of a number.

185. The ***Root*** of a number is one of the equal factors that will produce it. Thus, 4 is the root, or one of the two equal factors which will produce the number 16.

186. The symbol, $\sqrt{\ }$, called the *Radical Sign,* when placed before a number, indicates that its root is to be taken. To denote the degree of the root, a small figure, called the *index,* is written in the angle of the symbol. Thus, $\sqrt[3]{8}$ indicates the cube root of 8.

NOTE.—The symbol without any *index,* always denotes the square root.

187. Roots may also be expressed by fractional exponents; as $6^{\frac{1}{3}} = \sqrt[3]{6}$, and signifies the cube root of the first power of 6; $6^{\frac{2}{3}} = \sqrt[3]{6^2}$, and signifies the cube root of the

second power of 6. The numerator denotes the power to which the quantity is to be raised; and the denominator, the root that is then to be taken.

188. As any number whatever may be considered as a power whose root is to be extracted, it follows that there are two kinds of roots, the *Rational* and the *Surd.*

189. A ***Rational*** or exact root, is the root of a perfect power; as, 2 is the rational root of 4.

190. A ***Surd*** is the indicated root of an imperfect power; as, $\sqrt{5}$ denotes the square root of the imperfect power 5.

191. The *roots* of numbers are named from their corresponding powers, as follows:

The square root of 4 is 2, expressed $\sqrt{4} = 2$.

The cube root of 8 is 2, expressed $\sqrt[3]{8} = 2$.

The fourth root of 81 is 3, expressed $\sqrt[4]{81} = 3$.

The fifth root of 32 is 2, expressed $\sqrt[5]{32} = 2$.

SQUARE ROOT.

192. The ***Square Root*** of a number is one of its two equal factors; as, 9 is the square root of 81.

To assist the pupil in finding the number of places in the square root of any number, we give the following

TABLE.

$1^2 = 1$	1 Period.	$10^2 = 1{,}00$	2 Periods.
$9^2 = 81$		$99^2 = 98{,}01$	

PRINCIPLES.—*I. The square of any number contains twice as many figures, or twice as many less one, as the number itself.*

II. The square root of any number contains as many terms as there are periods of two figures each, in the number.

MENTAL EXERCISES.

1. What is the square root of 36 ?

SOLUTION.—To extract the square root of 36, we resolve it into its two equal factors, 6 and 6. Hence, the square root of 36 is 6.

2. What is the square root of 4 ? Of 9 ? Of 16 ? 25? 49 ? 64 ? 81 ? 100 ? 121 ? 144 ?

3. Find the square root of $\frac{1}{4}$. $\frac{1}{9}$. $\frac{9}{16}$. $\frac{16}{25}$. $\frac{49}{81}$. $\frac{25}{64}$.

4. What is the square root of .04 ? Of .25 ? 1.21 ? .0025 ? 1.44 ? .1600 ?

5. What is the square root of $2\frac{1}{4}$? Of $2\frac{7}{9}$? $6\frac{1}{4}$? $7\frac{1}{9}$?

WRITTEN EXERCISES.

1. What is the square root of 7056 ?

OPERATION.

$$\begin{array}{rcr|l} t^2 + 2tu + u^2 & = & 7056 & 80 \\ t^2 = 80^2 & = & 6400 & 4 \\ \hline 2tu + u^2 & = & 656 & 84\ \textit{Ans.} \\ 2tu = (160 \times 4) & = & 640 & \\ \hline u^2 & = & 16 & \\ u^2 & = & 16 & \\ \hline \end{array}$$

SOLUTION. — As the number contains two periods, the root has two figures, tens and units.

The largest square in 70 hundreds, is 64 hundreds, the root of which is 8 tens, the first figure of the required root.

Squaring the 8 tens and subtracting, we get a remainder of 656, which contains twice the product of the tens by the units, plus the square of the units.

Now, twice 8 tens are 16 tens, or 160; and 160 are contained four times in 656. The figure 4, therefore, is probably the unit figure of the root.

Four times 160 are 640, which taken from 656, leave 16; and this remainder, 16 units, is the exact square of the unit figure 4. Hence, the root of the given number is 84.

Short Method.

OPERATION.

```
     70'56 | 84
  8  64
     656
164  656
```

Solution.—Having subtracted the square of the tens from the first period of the dividend, divide the remainder by twice the tens, which will give 4 for the probable units figure of the root. Annex 4 to the trial divisor, multiply the latter by 4, and subtract the product from the remainder of the power.

2. Find the square root of 516961. Of 4.2025.

OPERATION.

```
        51'69'61 | 719
7² =    49
         269
141      141
         12861
1429     12861
```

OPERATION.

```
        4.'20'25 | 2.05
2² =    4
         .20 25
405      .20 25
```

Note.—As 0 occurs in the root, we annex 0 to the divisor, 4, and bring down the next period.

Rule.—*I. Beginning at units, separate the numbers into periods of two figures each.*

II. Having found the root of the left-hand period, subtract its square from the period, and to the remainder annex the next period for a dividend.

III. Divide the dividend, omitting the last figure, by double the root already found, and annex the quotient to the root and also to the divisor.

IV. Multiply the divisor as it now stands, by the last figure of the root, and subtract the product from the dividend.

V. Proceed in the same manner with the other periods, until all have been used.

NOTES.—1. If on multiplying twice the tens plus the units, by the units, the product be found too great, diminish the units of the root, in such a manner as to obtain the required product.

2. If a cipher occur in the root, annex 0 to the divisor and another period to the dividend, and proceed as before.

3. When the number contains an uneven number of decimal places, annex a cipher.

4. When the given number is not a perfect power, it is necessary to annex as many periods of ciphers as the number of decimal places required for the root.

5. To extract the square root of a common fraction, take the root of each term. If either, or both terms, be an imperfect power, reduce the fraction to a decimal before finding the root.

GEOMETRICAL METHOD.

What is the length of a square that contains 2809 square feet?

OPERATION.

	28'09	50
$50^2 =$	2500	3
$50 \times 2 =$ 100	309	53 *Ans.*
3		
103	309	

SOLUTION.—As the area of a square is equal to the square of its side, to find the length of its side, it is necessary to take the square root of the area.

The square root of 2809 sq. ft. contains two figures, *tens* and *units*, and must be some number between 50 and 60.

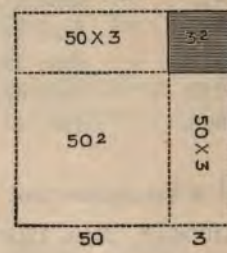
If from the given square, Fig. 1, we subtract a smaller square whose side is 50 ft., there will remain the area represented in Fig. 2. This area comprises the two rectangles, *A* and *B*, and the square *C*. The length of each of the rectangles is 50 ft., and their common width is equal to the side of the square *C*. Joining them by their widths, we will form another rectangle whose length is 100 ft., and whose approximate area is the difference between 2809 sq. ft. and 2500 sq. ft., or 309 sq. ft.

Fig. 1.

Since the area of a rectangle is equal to the product of its length and width, the width is equal to the area divided by the length. Therefore, the probable width of each of the two rectangles, *A* and

Fig. 2.

Fig. 3.



B, is equal to the approximate area, 309 sq. ft., divided by their united lengths, 100 ft.; which gives 3 ft.

After removing the rectangles, *A* and *B*, there remains the square *C*, the length of whose side being equal to the width of the rectangles, is 3 ft. By combining the two rectangles and the

Fig. 4.

square *C*, we may form another rectangle, Fig. 4, which is 103 ft. long, and 3 ft. wide. The area of this last rectangle is equal to 103 ft. multiplied by 3 ft., or 309 sq. ft.; which exactly corresponds with that portion of the given square that remained after subtracting the square 2500.

Hence, as shown in Fig. 3, the side of the original square is equal to 50 ft. + 3 ft. = 53 ft.

NOTE.—In order to secure the desired result, the teacher should have these geometrical figures drawn on the blackboard and different values assumed for each of them. The pupils should then be required to give the full explanation. Special attention should be given to the manner of finding the probable width of the rectangles.

WRITTEN EXERCISES.

Extract the square root of:

1. 441.	*Ans.* 21.	*6.* 4624.	*Ans.* 68.
2. 576.	*Ans.* 24.	*7.* 15625.	*Ans.* 125.
3. 1521.	*Ans.* 39.	*8.* 81796.	*Ans.* 286.
4. 3249.	*Ans.* 57.	*9.* 205209.	*Ans.* 453.
5. 1849.	*Ans.* 43.	*10.* 409600.	*Ans.* 640.

What is the square root of:

11. 1440000 ?	*Ans.* 1200.	*16.* 12.25 ?	*Ans.* 3.5.
12. 4020025 ?	*Ans.* 2005.	*17.* .2209 ?	*Ans.* .47.
13. 9647236 ?	*Ans.* 3106.	*18.* .7396 ?	*Ans.* .86.
14. 65610000 ?	*Ans.* 8100.	*19.* 1.0816 ?	*Ans.* 1.04.
15. 30151081 ?	*Ans.* 5491.	*20.* 6.7600 ?	*Ans.* 2.60.

Find the square root of:

21. 337.4569.	*Ans.* 18.37.	*26.* 15.6307.	*Ans.* 3.95+.
22. 1757.2864.	*Ans.* 41.92.	*27.* 74.0681.	*Ans.* 8.60+.
23. 2506.0036.	*Ans.* 50.06.	*28.* .992703.	*Ans.* .996+.
24. .00001296.	*Ans.* .0036.	*29.* .200039.	*Ans.* .447+.
25. .00498436.	*Ans.* .0706.	*30.* 10.1003.	*Ans.* 3.17+.

Extract the square root of:

31. $\frac{2704}{4489}$.	*Ans.* $\frac{52}{67}$.	*34.* $(\frac{3}{4})^3 \times (\frac{2}{5})^2$.	*Ans.* .259+.
32. $\frac{32761}{131044}$.	*Ans.* .5.	*35.* $(1\frac{1}{7})^2 \times (1\frac{3}{4})^2$.	*Ans.* 2.
33. $\frac{324}{17161}$.	*Ans.* $\frac{18}{131}$.	*36.* $(2\frac{2}{3})^2 \times (1\frac{7}{8})^2$.	*Ans.* 5.

37. $\frac{2}{5}$.	*40.* .006.	*43.* $16\frac{9}{25}$.
38. $\frac{3}{5}$.	*41.* $9\frac{1}{10}$.	*44.* $(.15)^3$.
39. .03.	*42.* $23\frac{1}{4}$.	*45.* $(2.07\frac{1}{2})^3$.

CUBE ROOT.

193. The ***Cube Root*** of a number is one of the three equal factors which produce it; as, 2 is the cube root of 8, because $2 \times 2 \times 2 = 8$.

The relation between the number of places in the cube root of any number, and in the number itself, may be shown by the following

TABLE.

$1^3 = 1$	1 Period.	$10^3 = 1{,}000$	2 Periods.
$3^3 = 27$		$30^3 = 27{,}000$	
$9^3 = 729$		$99^3 = 970{,}299$	

PRINCIPLES.—*I. The cube of any number contains three times as many figures, or three times as many less one or two, as the number itself.*

II. The cube root of any number contains as many terms as there are periods of three figures each, in the number.

MENTAL EXERCISES.

1. What is the cube root of 8?

SOLUTION.—To find the cube root of 8, resolve it into its three equal factors, $2 \times 2 \times 2$.

2. What is the cube root of 27? Of 64? 125? 1000? 8000? 27000?

3. Find the cube root of $\frac{1}{8}$. $\frac{8}{27}$. $\frac{27}{64}$. $\frac{1}{1000}$.

4. Find the cube root of $3\frac{3}{5}$. $1\frac{61}{64}$.

5. What is the side of a cube whose solidity is 512 cu. ft.?

6. If a cubical box contains 216 cu. in., what is the length of one of its sides?

7. Find the side of a cube whose solidity is $2\frac{19}{27}$ cu. ft.

8. If a cubical bin contains 343 cu. ft., what is the length of one of its sides?

WRITTEN EXERCISES.

1. Find the cube root of 24389.

OPERATION.

$t^2+3t^2\times u+3t\times u^2+u^3$	$=$		24,389	20
$t^3=20^3$	$=$		8 000	9
$3t^3\times u+3t\times u^2+u^3$	$=$		16 389	29, *Ans.*
$(3t^2+3t\times u+u^2)\,u$				
$3t^2$	$=$	1200		
$3t\times u$	$=$	540		
u^2	$=$	81		
$(3t^2+3t\times u+u^2)\,u$	$=$	$1821\times 9=\underline{16389}$		

SOLUTION.—As the given number contains two periods, its root will consist of two places, *tens* and *units*. Hence, $t^3+3t^2\times u+3t\times u^2+u^3=24389$.

The greatest cube in the period 24, is 8; and its root, 2, is the tens figure of the required root. Subtracting the cube of the tens, we get a remainder of 16389, which equals $3t^2\times u+3t\times u^2+u^3$.

$3t^2$, or the trial divisor, 1200, is contained in 16389, 9 times, and 9 is probably the units' figure of the root.

Then, $3t^2=1200$; $3t\times u=540$; $u^2=81$; and the sum $=1200+540+81=1821$, which multiplied by the u, $=1821\times 9=16389$, the exact remainder. The required root, therefore, is 29.

2. What is the cube root of 34201530936 ?

OPERATION.

			34'201'530'936 | 3246
	$3^3 =$		27
Trial divisor =	$3^2 \times 300 =$	2700	7201
	$3 \times 2 \times 30 =$	180	
	$2^2 =$	4	
Complete divisor =		2884	5768
Trial divisor =	$(32)^2 \times 300 =$	307200	1433530
	$32 \times 4 \times 30 =$	3840	
	$4^2 =$	16	
Complete divisor =		311056	1244224
Trial divisor =	$(324)^2 \times 300 =$	31492800	189306936
	$324 \times 6 \times 30 =$	58320	
	$6^2 =$	36	
Complete divisor =		31551156	189306936

SOLUTION.—In the above operation, instead of 3 times the square of the first term considered as tens, take 300 times the square, which gives the same result; and in completing the divisor, multiply the product of the first and second terms by 30.

To find the second trial divisor, take the square of the first two terms of the root considered as one. In like manner square the first three terms, for the third trial divisor.

RULE.—*I. Beginning at units, separate the number into periods of three places each.*

II. Having found the root of the left-hand period, subtract its cube from the period, and to the remainder annex the next period for a new dividend.

III. Take 300 times the square of the first term of the root for a trial divisor; divide and take the quotient for the probable second term of the root.

IV. To complete the divisor, multiply the product of the first and second terms of the root by 30; 2dly,

square the second term; and then add both results to the trial divisor.

V. Multiply the complete divisor by the second term, and find the remainder as before; bring down the next period, and thus proceed until all the periods are exhausted.

VI. To find a second, or third trial divisor, take 300 times the square of the entire root thus far found.

NOTES.—1. If the product of the complete divisor by its corresponding term of the root, should exceed the dividend, the term of the root should be diminished.

2. When the dividend does not contain the complete divisor, write a cipher in the root, bring down the next period, annex two ciphers to the trial divisor, and proceed as before.

3. If the number of decimal places in the given number be not divisible by 3, make it so by annexing ciphers.

4. If the given number be not a perfect cube, annex as many periods of ciphers as there are to be decimal places in the root.

GEOMETRICAL METHOD.

What is the length of the edge of a cube containing 300763 cubic feet?

OPERATION.

		300763	60
	$60^3 =$	216000	7
		84763	67
$60^2 \times 3 =$	10800		
$60 \times 3 \times 7 =$	1260		
$7^2 =$	49		
	12109	84763	

SOLUTION.—As the solidity of a cube is equal to the cube of its edge, to find the length of its edge, it is necessary to take the cube root of its solidity.

The cube root of 300763 contains two figures, and must be between 60 and 70.

If from the given cube, Fig. 1, we subtract a cube, Fig. 2, whose

Fig. 1.

Fig. 2.

edge is 60 ft., there will remain a solid containing 84763 cu. ft., as represented by Fig. 3. This solid consists chiefly of the three rectangular slabs *A*, *B*, and *C*, each of which is 60 ft. long and 60 ft.

Fig. 3.

wide. It is evident that the thickness of these slabs added to the length of the cube removed, will be equal to the edge of the given cube. Dividing the approximate sum of their volumes, 84733 cu.

Fig. 4.

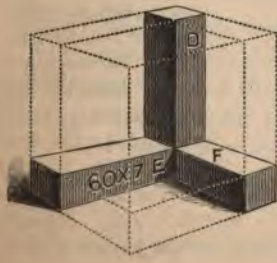
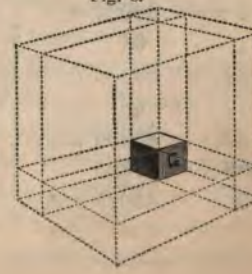


Fig. 5.



ft., by the sum of the areas of their bases, $60^2 \times 3$, we get 7 ft. for their probable thickness.

A, *B*, and *C* removed, there still remain three smaller rectangular solids, *D*, *E*, and *F*, as shown in Fig. 4. Each of these solids has for its length, the length of the cube removed, and for its thickness, that of the rectangular slabs. The area of the face of one of these solids is, therefore, equal to 60 ft. × 7 ft., or 420 sq. ft.; and the area of a face of the three of them is $60 \times 7 \times 3$, or 1260 sq. ft.

Finally, after removing the solids, *D*, *E*, *F*, there remains only the small cube *G*, Fig. 5, whose face being equal in area to the end of one of the smaller solids, is 7 ft. sq.

Now, by taking the sum of the areas of the faces of the rectangular solids, *A*, *B*, *C*, *D*, *E*, *F*, and that of the small cube, *G*, and multiplying this sum by their common thickness, 7 ft., we shall have a total area of $(10800 + 1260 + 49) \times 7$, or 84763 cu. ft. This result exactly corresponds with the volume of the solid that remained after removing the cube whose edge was 60 ft. Hence, the length of the given cube is equal to 60 ft., the edge of the cube removed, plus 7 ft., the thickness of the rectangular solids.

Horner's Method.

Extract the cube root of 1953125.

OPERATION.

1st col.	2d col.	1′953′125 | 125
1	1	
1	2	1
2	300 t. d.	953
1	64	
32	364 c. d.	728
2	68	225125
34	43200 t. d.	
2	1825	
365	45025 c. d.	225125

Solution.—Write 1, the first term of the root, in the first column, its square, 1, in the second column, and its cube, 1, under the first period. Subtract, and bring down the second period.

Add 1, the first term of the root, to 1, the first term in the first column, multiply the sum by the root, and add the product to the number

in the second column; which result, with two ciphers annexed, will give 300, the first *trial divisor*.

Before finding the second term of the root, add 1, the first term, to the number 2, in the first column. Divide the dividend by the trial divisor, and annex 2, the second term of the root, to the number 3 in the first column. Multiply the result by the second term of the root, and add the product to the trial divisor for the *complete divisor*, 364. Multiply, subtract, and annex the third period for a new dividend.

To find the next *trial divisor*, add the second term of the root, 2, to the last number in the first column, 32. Multiply the sum by 2, the second term of the root, add the product to the complete divisor, and annex two ciphers.

Again add 2, the second term of the root, to the last number in the first column, obtaining 36 for result. Then having found the next term of the root, 5, annex it to the last number, 36; in the first column. Multiply the result, 365, by 5, the last term of the root, and add the product to the trial divisor, 43200, which will give 45025, the *complete divisor*. Then multiply and subtract as before.

RULE.—*I. Having found the first term of the root in the usual way, write it in the first column, its square in the second column, and its cube under the first period. Subtract and bring down the next period for a new dividend.*

II. Add the root found to the number in the first column, multiply the sum by the root, and add the product to the second column; the result, with two ciphers annexed, gives the TRIAL DIVISOR.

III. After adding the root to the last term in the first column, find the second term of the root by dividing the dividend by the TRIAL DIVISOR.

IV. To the last number in the first column, annex the second term of the root, multiply the result by the second term of the root, and add the product to the trial divisor for the COMPLETE DIVISOR.

V. Multiply the COMPLETE DIVISOR *by the last term of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

VI. Follow the same method until all the periods have been used.

NOTE.—If a cipher occur in the root, bring down the next period, annex one cipher to the first column, two ciphers to the second column, and then divide as before.

WRITTEN EXERCISES.

Extract the cube root of:

1. 12167.	*Ans.* 23.	*7.* 658503.	*Ans.* 87.
2. 21952.	*Ans.* 28.	*8.* 8615125.	*Ans.* 205.
3. 39304.	*Ans.* 34.	*9.* 2000376.	*Ans.* 126.
4. 27000.	*Ans.* 30.	*10.* 52.734375.	*Ans.* 3.75.
5. 85184.	*Ans.* 44.	*11.* 66923.416.	*Ans.* 40.6.
6. 140608.	*Ans.* 52.	*12.* .000250047.	*Ans.* .063.

13. 1121622.319. *Ans.* 103.9.
14. 443.322265625. *Ans.* 7.625.
15. 28568426193. *Ans.* 3057.
16. 71009375221. *Ans.* 4141.
17. 903670125632. *Ans.* 9668.
18. 163129337344. *Ans.* 5464.
19. 205891132094649. *Ans.* 59049.
20. 347.577210791. *Ans.* 7.031.
21. Find the cube root of $\frac{2197}{3375}$. *Ans.* $\frac{13}{15}$.

NOTE.—To find the cube root of a common fraction, either take its terms separately, or first reduce it to a decimal.

22. What is the cube root of $\frac{8000}{17576}$? *Ans.* $\frac{10}{13}$.

23. Find the cube root of 22 correct to the third decimal place. *Ans.* 2.802.

24. Extract the cube root of $3605\frac{1}{27}$. *Ans.* $15\frac{1}{3}$.

25. What is the cube root of $.000166\frac{2}{3}$? *Ans.* .055.

26. Find the cube root of 6 to the fifth decimal place. *Ans.* 1.81712.

Find the value of each of the following expressions:

27. $\sqrt[3]{10^2}$. *Ans.* 4.641+.

28. $\sqrt[3]{2^{10}}$. *Ans.* 10.07.

*29.** $\sqrt[6]{64}$. *Ans.* 2.

30. $\sqrt[3]{\sqrt[2]{19683}}$. *Ans.* 3.

31. $\sqrt[3]{\sqrt[2]{11390625}}$. *Ans.* 15.

32. $\sqrt[8]{65536}$. *Ans.* 4.

33. $\sqrt[12]{\frac{531441}{4096}}$. *Ans.* $1\frac{1}{2}$.

34. $\sqrt[9]{\frac{19683}{262144}}$. *Ans.* $\frac{3}{4}$.

35. $\sqrt[6]{\frac{1}{1000000}}$. *Ans.* $\frac{1}{10}$.

36. $\sqrt[8]{\left(\frac{4}{25}\right)^4}$. *Ans.* $\frac{2}{5}$.

* NOTE.—The $\sqrt[6]{64} = \sqrt[3]{\sqrt[2]{64}}$; and the same is true of all other higher roots whose indices contain only the factors 2 or 3.

APPLICATION OF POWERS AND ROOTS.

1. How many yards in the side of a square field containing 3600 sq. yd.? *Ans.* 60 yd.

2. What is the area of a square farm whose side is 42 rods? *Ans.* 11 A. 4 sq. rd.

3. If an army of 93025 men be drawn up in the form of a square, how many men will there be on a side? *Ans.* 305 men.

4. What is the length of the edge of a cube containing 4913 cu. ft.? *Ans.* 17 ft.

5. What is the side of the largest possible square that can be made from a board 81 inches long and 16 inches wide? *Ans.* 36 in.

6. What will be the cost of 9 square yards of plastering at $3\frac{1}{2}$ cents a square foot? *Ans.* \$2.84.

7. Find the contents of a cubical block of marble each of whose faces contains 34 sq. ft. 4 sq. in.

Ans. 7 cu. yd. 9 cu. ft. 856 cu. in.

8. How many square feet in the entire surface of a cube that contains 4492.125 cu. ft.? *Ans.* $1633\frac{1}{2}$ sq. ft.

9. What is the width of a rectangular field that is three times as long as it is wide, and contains 10800 square rods? *Ans.* 60 rods.

10. Find three equal factors whose product multiplied by 12 will give 82308 for result. *Ans.* $19 \times 19 \times 19$.

11. A court-yard is 56.5 ft. square. What will it cost to pave it at \$1.80 per square yard? *Ans.* \$638.45.

12. What is the thickness of a square block of marble that is 24 ft. long, and contains 93312 cu. inches?

Ans. $1\frac{1}{2}$ ft.

13. Find the thickness of a square piece of timber that is 4 times as long as it is wide, and contains 32000 cubic inches. *Ans.* 20 in.

14. What is the depth of a cubical cistern of the same capacity as a cistern 25 ft. long, 12 ft. wide, and $11\frac{1}{4}$ ft. deep? *Ans.* 15 ft.

15. What will it cost to fence a square field containing 3721 square rods, at \$1.25 a yard? *Ans.* \$$1677\frac{1}{2}$.

16. Find the cost of cementing the bottom and sides of the cistern mentioned in No. 14, at 8¢ a sq. foot.

Ans. \$90.

17. The breadth of a room is twice its height, and half its length; and the room contains 5832 cubic feet of air. Find its length.

18. Find the side of a square court-yard, the expense of paving which, at 36 cents per sq. yd., is \$8.41.

ARITHMETICAL PROGRESSION.

194. A ***Series*** is a succession of numbers, each of which is derived from the one immediately preceding it by some fixed relation.

195. The ***Terms*** of a series are the numbers of which it is composed. The first and last terms are called the *Extremes*, and the others, the *Means*.

196. When the terms increase from left to right, they form an *Ascending Series;* and when they decrease from left to right, a *Descending Series*.

197. An ***Arithmetical Progression*** is a series of numbers that increase or decrease by a common difference.

Thus: 1, 3, 5, 7, 9, 11, is an ascending series;
11, 9, 7, 5, 3, 1, is a descending series.

The *common difference* in each series is 2.

198. In every arithmetical series are found the following five quantities: (1) The *first term;* (2) the *last term;* (3) the *common difference;* (4) the *number of terms;* (5) the *sum of all the terms*.

Any three of these quantities being given, the other two may be found, as shown in the following five cases:

CASE I.

199. To find the last term, when the first term, the common difference, and the number of terms are given.

1. The first term of an arithmetical series is 5, the common difference is 4, and the number of terms is 6; what is the last term?

OPERATION.

1st term	=		5
2d "	=	$5+4$	= 9
3d "	=	$5+(4\times2)$	= 13
4th "	=	$5+(4\times3)$	= 17
5th "	=	$5+(4\times4)$	= 21
6th "	=	$5+(4\times5)$	= 25

SOLUTION.—The first term being 5, the second term is equal to the first term, 5, plus the common difference, 4, which gives 9. The third term = 5, the first term, plus twice the common difference, 4, which gives 13. The fourth term = 5 plus 3 times the common difference, which equals 17. Hence, the sixth term = 5+5 times the common difference, which equals 25.

RULE.—*To find the last term, multiply the common difference by the number of terms less one, and add the result to the first term.*

NOTE.—In a descending series, instead of adding, subtract the product of the common difference by the number of terms less one.

WRITTEN EXERCISES.

2. In an arithmetical progression the first term is 8, the common difference is 3, and the number of terms 12; what is the last term? *Ans.* 41.

3. If the first term of an ascending series be 9, and the common difference 6, what is the 30th term? *Ans.* 183.

4. Given the first term 2, the common difference 6, and the number of terms 21; to find the last term. *Ans.* 122.

5. Find the last term of a descending series, in which the first term is 314, the common difference 7, and the number of terms 45. *Ans.* 6.

6. A woman bought a dozen of eggs, giving $2\frac{1}{2}$ cents for the first, 3 cents for the second, $3\frac{1}{2}$ for the third, and so on; what did she pay for the last egg? *Ans.* 8 cents.

CASE II.

200. To find the common difference, when the extremes and the number of terms are given.

1. What is the common difference in an arithmetical series, if the first term is 5, the last 152, and the number of terms 50 ?

OPERATION.

$$5+(50-1)\times d = 152$$
$$49\times d = 152-5$$
$$d = \frac{152-5}{49} = 3$$

SOLUTION.—In Case I. we found the last term 152, to be equal to 5, the first term, plus the common difference multiplied by (50−1), the number of terms less one. The common difference, therefore, is equal to the difference between the first and the last term, divided by the number of terms less 1.

RULE.—*To find the common difference, divide the difference of the extremes by the number of terms less one.*

WRITTEN EXERCISES.

2. If the number of terms is 30, the first term 13, and the last term 129, what is the common difference?

Ans. 4.

3. The extremes of an arithmetical series are $\frac{3}{4}$ and $16\frac{3}{4}$, and the number of terms 25; what is the common difference? *Ans.* $\frac{2}{3}$.

4. The first of a series of 20 steps, is 15 inches from the ground, and the last 205 inches; how far apart are the steps? *Ans.* 10 inches.

5. A family comprises 10 persons; the youngest is 8, and the eldest 44 years old; and their ages increase in arithmetical progression. Find the difference of their ages. *Ans.* 4 years.

Case III.

201. To find the number of terms, when the extremes and the common difference are given.

1. The extremes are 9 and 144, and the common difference is 5; what is the number of terms?

OPERATION.

$$5(n-1) = 144-9$$
$$(n-1) = \frac{144-9}{5}$$
$$n-1 = 27$$
$$n = 28$$

Solution.—By Case II. we have found that the common difference is equal to the difference between the extremes divided by the number of terms less one. Hence, the number of terms is equal to the quotient of the difference between the extremes divided by the common difference, plus one.

Rule.—*To find the number of terms, divide the difference between the extremes by the common difference, and add one to the quotient.*

2. The extremes are 11 and 102, and the common difference 7; what is the number of terms? *Ans.* 14.

Case IV.

202. To find the sum of the series, when the extremes and the number of terms are given.

1. If the extremes are 3 and 13, and the number of terms 6, what is the sum of the series?

OPERATION.

$$\text{Sum} = 3+5+7+9+11+13$$
$$\text{Sum} = 13+11+9+7+5+3$$
$$2\ \text{Sum} = 16+16+16+16+16+16$$
$$2\ \text{Sum} = 16\times 6 = (3+13)\times 6$$
$$\text{Sum} = \frac{(3+13)\,6}{2} = 48.$$

Solution.—Having found the common difference by Case II., take the sum of the series in its natural order and also in its inverted order. By adding we get twice the sum equal to 6 times 16 = 6 times the sum of the extremes 3 and 13. Hence, the

sum of the series is equal to 6 times the sum of the extremes divided by 2, = 48.

RULE.—*To find the sum of an arithmetical series, multiply the sum of the extremes by the number of terms, and divide the product by two.*

2. The first term is 6, the last term 64, and the number of terms 30; what is the sum of the series?

Ans. 1050.

3. A boy earned \$3½ the first week, \$4¼ the second, \$5 the third, and so on; how much did he earn in 8 weeks?

Ans. \$49.

4. How far would a man walk in 12 days, walking 8 miles the first day, and increasing at the rate of 3½ miles a day? *Ans.* 327 miles.

CASE V.

203. To find the first term, when the last term, the common difference, and the number of terms are given.

1. What is the first in a series of 32 terms, the last of which is 125, and the common difference 4?

OPERATION.

$$125 = a + (32-1)\,4$$
$$a + (32-1)\,4 = 125$$
$$a = 125 - (32-1)\,4$$
$$a = 125 - 124 = 1$$

SOLUTION.—According to Case I., the last term is equal to the first plus the product of the common difference multiplied by the number of terms less one. Hence, the first term is equal to $125 - (32 - 1)4 = 125 - 124 = 1$.

RULE.—*To find the first term, subtract the product of the common difference by the number of terms less one, from the last term.*

2. Find the first term, the last being 242, the common difference 8, and the number of terms 30. *Ans.* 10.

3. A servant was employed for 6 days, receiving an increase of 40 cents each day. On the last day he received $2.50; how much did he receive on the first?

Ans. 50 cents.

4. If the last term is $2\frac{1}{2}$, the common difference $\frac{5}{24}$, and the number of terms 18, what is the first term?

GEOMETRICAL PROGRESSION.

204. A ***Geometrical Progression*** is a series of numbers which increase or decrease by a common multiplier; as 1, 2, 4, 8, 16, etc.

205. The multiplier is called the *Ratio.* When it is greater than a unit, the series is called *ascending;* when less, the series is *descending.*

Thus, 5, 10, 20, 40, 80, is an ascending series, in which the ratio is 2; but 80, 40, 20, 10, 5, is a descending series, in which the ratio is $\frac{1}{2}$.

206. An ***Infinite Series*** is a series in which the number of terms is infinite.

The last term of a descending series continued to infinity, becomes so small that it may be considered zero.

207. The ***Quantities*** considered in Geometrical Progression are as follows: (1) the *first term;* (2) the *last term;* (3) the *number of terms;* (4) the *ratio;* (5) the *sum of the terms.* Any three of these terms being given, the others may be found.

CASE I.

208. To find any term, when the first term, the ratio, and the number of terms are given.

1. The first term of a geometrical series is 6, and the ratio 2; what is the 5th term?

OPERATION.

2d term $= 6 \times 2$
3d " $= 6 \times 2^2$
4th " $= 6 \times 2^3$
5th " $= 6 \times 2^4 = 96$

SOLUTION.—As the first term is 6, and the ratio 2, the second term is equal to 6×2; the third is equal to $6 \times 2 \times 2$, or 6×2^2, which expresses the first term into the second power of the ratio. Hence, the fifth term equals 6, the first term, into 2^4, the fourth power of the ratio, which gives 96.

RULE.—*To find any term, multiply the first term by that power of the ratio denoted by the number of terms less one.*

2. The first of a geometrical series is 4, and the ratio 5; what is the fourth term? *Ans.* 500.

3. The first term of a descending series is 128, and the ratio is $\frac{1}{4}$; what is the sixth term? *Ans.* $\frac{1}{8}$.

4. What is the amount of $10 at compound interest, for 5 years, at 6 per cent. per annum?

Ans. $13.3822+.

NOTE.—In compound interest problems, raise (100 + rate) to the power indicated by the number of years.

5. If a boy save 1 cent in January, 3 cents in February, 9 cents in March, and so on, how much will he save in December? *Ans.* $1771.47.

CASE II.

209. To find the sum of the series, when the extremes and rate are given.

1. The first term is 1, the last term 64, and the rate 2; what is the sum of all the terms?

OPERATION.

$$\begin{array}{lll} 1.\ \text{Sum} & = 3 + 9 + 27 + 81 \\ 2.\ \text{Sum} \times 3 = & \quad 9 + 27 + 81 + 243 \\ \hline \text{Twice Sum} & = \quad 243 - 3 \\ \text{Sum} & = \dfrac{243 - 3}{2} = 120 \end{array}$$

SOLUTION.—Subtracting the sum of the series as expressed in equation 1, from three times the sum, as expressed in equation 2, we find twice the sum equals 243 − 3. Hence, the sum equals 243 − 3 divided by 2, which gives 120.

RULE.—*To find the sum of the series, multiply the greater extreme by the ratio, subtract the less extreme from the product, and divide the remainder by the ratio minus one.*

NOTE.—1. In a descending series, subtract the product of the last term by the rate, from the first term, and divide the remainder by one minus the rate.

2. In an infinite descending series, divide the first term by one minus the rate.

2. The first term is 5, the last term 163840, and the rate 8; what is the sum of the terms? *Ans.* 187245.

3. The first term of a descending series is 384, the last term 3, and the rate $\frac{1}{2}$; what is the sum of the series? *Ans.* 765.

4. What is the sum of $\frac{1}{4}$, $\frac{1}{12}$, $\frac{1}{36}$, $\frac{1}{108}$, etc., continued to infinity? *Ans.* $\frac{3}{8}$.

5. The extremes are 1 and $5\frac{1}{16}$, and the rate $1\frac{1}{2}$; what is the sum of the series? *Ans.* $13\frac{3}{16}$.

6. In discharging a debt, a corporation paid $5 the first month, and tripled the amount each succeeding month for a year. What was the amount of the debt? *Ans.* $1328600.

MENTAL REVIEW.

I.

1. A man bought a cow for $60, which was $\frac{1}{8}$ of 4 times as much as he sold it for; how much did he gain?

SOLUTION.—Since $\frac{1}{8}$ of 4, or $\frac{1}{2}$ times the selling price, is equal to $60, the cost, the whole selling price is equal to twice $60 = $120. Hence, if he bought the cow for $60 and sold it for $120, he gained $60.

2. Margaret paid 9 cents for some needles, which was $\frac{2}{3}$ times $\frac{1}{2}$ of all her money; how much money had she?

3. Joseph bought 24 marbles, which was $\frac{2}{5}$ of 5 times as many as William bought; how many did William buy?

4. A carriage was sold for $65, which was $\frac{3}{5}$ of $\frac{5}{8}$ of the cost; what was the loss?

5. A drover, after selling $\frac{7}{8}$ of his sheep, had $\frac{2}{3}$ of 120 left; how many had he at first?

6. $\frac{5}{7}$ of Michael's age increased by 20 years, equals $2\frac{1}{7}$ times his age; how old is he?

7. Having been robbed of the $\frac{4}{7}$ of my money, I spent $\frac{2}{3}$ of the remainder in prosecuting the thief, and had $8 left. How much had I at first?

8. Mr. Sinclair was 30 years of age when he was married, and his age was $1\frac{1}{3}$ times $\frac{5}{8}$ of his wife's age. How old was she?

9. If to the $\frac{9}{10}$ of my sheep you add 40, the same will lack 8 of being equal to $1\frac{1}{2}$ times the number. How many sheep have I?

10. During my first year of business I lost $\frac{7}{8}$ of my capital, but during the second year I gained $1\frac{1}{7}$ times as

much as I had lost during the first. I then had $18000; what was my capital at commencing business?

11. $\frac{1}{8}$ of a barrel of cider was used, $\frac{3}{4}$ of it was sold, and the remaining 5 gallons leaked out; how many gallons did the barrel contain at first?

12. After spending $\frac{3}{5}$ of my money and $\frac{1}{2}$ of the remainder, I had $6000; how much had I at first?

II.

1. A and B hired a horse and carriage for $30; A used it 4 days and B 6 days; how much should each pay?

SOLUTION.—If A used the horse and carriage for 4 days, and B for 6 days, they both used them for 10 days. If the use of the horse and carriage for 10 days cost $30, for 1 day it will cost $3; and for 4 days, $12; and for 6 days, $18. Hence A should pay $12, and B $18.

2. Two men hired a pasture for $45; the first put in 5 sheep, and the second 4 sheep; how much should each pay?

3. C and D built a boat for $210; C engaged 3 men 10 days, and D engaged 5 men 8 days; how much should C and D respectively receive?

4. A and B enter into partnership; A invests $12 for 5 months and B invests $20 for 6 months; they gain $180; how much should each receive?

5. A, B, and C buy 80 barrels of flour for $750; of which A pays $\frac{2}{5}$, B $\frac{1}{5}$, and C the remainder; how many barrels does each receive?

6. Divide $60 between M and O, giving M $1 as often as O $\frac{5}{7}$ of a dollar.

7. Two men hired a pasture for $49. One put in 8 horses for 9 days, and the other 5 horses for 15 days; how much should each pay?

8. C, D, and E bought a horse. C paid $10 as often as D $8 and E $6. They gained $72 in selling him; what is each man's share of the gain?

9. A merchant divided $700 among his 3 sons and 5 daughters, giving each daughter $\frac{1}{3}$ as much as each son; how much did each receive?

10. Three men hire a pasture field for $31; the first puts in 8 cows, the second 12 calves, and the third 6 colts. What should each pay, if 4 cows eat as much as 5 calves, and 2 colts consume as much as 3 calves?

11. M and R agree to reap a field for $56. M sends 4 men 3 days, and R 6 boys 8 days; how much should each receive, if 3 boys do as much as 1 man?

12. A gardener divided 48 pears among some children, giving 2 apiece to the boys and four apiece to the girls; how many children were there, if the boys received as many pears as the girls?

III.

1. Joseph bought a certain number of lemons at the rate of 3 cents each; if he had paid 2 cents each, they would have cost 15 cents less; how many did he buy?

SOLUTION.—If he had paid 2 cents apiece, each lemon would have cost 1 cent less; and since the whole number would have cost 15 cents less, there would have been as many lemons as 1 cent is contained times in 15 cents, which are 15.

2. A mother gave $7 to each of her children and had $12 remaining; had she given them $9 each, she would have had nothing left; how many children had she?

3. John gave 70 cents for an equal number of apples and bananas, giving 2 cents each for the apples, and 5 cents each for the bananas; how many of each did he purchase?

4. A teacher having given each of his pupils 4 questions in history, had 20 remaining; had he given them 5 questions each, he would have had none remaining; how many pupils had he?

5. Thomas has 5 apples more than Henry, and they both together have 29; how many has each?

SOLUTION.—Since Thomas has 5 more than Henry, his number is equal to Henry's plus 5. Hence, twice Henry's plus 5 = 29.

6. William in one week spent \$4 more than $\frac{1}{2}$ of his money, and the next week he spent the remaining \$8; how much money had he?

7. The sum of two numbers is 53, and their difference is 16; what are the numbers?

8. If 5 cents be taken from $\frac{1}{3}$ of Michael's money, the remainder will be equal to $\frac{1}{2}$ of John's; if both together have 65 cents, how much has each?

9. Two boys had each an equal number of marbles; one of them lost 6, and they then together had only 26 remaining; how many had each at first?

10. C and D agree to perform a piece of work, C receiving \$3 and D \$5 a day. If C works twice as many days as D, and they together receive \$110, how many days does each work?

11. A girl being asked her age, replied that 4 times her age minus 13 years were 47 years; how old was she?

12. E and F dig a ditch for \$40; E receives \$$2\frac{1}{2}$, and F \$3 a day; E works twice as many days as F; what does each receive?

13. Mary bought 45 oranges at the rate of 3 for 2 cents. She sold $\frac{3}{5}$ of them at the rate of 3 for 5 cents, and the remainder at the rate of 2 for a cent; how much did she gain or lose?

14. Three boys together have 82 plums; John has 5 more than Francis, and Henry has as many as John and Francis together; how many has each?

15. Helen had $\frac{1}{4}$ as many cherries as her sister Sarah. After each had eaten 10, they together had 80; how many had each at first?

16. C and D together have 90 sheep; $\frac{3}{4}$ of C's flock increased by 6 sheep, is equal to D's; how many sheep has each?

17. William has 8 pears more than Charles, and $\frac{2}{5}$ of William's pears equal $\frac{1}{2}$ of Charles's; how many has each?

18. Divide 42 peaches between Susan and Alice, so that $\frac{3}{5}$ of Alice's may be equal to $\frac{1}{3}$ of Susan's.

19. Philip walked 63 miles in 3 days. On the second day he walked $\frac{1}{2}$ as far as on the first and third; and on the last two days twice as far as on the first; how many miles did he travel each day?

IV.

1. If A can do a piece of work in 6 days, and B can do the same in 9 days, in what time can both do it working together?

SOLUTION.—If A can do the work in 6 days, in 1 day he can do $\frac{1}{6}$ of the work; and B in 1 day can do $\frac{1}{9}$ of the work. Working together they can do $\frac{1}{6}+\frac{1}{9}=\frac{5}{18}$ of the work in 1 day. Hence, if by working together they can do $\frac{5}{18}$ of it in 1 day, it will require $\frac{1}{5}$ of a day to do $\frac{1}{18}$ of it; and $\frac{18}{5}$, or $3\frac{3}{5}$ days, to do the whole.

2. If C can build a barn in 16 days, and D can do the same in 20 days, what part of the work can they do together in one day?

3. A farmer and his son can mow a field in 12 days; the son can mow it in 25 days; how long would it take *the farmer to mow it* alone?

4. P and Q can make a book-case in 5 days; with the assistance of R they can finish it in 3 days; in what time could R make it working alone?

5. If a man can drink a gallon of beer in 6 days, and another can drink a gallon in 4 days, how long would a gallon last both of them?

6. Jane can knit a pair of mittens in $\frac{2}{3}$ of a day, and Julia can knit a pair in $\frac{3}{4}$ of a day; how many pairs can both together knit in 6 days?

7. A, B, and C can do a piece of work in 10 days, A and B, in 20 days, and A and C, in 15; how long would it take each separately?

8. Lawrence can make a pair of shoes in $\frac{3}{8}$ of a day and Samuel in $\frac{1}{2}$ a day; how long would it take them to make 11 pairs?

9. It requires 10 hours for one pipe to fill a cistern, and $\frac{3}{4}$ of that time for a second; in what time would both together fill it?

10. A barrel of flour will last a man and his son 4 months, and the son alone 10 months; how long would it last the man alone?

11. If 2 women or 4 girls can do some sewing in 8 days, in what time can 2 women and 2 girls do it?

12. Joseph can dig a ditch in 15 days, and with Henry's assistance, in 10 days; how long will it take Henry to finish what remains, after Joseph has dug $\frac{3}{5}$ of it?

13. A, B, and C can hoe a field of corn in 8 days, A and B in 12 days, and B and C in 15 days; how long will it take each to hoe it?

14. A certain cistern can be emptied by 3 pipes in 5 hours; the first and second will empty it in 10 hr.; the first and third in 9; in what time will each pipe empty it?

15. If a man can chop a cord of wood in $\frac{5}{8}$ of a day, and another man can do the same in $\frac{3}{4}$ of a day, how many days would both require to chop 22 cords?

16. Matthew can make 40% of a fence in two weeks, and Alfred 30% of it in the same time. After they had both worked one week, how long would it take Alfred to finish it?

V.

1. If 45 pears be divided among some children, so that $\frac{1}{2}$ of them receive 3 apiece, and the remainder 6 apiece, how many children are there?

SOLUTION.—According to the conditions of the problem the children may be considered as divided into two groups. As often as a child in the first group receives 3 pears another in the second group receives 6. Therefore every two children receive 9 pears. If they all receive 45 pears, there are as many times two children as 9 pears are contained times in 45 pears, which are 5 times. Hence there are 5 times 2 children.

2. Thomas gave his chickens 90 grains of corn, giving $\frac{1}{2}$ of them 5 grains each, and the others 1 grain each; how many chickens had he?

3. 7 times $\frac{1}{8}$ of a certain number, plus 3 times $\frac{3}{8}$ of the same number, is 32; what is the number?

4. 6 times $\frac{1}{2}$ of a certain number, plus 3 times $\frac{3}{8}$ of the same number, is 33; what is the number?

5. A drover received \$5 a head for $\frac{1}{4}$ of his flock, \$3 a head for as many more, \$6 a head for $\frac{1}{4}$ of the remainder, and \$2 a head for the rest; how many sheep had he, if for the whole flock he received \$280?

6. A boy being asked how many marbles he had, said that 3 times the number was 42 less than 5 times the number. How many marbles had he?

7. The sum of E's and F's ages is 40 years; 5 years ago, F's age was $\frac{1}{2}$ of E's; what are their ages?

8. A horse and carriage cost $210; the horse cost $30 more than twice the cost of the carriage; what was the cost of each?

9. There are 79 sheep in two pens; in one there are 11 sheep less than twice as many as in the other; how many in each?

10. Two men being $52\frac{1}{2}$ miles apart, travel toward each other; when they meet, one has travelled $12\frac{3}{4}$ miles more than the other; how far has each travelled?

11. The sum of two numbers is 22; their difference is equal to $\frac{5}{8}$ of the greater; what are the numbers?

12. Henry paid 50 shillings for a pair of skates and a knife; three times the difference between the cost of the two, plus $1\frac{1}{2}$ times the cost of the knife, equals twice the cost of the skates; what did he pay for each?

13. By selling a quantity of cloth for $350, I gained $\frac{2}{5}$ of what it cost; how much did it cost?

14. Michael and John worked together for 15 days; Michael received 3 dimes a day more than John, and they both received $94.50; how much did each receive a day?

15. Two men 150 miles apart, travel towards each other, one at the rate of 6 miles an hour, and the other 9 miles an hour; how long will it take them to meet?

16. A said that he was 3 times as old as B, and 20 years older than C; C replied that he was twice as old as B; how old was each?

VI.

1. A man engaged to work for 40 days, agreeing that he should receive $\$2\frac{1}{2}$ for every day he labored, and for-

feit $1½ for every day he was idle. At the settlement he received $72; how many days was he idle?

SOLUTION.—At $2½ a day, for 40 days' labor he would have received $100. He, therefore, lost $100 − $72 = $28. Every day he was idle he lost his wages $2½ + his forfeit, $1½ = $4. Hence, if he lost $4 in one day, to lose $28, it required as many days as $4 are contained times in $28, which are 7 days.

2. A boy was hired for 5 dimes a day, on condition that for every day that he was idle he should pay 3 dimes for his board. At the end of 30 days he received $7; how many days did he labor?

3. A farmer bought an equal number of ducks and turkeys, paying for the former 2 dimes each, and for the latter 4 dimes each; how many of each did he buy, if the turkeys cost $2 more than the ducks?

4. A flour merchant bought 30 barrels of flour for $155, giving $5½ a barrel for first quality, and $3½ for second quality; how many barrels of second quality did he buy?

5. Mr. Larkins sold 29 sheep for $86, receiving $4 a head for old sheep, and $2 for yearlings; how many yearlings did he sell?

6. Thomas agreed to carry 28 apples to market for ½ a cent each, and to forfeit 2½ cents for every one he ate. He received 11 cents; how many did he eat?

7. A man receives $3 a day for his labor, and pays $6 a week for his board. At the end of 5 weeks he has saved $25. How many days has he worked?

8. A farmer sold ⅔ of 1½ times his flock, and had 24 sheep remaining; how many sheep had he at first?

9. Vincent having some peaches, gave ⅔ of them to his brother, who after eating 4, gave ⅙ of what remained to his sister, and still had 5 remaining; how many had Vincent?

VII.

1. A man travelled 70 miles in two days; the first day he travelled $2\frac{1}{2}$ times as far as the second; how far did he travel each day?

SOLUTION.—The second day's journey is divided into 2 parts, 5 times one of which is equal to the first day's journey. The sum of the first and the second day's journey is equal to 7 equal parts, one of which is equal to $\frac{1}{7}$ of 70 miles, or 10 miles; 2 parts are equal to 20 miles, the second day's journey; and 5 parts are equal to 50 miles, the first day's journey.

2. A pole 66 feet long was broken so that $\frac{3}{8}$ of the longer piece was equal to the shorter; what was the length of each piece?

3. James and William together have 84 cents. William has $\frac{5}{7}$ as much as James; how much has each?

4. Thomas caught a fish 22 inches long; the tail was $\frac{3}{4}$ as long as the body, and the head was 8 inches; what was the length of the tail?

5. Divide 52 into two such parts that $1\frac{8}{9}$ times one shall be equal to the other.

6. Charles and Arthur together receive 63 cents; $\frac{3}{5}$ of what Charles receives plus 7 cents, equals what Arthur receives; how much does each receive?

SOLUTION.—Arthur receives 7 cents more than $\frac{3}{5}$ as much as Charles: hence the sum 63 cents is 7 cents greater than if Arthur had received only $\frac{3}{5}$ as much as Charles. Since Charles's money is divided into 5 equal parts, 3 of which are equal to Arthur's, 8 equal parts are equal to 63 cents minus 7 cents = 56 cents; one equal part equals 7 cents, 3 parts plus 7 cents = 28 cents, Arthur's money; and 5 parts = 35 cents, Charles's money.

7. A hat and a coat cost \$26; $\frac{1}{4}$ of the cost of the coat *plus* \$2 equals the cost of the hat; what is the cost of each?

8. The head of a fish is 6 inches long, the tail is as long as the head plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail together; what is the length of the fish?

9. If a horse and a colt were worth $92, and the horse was worth $12 more than $1\frac{1}{2}$ times the value of the colt, what was the value of each?

10. A dish and cover weigh 20 pounds; $\frac{1}{3}$ of the weight of the cover increased by 16 pounds equals the weight of the dish; what is the weight of each?

11. Divide the number 124 into two such parts that $\frac{4}{5}$ of the first plus 7 shall equal the second.

12. If a traveller pay $1.50 for his breakfast and dinner, how much does he pay for each, provided his breakfast cost $\frac{3}{7}$ as much as his dinner?

13. George has 75 apples and oranges; $\frac{2}{3}$ of the number of oranges are equal to $\frac{4}{5}$ of the number of apples; how many of each has he?

SOLUTION.—If $\frac{2}{3}$ of the number of oranges are equal to $\frac{4}{5}$ of the number of apples, $\frac{1}{3}$ of the oranges equals $\frac{2}{5}$ of the apples; and $\frac{3}{3}$, or all the oranges, equal $\frac{6}{5}$ or $\frac{3}{5}$ of the apples. Hence the whole number of apples and oranges is divided into 5 equal parts, 2 of which are equal to the apples, and 3 to the oranges.

14. George and Andrew received together 80 cents; $\frac{1}{2}$ of Andrew's money was equal to $\frac{5}{6}$ of George's; how much had each?

15. The sum of two numbers is 124; $\frac{4}{5}$ of one is equal to $\frac{3}{4}$ of the other; what are the numbers?

16. A tree 38 feet high was broken in a storm; $\frac{3}{5}$ of the length broken off was equal to $\frac{3}{7}$ of the length remaining; how much was broken off?

17. $2\frac{1}{3}$ times Robert's age is $1\frac{2}{5}$ times the age of Edmund, and the sum of their ages is 56 years; what is the age of each?

18. What is the time of day, if $\frac{1}{3}$ of the time to midnight is equal to the time past noon?

SOLUTION.—The time to midnight is divided into 3 equal parts, one of which equals the time past noon. Then 3 parts plus 1 part equals 4 parts = 12 hours, the time from noon to midnight; and 1 part = 3 hours. Hence it is 3 o'clock.

19. What is the hour, if the time past noon is $\frac{1}{5}$ of the time to midnight?

20. What is the hour, provided $\frac{5}{7}$ of the time past midnight equals the time to midnight again?

VIII.

1. John and James together have 78 cents; $\frac{2}{3}$ of John's money equals $\frac{2}{5}$ of James's, plus 4 cents; how much has each?

SOLUTION.—If $\frac{2}{3}$ of John's = $\frac{2}{5}$ + 4 cents, $\frac{1}{3}$ = $\frac{1}{5}$ + 2 cents; and $\frac{3}{3}$, or all John's = $\frac{3}{5}$ + 6 cents. Since John's = $\frac{3}{5}$ of James's + 6 cents, both together = $\frac{8}{5}$ + 6 cents = 78 cents. Hence, John has 33 cents, and James 45 cents.

2. Two men built 60 rods of fence; $\frac{2}{3}$ of what one built minus 6 rods, is equal to $\frac{3}{4}$ of what the other built; how many rods did each build?

3. The tail of a fish is 14 inches long, the head is $\frac{1}{2}$ as long as the tail minus 3 inches; and the body is as long as the head and the tail; what is the length of the fish?

4. After selling $\frac{3}{4}$ of my wheat, and $\frac{3}{4}$ of what then remained, I had 6 bushels left; how much had I at first?

5. Thomas being asked his age, said that $\frac{1}{2}$ of his age exceeded its $\frac{1}{3}$ by 4 years; how old was he?

6. After losing $\frac{3}{5}$ of my money I earned $\frac{2}{5}$ as much as I had lost, and then had $18 less than what I had at first; how much had I at first?

7. A tree was broken into 3 pieces; the second piece was 10 feet long, the third was as long as the second plus $\frac{1}{5}$ of the first, which was as long as the other two; what was the length of the tree?

SOLUTION.—The first piece equals $\frac{1}{2}$ of the tree; the third equals $\frac{1}{5}$ of $\frac{1}{2}$, plus 10 feet; and the whole tree equals $\frac{1}{2} + \frac{1}{10} + 20$ ft. Hence, $\frac{6}{10} + 20 = \frac{10}{10}$; $\frac{1}{10} = 5$ ft.; and $\frac{10}{10}$, or the whole tree, $= 50$ ft.

8. Three boys sell some papers; the first sells $\frac{2}{5}$ of the whole number less 10 papers; the second sells 20, and the third sells as many as the other two; how many does each sell?

9. Henry is 7 years more than $\frac{1}{2}$ as old as John, William is $\frac{1}{2}$ as old as Henry, and John's age is equal to the sum of William's and Henry's age; what is the age of each?

10. A hare being 30 leaps before a hound, takes 5 leaps to the hound's 3. If 4 of the hound's leaps are equal to 8 of the hare's, how many leaps must the hound take to catch the hare?

11. Michael is 6 years old, and Thomas is 20; in how many years will Thomas be twice as old as Michael?

SOLUTION.—If Thomas is now 20 years of age, 6 years ago he was 14, and 14 years from that time, or 8 years from the present, he will be 28 years old, or twice as old as Michael, who will then be 14 years of age.

12. Mary is 8 years old, and her sister is 30; in how many years will Mary be $\frac{1}{2}$ as old as her sister?

13. When I first met Mr. Scott, his age was twice mine, and in 10 years after that, my age was equal to $\frac{3}{5}$ of his; what was the age of each when we first met?

14. Mr. Garland's age increased by the difference between its $\frac{3}{4}$ and $\frac{5}{6}$, is 38 years more than $\frac{2}{3}$ of his age; how old is he?

15. Joseph earned $30 in September; in October, as much as in September and $\frac{1}{4}$ as much in November; and in November as much as in September and October together. How much did he earn in all?

16. A hound takes 2 leaps to a fox's 3, and 2 of the hound's leaps are equal to 4 of the fox's; how many leaps must the hound take to gain 1 on the fox?

17. I bought 6 yards of cloth for $8. How many more yards must I buy at $\$1\frac{1}{5}$ and $1 a yard that the average price of the whole may be $\$1\frac{1}{4}$?

18. One of two numbers is 5 times the other; but if 30 be added to each, one will be double the other; what are the numbers?

IX.

1. A farmer has 50 more sheep than cows; yet $\frac{3}{4}$ of the cows are equal to $\frac{1}{2}$ of the sheep; how many sheep has he?

2. Thomas bought 12 marbles more than his brother; and $1\frac{2}{5}$ times his brother's equalled $\frac{7}{8}$ of his; how many did each buy?

3. The difference between the cost of a coat and that of a whole suit is $12; and $\frac{2}{5}$ of the cost of the coat is equal to $\frac{1}{4}$ of the cost of the suit; what is the cost of the coat?

4. Susan is $\frac{1}{4}$ as old as her father; 4 years ago she was only $\frac{1}{6}$ as old; in how many years will she be $\frac{1}{3}$ as old?

5. A gains $6 while B gains $9; how much will each have gained when the difference between their gains is $.48?

6. Mark is 50 rods in advance of Philip, who follows at the rate of 5 rods a minute; if Mark runs 3 rods a minute, how long will it take Philip to overtake him?

7. A. B, and C bought a horse; B paid $30, which was $\frac{1}{5}$ of what A and C paid; A paid $\frac{1}{2}$ as much as B and C; what did A and C pay?

8. What is the time, if it is between 4 and 5 o'clock and the hands are directly opposite?

9. Henry and William had each 50 cents; Henry gave William 15 cents, and William gave Henry 5 cents; after which William lost such a number that he had then only 2 cents more than Henry. How many cents did William lose?

10. Michael has $\frac{4}{5}$ times as many marbles as his brother, who has 100. If each gives the other $\frac{1}{2}$ of the number he now has, how many will each then have?

WRITTEN REVIEW.

I.

1. After selling grain on commission at $5\frac{1}{2}\%$, through an agent, who charged me 3%, my commission was $250; what amount did I remit my employer? *Ans.* $9450.

2. Three men start together and travel in the same direction round a circular island whose circumference is 54 miles. A travels 6 miles an hour, B 9, and C 12. In what time will they all come together again, if they travel 10 hours a day? *Ans.* $1\frac{4}{5}$ da.

3. A can do a piece of work in 5 days, B can do twice as much in 6 days, and C 3 times as much in 10 days; in what time can they all together do the first piece of work?

4. What sum of money loaned for 4 years, at 5 per cent., will produce $45.50 interest?

5. C and D agreed to divide $154 in the ratio $\frac{3}{4}$ to $\frac{5}{8}$; how much did each receive? *Ans.* D, $70; C, $84.

6. A farmer sells a merchant 120 quarts of milk at 7 cents a quart, and makes a profit of 40%; the merchant sells the farmer 120 yards of cloth at 15 cents a yard, and makes a profit of 20%; which of the two makes the greater gain, and how much? *Ans.* Merchant, $0.60.

7. Smith and Scott find their stock to be worth $7200, of which Scott owns $4000. If they have gained 60% on their first capital, how much did each invest?

Ans. $2000, $2500.

8. A gentleman promised to pay a merchant $1200 in 2 years; but after 6 months he agreed to pay $500, if the merchant would wait long enough for the balance to compensate for the advance. How long should the merchant wait after the expiration of the two years?

9. If a man paid $240 premium on a house, at the rate of $3\frac{1}{2}$ per cent., for what amount was the policy given?

10. In 2 years and 6 months $5230 amounted to $6145.25, at simple interest; what was the rate per cent.? *Ans.* 7%.

11. A man has three notes payable as follows: one for $400, due Mar. 1, 1883, another for $250, due Nov. 1, and another for $600, due Feb. 1, 1884; what is the average time of maturity?

12. A and B commenced business with equal capitals. At the end of ten months A's gain was equal to $\frac{1}{4}$ of his capital, and B's $\frac{1}{5}$ of his; the difference between their gains was $1000; what was the capital of each?

Ans. $20000.

13. Mr. Gibbin bought goods to the amount of $3000, and gave his note, dated Feb. 4, 1881, on interest after 2 months. Three months after the note was given, he paid $620, and 5 months after the first payment, he paid $540; what was due Nov. 1, 1883?

14. If 6 pounds of coffee be equal in value to 10 pounds of sugar, and 12 pounds of sugar to 5 pounds of butter, and 7 pounds of butter to 10 pounds of cheese, how many pounds of coffee are equal in value to 125 pounds of cheese? *Ans.* 126.

15. What proportional quantity of wine at \$.90 a gallon, cider at \$.20, brandy at \$1.50, and water, would be required to make a mixture worth \$1.00 a gallon?

16. What is the difference between the interest and the discount of \$3500 at 6%, for 3 yr. and 5 months?

17. For what sum must a note be drawn at 8 months, that the proceeds of it, when discounted at a bank, at 6%, without grace, shall be \$690? *Ans.* \$718.75.

18. By selling goods for \$850 a person loses 15%; for how much should the goods have been sold to gain 25½%? *Ans.* \$1255.

19. When money is worth 5%, which is the more profitable, to sell coal at \$6 a T., on 9 mo., or \$5.62½ cash?

20. If the interest of \$342.25 be \$102.675 for 5 years, what will be the interest of \$800 for 8 years and 6 months? *Ans.* \$408.

21. What will a person gain or lose by exchanging 180 shares of 6% stock, at 85%, for stock bearing 8%, at 110%?

22. A man bought 150 cords of wood for \$555, paying \$4.00 for first quality, and \$3.50 for second quality; how many cords of the first quality did he buy? *Ans.* 60.

23. To what would \$500, deposited in a savings bank, amount in 6 years, interest being allowed semi-annually at 4% per annum?

24. A merchant in London gave £800 for a bill of exchange of \$4000 to remit to Philadelphia; what was the rate in favor of London?

25. At what rate should stock be bought to pay 12% on the investment, if, when bought at 10% discount, it pays 6% ?

26. I paid $960.30 duty, at 30%, on sugar in barrels weighing 300 lb. gross, and invoiced at 5½ cents per pound; how many barrels were there, allowing 3% for tare ?

27. How much must a person 18 years of age invest at 6% compound interest, to be worth $10061 when he is 30 years old ?

28. A gentleman bequeathed $24000 to his three sons in the ratio of 2, 6, and 8; but the first having died, the whole estate was equitably divided between the other two. How much did each receive ?

29. For what amount must a factory valued at $60000, be insured, so that in case of loss, the owners may recover both its value and the premium of 2% ?

30. How shall I mark cloth that cost $1.25 per yard, in order that I may be able to sell it at 20% less than the marked price, and still make 40% ?

31. Two men trade in company. A invests $4000, but after 6 months takes out a certain sum. B invests $3600, and after 4 months puts in $2000 more. At the end of the year, A's gain is $1296, and B's $1776; what did A take out after the 6 months ?

32. If 80 gallons of cider contain 10 gallons of water, how many gallons of water must be added in order that 4 gallons of the mixture may contain 3 quarts of water ?

33. A merchant sold 60 barrels of sugar which cost him $30 a barrel, at 15 cents a pound, thereby gaining 33⅓%; how many pounds did he sell ? *Ans.* 16000 lb.

34. If 8 cows or 12 horses eat 6½ tons of hay in 96 days, how long will 10 tons last 15 horses and 10 cows ?

35. A horse was sold for $160; had it been sold for $200, the gain would have been equal to the loss at the former price. What did the horse cost?

36. At what per cent. will the interest on $30.50 be equal to the principal in 12 years and 6 months?

37. A train ran 20 miles the first hour, and increased its speed at the rate of 3 miles an hour. How far did it run in 10 hours? *Ans.* 335 mi.

38. What % do I derive from buying 6% bonds at 105, and after 5 years, having them redeemed at par?

39. If 25% more than the selling price be 40% loss, what is the loss %?

40. I sold, at cost, goods which were bought on 8 months' credit; what did I gain %, money being worth 6%?

41. If stock bought at 12% premium yield 6% on the investment, what will it yield if purchased at 20% discount?

42. Three men receive, respectively, $1, $3, and $5 a day, working separately, 92 days in all, and earning equal sums of money; how many days did each man work? *Ans.* 60, 20, 12.

43. What is the face of a draft in Paris, that costs $2400, when exchange is 5.18 francs to the dollar?

44. How shall teas worth 60, 80, and 90 cents a pound, be mixed, so that 50 pounds of the mixture shall be worth $40?

45. What is the amount of a ground-rent of $800 which has remained unpaid for 5 years, compound interest at 6%?

46. At what time between 6 and 7 o'clock are the hour and the minute hand of a clock at right angles.

47. The proceeds of a 90-day note, discounted in New

York on the day of date, are $8860.50. For what sum must a 60-day note discounted at 4%, be drawn to take up the first note?

48. A man bought 3 carriages for $484, and sold them for equal sums, as follows: on the first he lost 25%, on the second he gained 20%, and on the third he gained 40%; what was the cost of each?

49. Divide $490 into two such parts that one shall be 60% more than 60% of the other.

50. The net proceeds are $4775, the rate of commission $4\frac{1}{2}$%; what is the commission?

51. What will I gain or lose in 2 years by borrowing $40000 from a bank at 6% discount, and lending the proceeds at $6\frac{1}{2}$% interest?

52. What is the equated time for paying a debt, $\frac{1}{3}$ of which is now due, $\frac{1}{4}$ in 60 days, and the remainder in 80 days?

53. An agent received $620 commission, at 4% for selling, and $2\frac{1}{2}$% for buying with the net proceeds; what was the amount of his sales?

54. A woman bought eggs at $2\frac{1}{2}$ cents apiece, and twice as many at 2 cents each; she sold them all at $2\frac{3}{4}$ cents apiece, making 84 cents profit; how many eggs did she buy? *Ans.* 144.

55. If 20 horses can be kept 15 days for $60, what sum will keep 25 horses for 45 days?

56. A mail-train, running at the rate of $36\frac{1}{2}$ mi. per hour, is 50 mi. ahead of an express-train, which runs at the rate of 45 mi. an hour; how long will it take the latter to overtake the former?

57. A, B, and C trading in company, gain $1665; A contributed $900 for $6\frac{1}{3}$ months, B $1500 for 10 months, and C $600 for 21 months; what is each man's share?

58. Find the average time of credit of $2000 due at once, $800 due in 5 mo. 10 da., and $1200 due in $6\frac{1}{2}$ months.

59. By selling wine at 95 cents a gallon I gain 3 times as much as I do by selling it at 75 cents a gallon; what does the wine cost me? *Ans.* $.65 per gallon.

60. Milk worth 24 cents a gallon has been reduced in value to 14 cents a gallon by the addition of water; what per cent. of water has been added?

61. A clock that gains 6 minutes in 3 days, is set right at 7:30 A.M. on Monday; what will be the correct time when the clock indicates 4 P.M. on Tuesday?

62. If 8 men or 12 boys can do a piece of work in $17\frac{1}{2}$ days, in what time can 5 men and 10 boys do it?

63. Two men enter into partnership with a capital of $9600, of which B furnishes $4000. How much must A put in at the end of 5 months, that he may receive $\frac{2}{5}$ of the year's gain?

64. Mr. Grace invested $7280 in bonds at $103\frac{1}{2}$, brokerage $\frac{1}{2}$%, and sold the same at 110, brokerage $1\frac{1}{4}$%; what was his gain?

65. A and B can do a piece of work in $3\frac{1}{3}$ days, B and C in $2\frac{1}{2}$ days, and A and C in $1\frac{2}{3}$ days. In what time can each do it working alone?

66. Which is the more profitable, to invest $7000 in $4\frac{1}{2}$% stock, at a discount of 30%, or to loan the same at 6% simple interest?

67. A loaned B $240 for 3 years and 4 months. How long should B lend A $500 to balance the favor?

68. A farmer sold 50 bushels of wheat and 45 bushels of corn for $48, receiving 20 cents per bushel more for wheat than for corn; what did he receive for each per bushel? *Ans.* $.60, $.40.

69. Having engaged an agent at $2\frac{1}{2}\%$ commission, to collect a bill of \$840, I received \$511.87$\frac{1}{2}$; what % of the bill did he collect? *Ans.*, $62\frac{1}{2}\%$.

70. A drover bought 20 sheep at \$5$\frac{1}{2}$ a head; how many more must he buy at \$8 and \$6 per head, that he may sell them all at \$6$\frac{1}{3}$ per head and lose nothing?

Ans. 12 at \$8, 10 at \$6.

71. Mr. Arthur bought $\frac{8}{9}$ of a farm; but the property having depreciated 12% in value, he sold 25% of his share for \$8222.50; what was the original value of the farm?

Ans. \$420467$\frac{7}{8}$.

72. How much must I invest in railroad stock paying 6% annually, to obtain a quarterly dividend of \$485.25?

73. What do I gain % by selling 36 hats for as much as I paid for 80?

74. A man buys wine at \$1.50 a gallon and brandy at \$2.80 a gallon; he mixes them in the proportion of 3 to 10, and sells the mixture at \$5.00 a gallon; what does he gain %?

75. From 3 bushels of pears and 30 pounds of sugar, a farmer made 90 jars of preserves. If the pears cost \$1.50 a bushel, and the sugar 10¢ a pound, at what price per jar must he sell the preserves, in order to gain 20%?

76. If the duty on imported goods at 30% ad valorem, be \$325.50, what was the cost of the goods?

77. A man paid \$102.30 for 3 barrels of ink, each containing 31 gallons. How much water must he mix with the ink that he may be able to sell it at \$1.10 a gallon and gain 40%?

78. If a bar of metal 6.75 ft. long, 2.5 ft. wide, and 1.6 ft. thick, be melted and cast into cubic form, what will be the edge of the cube? *Ans.* 3 ft.

79. If 50 yards of cloth, 1 yard wide, shrink 3% in

length and 4% in width by sponging, what will be the loss in square yards?

80. A woman buys hens' and ducks' eggs at a mean price of 20¢ a dozen. If 3 dozen of the latter are equal to 5 dozen of the former, what is the price of each per dozen?

81. Mr. Warren, who purchased $2000 worth of goods on 12 months' credit, paid $\frac{3}{4}$ of it so long before the time agreed on that he could equitably retain the balance for 18 months. When did he make the first payment?

82. If 25 men require 60 days to do a piece of work, how many men will be required to do 4 times as much in $\frac{1}{6}$ of the time?

83. On March 2, 1884, a man paid $4.50 for getting a note of $200 discounted at 5%. When did said note mature?

84. Two brothers, John and James, have the same income. John saves $\frac{1}{4}$ of his; but James, spending $600 a year more than John, at the end of 4 years finds himself $800 in debt. What is the income of each?

85. If 14 oxen eat as much as 12 horses, and 6 oxen and 4 horses eat 8 tons of hay in 50 days, how much hay will 5 horses and 12 oxen eat in 72 days?

86. A speculator bought $10000 in gold at $112\frac{1}{2}$ on May 20, paying $\frac{1}{2}$% commission. On June 20, he sold it at $115\frac{1}{4}$; what did he gain, interest being 7%?

87. Mr. Dillon sold some Mdse., at a loss of 5%; and Mr. Warren sold what cost him the same money, at a profit of 12%, thereby receiving $1360 more than Mr. Dillon. What did Mr. Dillon receive? *Ans.* $7600.

88. C's money was to D's as 9 to 11; but C having spent $400, and D $600, C's is to D's as 5 to 6. What had each at first? *Ans.* C, $5400; D, $6600.

89. A and B together had $12000; but when A had invested $\frac{3}{5}$ of his, and B $\frac{1}{3}$ of his, they had equal sums remaining. How much had each at first?

Ans. A, $7500; B, $4500.

90. How shall I sell potatoes, which I bought 8 months ago, on 3 months' credit, for $5.25 per barrel, so as to lose 20%, when money is worth 6%?

91. How many men, in 20 days of 10 hours each, will perform 5 times as much work as 40 boys will perform in 15 days of 12 hours each, 4 men being equivalent to 9 boys?

92. I sold $8000 in gold at 114, and invested the proceeds in bank stock at 75, which I sold soon after at 82½. How much did I gain?

93. The property of a certain town valued at $1400000, is to be taxed $11575. What will be the tax on the dollar, if there are 300 polls each taxed $1.25?

Ans. .008.

94. A note of $3500 is dated Feb. 4, 1880, on which was indorsed May 6, 1880, $300; and Dec. 10, 1880, $540. What was due March 25, 1881, interest being 6%?

95. I owe a note of $5280 in New Orleans, which has been on interest 6 mo. at 6%. If exchange is 102½, what will be the face of a sight draft to pay the note and interest?

96. The duty on 500 dozen London porter, at 40%, was $295.50; breakage, 1½%. What was the invoice price per dozen? *Ans.* $1.50.

97. Michael Hunter sold John Smith, June 10, 1884, goods to the amount of $1300 on 6 months' credit. Aug. 20, Mr. Smith paid $200; Sept. 18, $250; Oct. 30, $350. When, in equity, ought the balance to be paid?

Ans. April 4, 1885.

APPENDIX.

ANNUITIES.

210. An ***Annuity*** is a sum of money which is payable annually, or at some other regular interval of time.

211. A ***Perpetual Annuity*** is one which is unlimited in duration.

212. Perpetual Annuities can be terminated only by the grantor on the payment of a sum whose interest will equal the annuity.

213. A ***Certain Annuity*** is one which begins and ends at a fixed time.

214. A ***Contingent Annuity*** is one whose date of commencing or ending, or both, depends upon some uncertain circumstance, such as the birth or the death of one or more individuals.

215. A ***Deferred Annuity*** is one that will begin at some future time.

216. The ***Final Value*** of an annuity is the sum, or amount of all the payments on interest from the time each is due to the end of the annuity.

217. The ***Present Value*** is such a sum as put on interest for the given time and rate, will amount to the final value.

CASE I.

218. To find the final value of an annuity at simple interest.

1. What will be the final value of an annuity of $100 for 3 years, at 6 per cent.?

OPERATION.

$$\$100 + (6 \times 2) = \$112 = \text{Last term.}$$

$$\frac{\$100 + \$112}{2} \times 3 = \$318, \textit{ Ans.}$$

SOLUTION.—In this problem the annuity, $100, may be taken as the first term of an arithmetical series, in which the common difference is the interest for one year, the number of terms is the number of years, and the sum of the terms is the final value, or amount. Hence, by applying the rule for finding the sum of the series in Arithmetical Progression, we find the final value of the annuity.

WRITTEN EXERCISES.

2. What is the final value of an annuity of $300 for 5 years, at 4 per cent.? *Ans.* $1620.

3. An annuity of $800 was unpaid for 4 years; what was then due, interest at 5 per cent.? *Ans.* $3440.

4. What is the present value of an annuity of $150 for 8 years, at 4%? *Ans.* $1036.36.

NOTE.—The present value equals the final value, 1368, ÷ 1.32 = $1036.36.

5. Find the present value of an annuity of $4000 for 8 years, at 6%? *Ans.* $26162.16.

6. What must I pay for an annuity of $600 for 2 years, at 7 per cent.? *Ans.* $1242.

7. The rent of a store which is $1000 per year, has remained unpaid for 6 years. What amount is now due, allowing interest at 6%? *Ans.* $6900.

TABLE

Showing the amount of an annuity of $1 per annum, at compound interest for any number of years not exceeding forty-five.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.030000	2.035000	2.040000	2.050000	2.060000	2.070000
3	3.090900	3.106225	3.121600	3.152500	3.183600	3.214900
4	4.183627	4.214943	4.246464	4.310125	4.374616	4.439943
5	5.309136	5.362466	5.416323	5.525631	5.637093	5.750739
6	6.468410	6.550152	6.632975	6.801913	6.975319	7.153291
7	7.662462	7.779408	7.898294	8.142008	8.393838	8.654021
8	8.892336	9.051687	9.214226	9.549109	9.897468	10.259803
9	10.159106	10.368496	10.582795	11.026564	11.491316	11.977989
10	11.463879	11.731393	12.006107	12.577893	13.180795	13.816448
11	12.807796	13.141992	13.486351	14.206787	14.971643	15.783599
12	14.192030	14.601962	15.025805	15.917127	16.869941	17.888451
13	15.617790	16.113030	16.626838	17.712983	18.882138	20.140643
14	17.086324	17.676986	18.291911	19.598632	21.015066	22.550488
15	18.598914	19.295681	20.023588	21.578564	23.275970	25.129022
16	20.156881	20.971030	21.824531	23.657492	25.670528	27.888054
17	21.761588	22.705016	23.697512	25.840366	28.212880	30.840217
18	23.414435	24.499691	25.645413	28.132385	30.905653	33.999033
19	25.116868	26.357180	27.671229	30.539004	33.759992	37.378965
20	26.870374	28.279682	29.778079	33.065954	36.785591	40.995492
21	28.676486	30.269471	31.969202	35.719252	39.992727	44.865177
22	30.536780	32.328902	34.247970	38.505214	43.392290	49.005739
23	32.452884	34.460414	36.617889	41.430475	46.995828	53.436141
24	34.426470	36.666528	39.082604	44.501999	50.815577	58.176671
25	36.459264	38.949857	41.645908	47.727099	54.864512	63.249030
26	38.553042	41.313102	44.311745	51.113454	59.156383	68.676470
27	40.709634	42.759060	47.084214	54.669126	63.705766	74.483823
28	42.930923	46.290627	49.967583	58.402583	68.528112	80.697691
29	45.218850	48.910799	52.966286	62.322712	73.639798	87.346529
30	47.575416	51.622677	56.084938	66.438848	79.058186	94.460786
31	50.002678	54.429471	59.328335	70.760790	84.801677	102.073041
32	52.502759	57.334502	62.701469	75.298829	90.889778	110.218154
33	55.077841	60.341210	66.209527	80.063771	97.343165	118.933425
34	57.730177	63.453152	69.857909	85.066959	104.183755	128.258765
35	60.462082	66.674013	73.652225	90.320307	111.434780	138.236878
36	63.271944	70.007603	77.598314	95.836323	119.120867	148.913460
37	66.174223	73.457869	81.702246	101.628139	127.268119	160.337400
38	69.159449	77.028895	85.970336	107.709546	135.904206	172.561020
39	72.234233	80.724906	90.409150	114.095023	145.058458	185.640292
40	75.401260	84.550278	95.025516	120.799774	154.761966	199.635112
41	78.663298	88.509537	99.826536	127.839763	165.047684	214.609570
42	82.023190	92.607371	104.819598	135.231751	175.950645	230.632240
43	85.483892	96.848629	110.012382	142.993339	187.507577	247.776496
44	89.048409	101.238331	115.412877	151.143006	199.758032	266.120851
45	92.719861	105.781673	121.029392	159.700156	212.743514	285.749311

TABLE.

Showing the present worth of an annuity of $1 per annum, to continue for any number of years not exceeding forty-five.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	0.970874	0.966184	0.961538	0.952381	0.943396	0.934579	1
2	1.913470	1.899694	1.886095	1.859410	1.833393	1.808017	2
3	2.828611	2.801637	2.775091	2.723248	2.673012	2.624314	3
4	3.717098	3.673079	3.629895	3.545951	3.465106	3.387209	4
5	4.579707	4.515052	4.451822	4.329477	4.212364	4.100195	5
6	5.417191	5.328553	5.242137	5.075692	4.917324	4.766537	6
7	6.230283	6.114544	6.002055	5.786373	5.582381	5.389286	7
8	7.019692	6.873956	6.732745	6.463213	6.209744	5.971295	8
9	7.786109	7.607687	7.435332	7.107822	6.801692	6.515228	9
10	8.530203	8.316605	8.110896	7.721735	7.360087	7.023577	10
11	9.252624	9.001551	8.760477	8.306414	7.886875	7.498669	11
12	9.954004	9.663343	9.385074	8.863252	8.383844	7.942671	12
13	10.634955	10.302738	9.985648	9.393573	8.852683	8.357635	13
14	11.296073	10.920520	10.563123	9.898641	9.294984	8.745452	14
15	11.937935	11.517411	11.118387	10.379658	9.712249	9.107898	15
16	12.561102	12.094117	11.652296	10.837770	10.105895	9.446632	16
17	13.166118	12.651321	12.165669	11.274066	10.477260	9.763206	17
18	13.753513	13.189682	12.659297	11.689587	10.827603	10.059070	18
19	14.323799	13.709837	13.133939	12.085321	11.158116	10.335578	19
20	14.877475	14.212403	13.590326	12.462210	11.469421	10.593997	20
21	15.415024	14.697974	14.029160	12.821153	11.764077	10.835527	21
22	15.936917	15.167125	14.451115	13.163003	12.041582	11.061241	22
23	16.443608	15.620410	14.856842	13.488574	12.303379	11.272187	23
24	16.935542	16.058368	15.246963	13.798642	12.550358	11.469334	24
25	17.413148	16.481515	15.622080	14.093945	12.783356	11.653583	25
26	17.876842	16.890352	15.982769	14.275185	13.003166	11.825779	26
27	18.327031	17.285365	16.329586	14.643034	13.210534	11.986709	27
28	18.764108	17.667019	16.663063	14.898127	13.406164	12.137111	28
29	19.188455	18.035767	16.983715	15.141074	13.590721	12.277674	29
30	19.600441	18.392045	17.292033	15.372451	13.764831	12.409041	30
31	20.000428	18.736276	17.588494	15.592811	13.929086	12.531814	31
32	20.388766	19.068865	17.873552	15.802667	14.084043	12.646555	32
33	20.765792	19.390208	18.147646	16.002549	14.230230	12.753790	33
34	21.131837	19.700684	18.411198	16.192204	14.368141	12.854009	34
35	21.487220	20.000661	18.664613	16.374194	14.498246	12.947672	35
36	21.832252	20.290494	18.908282	16.546852	14.620987	13.035208	36
37	22.167235	20.570525	19.142579	16.711287	14.736780	13.117017	37
38	22.492462	20.841087	19.367864	16.867893	14.846019	13.193473	38
39	22.808215	21.102500	19.584485	17.017041	14.949075	13.264928	39
40	23.114772	21.355072	19.792774	17.159086	15.046297	13.331709	40
41	23.412400	21.599104	19.993052	17.294368	15.138016	13.394120	41
42	23.701359	21.834883	20.185627	17.423208	15.224543	13.452449	42
43	23.981902	22.062689	20.370795	17.545912	15.306173	13.506962	43
44	24.254274	22.282791	20.548841	17.662773	15.383182	13.557908	44
45	24.518713	22.495450	20.720040	17.774070	15.455832	13.605522	45

8. Mr. Larkin's hotel expenses are $60 a month; what sum paid at the beginning of the year, would pay it, interest at 6%? *Ans.* $697.92.

9. A gentleman bought a farm for $15000, agreeing to pay $3000 annually; but having neglected the yearly payments, he was obliged to pay the whole at the end of 5 years, with interest at 5 per cent. What amount did he pay? *Ans.* $16500.

CASE II.

219. To find the final value of an annuity at compound interest.

1. What is the final value of an annuity of $600 for 2 years, at 6%, compound interest?

OPERATION.

$2.06 = Am't of $1 for 2 yr.
600
$1236.00 *Ans.*

SOLUTION.—The amount of $1.00 for 2 yr., at 6% compound interest, as seen in the table, is $2.06. Hence, the amount of $600 for the same time and rate, is 600 times $2.06 = $1236, which is the final value of the given annuity.

RULE.—*Multiply the amount, or final value, of an annuity of $1 for the given time and rate, as found in the table, by the given annuity.*

WRITTEN EXERCISES.

2. Find the final value of an annuity of $240 for 3 years, at 5%, compound interest. *Ans.* $756.60.

3. What is the final value of an annuity of $750 for 6 years, at 3%, compound interest? *Ans.* $4851.31.

4. How much should I receive for an annuity of $900, unpaid for 5 years, at 7%, compound interest? *Ans.* $5175.67.

5. A man spends $5.00 a month for tobacco; how much could he save in 12 years, by investing this sum at 4%, compound interest? *Ans.* $901.55.

CASE III.

220. To find the present value of an annuity at compound interest.

1. What is the present value of an annuity of $200, to continue for 3 years, at 6%, compound interest?

OPERATION.

$2.673012 = Present value $1.
200
$534.602400

SOLUTION. — From the table we find that the present value of $1 for 3 years, at 6%, compound interest, is $2.673012; hence, the present value of $200 = $2.673012 × 200 = $534.60.

RULE.—*Multiply the present value of an annuity of $1 for the given time and rate, by the given annuity.*

NOTE.—The present value of an annuity may also be found by taking the present worth of the final value.

WRITTEN EXERCISES.

2. What is the present value of an annuity of $1500, to continue 10 yr., at 5%, compound interest?

Ans. $11582.60.

3. If I desire to purchase an annuity that shall secure to my son, at 6%, compound interest, $500 for 12 yr., what amount must I deposit in the annuity office?

Ans. $4191.92.

4. How much must I deposit in order to secure an annuity of $6400 for 3 yr. at 4%, compound interest?

Ans. $17760.58.

5. Which is the more profitable, to sell a factory for $25000 cash, or on equal installments of $5000 a year for 7 years, money being worth 6%?

Ans. The latter, $2911.91 more.

CASE IV.

221. To find the present value of an annuity in perpetuity.

1. Find the present value of a perpetual leasehold of $300 a year, at 6%.

OPERATION.

$300 ÷ .06 = $5000.

SOLUTION.—The interest of $1 at the given rate, is .06; hence, to produce an interest of $300, it will require as many times $1.00 as .06 are contained times in $300, which are $5000.

RULE.—*Divide the given annuity by the interest of $1 for one year.*

WRITTEN EXERCISES.

2. What is the present worth of a ground-rent of $150 a year, at 4%? *Ans.* $3750.

3. What is the present value of a perpetuity of $2000 a year, at 5%? *Ans.* $40000.

4. A ground-rent in the city of New York yields an annual income of $963, at 6%; what is the value of the estate? *Ans.* $16050.

CASE V.

222. To find the present value of a deferred annuity.

1. What is the present value of an annuity of $250, deferred 5 years and to continue 6 years, allowing 6% compound interest?

OPERATION.

$7.886875 =	Present	value	of	$1	for	11	yr.	
4.212364 =	"	"	"	"	"	5	"	
3.674511 =	"	"	"	"	"	6	"	deferred 5 yr.
250								
$918.62775 =	"	"	$250	"	"	"	"	" " "

SOLUTION.—The present value of an annuity of $1 for 11 years, is equal to its present value for 5 years, plus its present value for the 6 succeeding years. Hence, the present value of an annuity of $1 for 6 years, deferred 5 years, is equal to its present value for 11 years, minus its present value for 5 years. And the present value of $250 is evidently 250 times as much.

WRITTEN EXERCISES.

2. Find the present value of $800, to begin in 3 years and continue 7 years, interest at 5%. *Ans.* $3998.79.

3. Find the present worth of a reversionary perpetuity of $400, to begin 9 years hence, at 7% compound interest. *Ans.* $3108.19.

NOTE.—The present worth of $1 in perpetuity is found by the rule under Case IV.

4. What is the present worth of a lease of $700 deferred 4 years, and to run 12 years, at 4% compound interest? *Ans.* $5615.68.

5. If a reversionary annuity of $340 for 10 years, is to commence in 4 years, what is its present worth, at 6% compound interest? *Ans.* $1982.16.

CASE VI.

223. To find the annuity, the present or final value, time and rate being given.

1. An annuity running 12 years, at 5% compound interest, is worth $44316.26; what is the annuity?

OPERATION.

8.863252) 44316.26
$5000

SOLUTION.—The present worth of an annuity of $1 for the given time and rate, is $8.863252; hence, $44316.26 will give an annuity equal to 44316.26 ÷ 8.863252.

RULE.—*Divide the present or final value of the given annuity by the present or final value of an annuity of $1, for the given time and rate.*

WRITTEN EXERCISES.

2. Find an annuity that, at 4%, for 7 years, will amount to $1816.60762. *Ans.* $230.

3. What annuity, payable semi-annually, for 3 years, at 7% compound interest, will amount to $589.51368? *Ans.* $90.

4. A yearly pension, unpaid for 5 years, at 6% compound interest, amounted to $6764.5116; what was the pension? *Ans.* $1200.

5. I gave, in payment for a house, an annuity valued at $2058.50, deferred 3 years, and to run 9 years; what is the payment, interest at 3½%? *Ans.* $300.

PARTIAL PAYMENTS.

The principal rules for Partial Payments, as already mentioned, are the United States and the Merchants Rule. It may, however, be desirable to give the student a knowledge of the following particular rules, as stated in Kirby's Reports.

CONNECTICUT RULE.—"*Compute the interest to the time of the first payment, if that be one year or more from the time that interest commenced; add*

it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another till all the payments are absorbed; PROVIDED, *the time between one payment and another be one year or more. But if any payments be made before one year's interest hath accrued, then compute the interest, or the principal sum due to the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.**

"If any payments be made for a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period."

* NOTE.—"If a year does not extend beyond the time of payment; but if it does, then find the amount of the principal remaining unpaid up to the time of settlement, likewise the amount of the payment or payments from the time they were paid to the time of settlement, and deduct the sum of these several amounts from the amount of the principal."

VERMONT RULE.—*I. "When payments are made on notes, bills, or similar obligations, whether payable on demand or at a specified time, with interest, such payments shall be applied:* FIRST, TO LIQUIDATE THE INTEREST *that has accrued at the time of such payments, and* SECONDLY, TO THE EXTINGUISHMENT OF THE PRINCIPAL.

II. "The annual interests that shall remain un-

paid on notes, bills, or similar obligations, whether payable on demand or at a specified time, 'with interest annually,' shall be SUBJECT *to simple* INTEREST *from the time they become due to the time of final settlement.*

III. "If payments have not been made in any year, reckoning from the time such annual interest began to accrue, the amount of such payments at the end of the year, with interest thereon from the time of payment, shall be applied; FIRST, *to* LIQUIDATE *the* SIMPLE INTEREST *that has accrued from the* UNPAID ANNUAL INTERESTS; SECONDLY, *to* LIQUIDATE *the* ANNUAL INTERESTS *that have become due;* THIRDLY, *to the* EXTINGUISHMENT *of the* PRINCIPAL."

NOTE.—When partial payments are made on notes "with interest annually," the New Hampshire and the Vermont Rule are essentially the same. But when payments are made expressly on account of interest accruing but not then due, in New Hampshire, they are applied when the interest falls due, *without interest* on such payments.

MENSURATION.

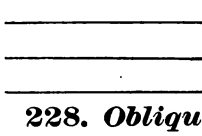
224. ***Mensuration*** is that branch of mathematics which treats of the measurements of lines, surfaces, and solids.

225. A ***Line*** is that which has length without breadth or thickness.

226. There are two kinds of lines; viz., the *straight* and the *curved.*

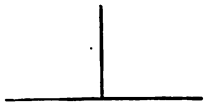
227. ***Parallel Lines*** are those which have the

same direction. Since they lie in the same plane, and are equally distant from each other, they can never meet.



228. ***Oblique Lines*** are those which approach each other, and will meet if sufficiently produced.

229. A ***Perpendicular Line*** is a line drawn to meet another in such a manner as to incline no more to the one side than to the other.



230. An ***Angle*** is the opening between two lines meeting each other in a common point called the *vertex.*

231. A ***Right Angle*** is an angle formed by two lines which are perpendicular to each other.

232. An angle which is less than a right angle, is termed *Acute;* and one which is greater, *Obtuse.*

SURFACES.

233. A ***Surface*** is that which has only two dimensions; viz., *length* and *breadth.*

234. Surfaces are either *plane* or *curved.*

235. A ***Plane Surface*** is one with which a straight line joining any two of its points, will exactly coincide.

236. A ***Curved Surface*** is one which changes its direction at every point.

237. A ***Polygon*** is a plane surface bounded by straight lines.

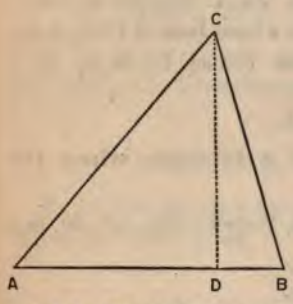
238. Polygons are named according to the number of their sides; thus, a polygon of three sides is called a *Triangle,* a polygon of four sides, a *Quadrilateral,* etc.

239. A ***Diagonal*** of a polygon is a line joining the vertices of two opposite angles.

240. The ***Perimeter*** of a polygon is the sum of all its sides.

241. The ***Area*** of a polygon is the surface included within its sides.

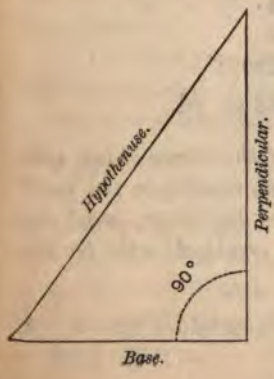
TRIANGLES.



242. A ***Triangle*** is a polygon having three sides and three angles; as, *ABC*.

243. The ***Base*** of a triangle is the side upon which it stands; as, *AB*. Any of the three sides of a triangle may be considered as the base.

244. The ***Altitude*** of a triangle is the perpendicular distance from the base to the opposite angle; as, *CD*.



245. A ***Right-Angled Triangle*** is a triangle having one of its angles a right angle. The side opposite the right angle is called the *Hypothenuse*.

CASE I.

246. To find the area of a triangle, when the base and the altitude are given.

1. Find the area of a triangle whose base is 16 ft., altitude 9 ft.

OPERATION.

$$\frac{16 \text{ ft.} \times 9 \text{ ft.}}{2} = 72 \text{ sq. ft.}$$

RULE.—*To find the area of a triangle, multiply the base by the altitude, and divide the product by 2.*

2. What is the area of a triangle whose altitude is 28 ch. 30 links, and base 24 ch. 48 links?

Ans. 34 A. 102.272 sq. rd.

3. Find the area of a triangle whose base is 18 ft. 4 in., and height 11 ft. 10 in. *Ans.* 108 sq. ft. 68 sq. in.

CASE II.

247. To find the area of a triangle when the three sides are given.

1. The sides of a triangle are, respectively, 25, 39, and 40 yd.; what is the area?

OPERATION.

$$25 + 39 + 40 = 104 \div 2 = 52$$
$$52 - 25 = 27$$
$$52 - 39 = 13$$
$$52 - 40 = 12$$
$$52 \times 27 \times 13 \times 12 = 219024$$
$$\text{Area} = \sqrt{219024} = 468 \text{ sq. yd.}$$

RULE.—*From half the sum of the three sides, subtract each side separately; then multiply the half sum and the three remainders together; and the square root of the continued product will be the area.*

2. Find the area of a triangle whose sides are 63, 60, and 39. *Ans.* 1134.

3. What is the area of a triangle whose sides are 78, 84, and 90? *Ans.* 3024.

CASE III.

248. Any two sides of a right-angled triangle being given, to find the third side.

1. If the base of a right-angled triangle is 30 ft., and the perpendicular 40 ft., what is the hypothenuse?

OPERATION.

$$\sqrt{30^2+40^2} = \sqrt{2500} = 50 \text{ ft., } \textit{Ans.}$$

2. In the triangle mentioned above, find the base, supposing the hypothenuse and perpendicular are given.

OPERATION.

$$\sqrt{50^2-40^2} = \sqrt{900} = 30 \text{ ft., } \textit{Ans.}$$

3. In like manner find the perpendicular, having the other two sides given.

OPERATION.

$$\sqrt{50^2-30^2} = \sqrt{1600} = 40 \text{ ft., } \textit{Ans.}$$

RULE.—*I. To find the hypothenuse, extract the square root of the sum of the squares of the other two sides.*

II. To find the base or the perpendicular, extract the square root of the difference between the squares of the two given sides.

4. The foot of a ladder is placed in the street 20 ft. from the wall of a house, and its top reaches directly to a window 48 feet above the level of the street; what is the length of the ladder? *Ans.* 52 ft.

5. How far from a wall 29 ft. high must a ladder 25 ft. long be placed so that a person may ascend to within 5 ft. of the top? *Ans.* 7 ft.

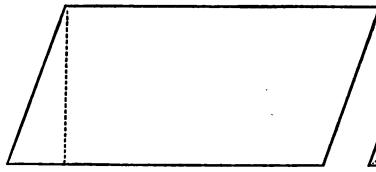
QUADRILATERALS.

249. A ***Quadrilateral*** is a polygon having four sides and four angles.

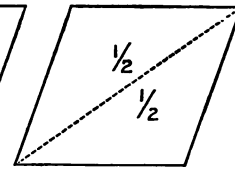
250. Quadrilaterals are divided into *Parallelograms, Trapezoids,* and *Trapeziums.*

251. A ***Parallelogram*** is a quadrilateral having its opposite sides parallel. The perpendicular distance between its opposite sides is called the *Altitude.*

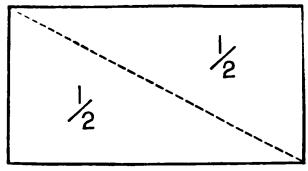
252. According to the nature of their sides and angles, parallelograms are divided into *Rhomboids, Rhombuses, Rectangles,* and *Squares.*



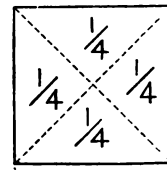
Rhomboid.



Rhombus.



Rectangle.



Square.

253. By closely observing and comparing these figures, it will be seen :

1. That each is a parallelogram having its opposite sides and opposite angles equal.

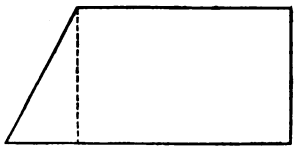
2. Either diagonal bisects the area; and if both be drawn, they will bisect each other, and divide the area into four equal or equivalent parts.

3. A *Rhomboid* is a parallelogram having its angles oblique, and its adjacent sides unequal.

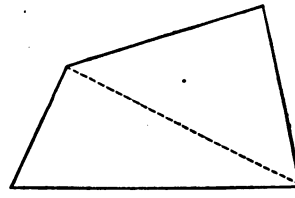
4. A *Rhombus* is an oblique-angled parallelogram having all its sides equal.

5. A *Rectangle* is a parallelogram having all its angles right angles.

6. A *Square* is an equilateral rectangle.



Trapezoid.



Trapezium.

254. A ***Trapezoid*** is a quadrilateral having only two of its sides parallel. Its altitude is the perpendicular distance between the parallel sides.

255. A ***Trapezium*** is a quadrilateral having none of its sides parallel.

Case I.

256. To find the area of a parallelogram, when the base and the altitude are given.

1. If the base of a parallelogram is 500 yd., and the altitude 363 yd., what is the area?

OPERATION.

500 yd. × 363 yd. = 181500 sq. yd.
181500 sq. yd. ÷ 4840 = 37.5 Acres., *Ans.*

RULE.—*To find the area of a parallelogram, multiply the base by the altitude.*

NOTE.—The area of a rectangle is found by multiplying the length by the breadth; and that of a square is equal to the side multiplied by itself.

2. What is the area of a parallelogram whose base is 125 ft., and altitude $37\frac{1}{2}$ ft.? *Ans.* $4687\frac{1}{2}$ sq. ft.

3. How many square feet of flooring will be required for a room $72\frac{1}{2}$ ft. long, and 53 ft. wide?

Ans. $3842\frac{1}{2}$ sq. ft.

4. How many acres in a garden in the form of a rhombus, the base being 150 yd., and the perpendicular height 30 rd.? *Ans.* 5 A. 550 sq. yd.

5. Find the area of a rhomboid whose length is 10.51 ch., and altitude 4.28 ch. *Ans.* 4 A. 1 R. 39.7 P.

6. A man bought a lot 121 yd. long, 40 yd. wide, and agreed to give $100 an acre. What did the lot cost?

Ans. $100.

CASE II.

257. To find the area of a trapezoid.

1. What is the area of a trapezoid whose parallel sides are 12 ft. and 18 ft., respectively, and whose altitude is 8 ft.?

OPERATION.

$$\frac{12 \text{ ft.} + 18 \text{ ft.}}{2} \times 8 \text{ ft.} = 120 \text{ sq. ft.}, \textit{Ans.}$$

RULE.—*Multiply one-half the sum of the parallel sides by the altitude.*

2. Find the area of a trapezoid whose parallel sides are $16\frac{3}{4}$ and $14\frac{5}{12}$ ft., respectively, and altitude 40 ft.

Ans. $615\frac{5}{6}$ sq. ft.

3. At \$12½ an acre, what is the value of a farm in the form of a trapezoid whose altitude is 22 chains, and its parallel sides respectively, 40 and 22 chains?

Ans. \$852.50.

CASE III.

258. To find the area of a trapezium.

1. What is the area of a trapezium whose diagonal is 32 ft., and the perpendiculars to this diagonal 16 and 18 ft.?

OPERATION.

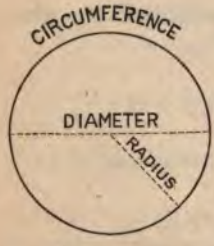
$$\left(\frac{16 \text{ ft.} + 18 \text{ ft.}}{2}\right) \times 32 \text{ ft.} = 544 \text{ sq. ft.}, \textit{ Ans.}$$

RULE.—*Multiply the diagonal by half the sum of the perpendiculars drawn to it from the vertices of the opposite angles.*

2. What is the area, in acres, of a field in the form of a trapezium, whose diagonal is ¼ of a mile, and the perpendiculars to this diagonal 5 ch. and 6 ch.?

Ans. 11 acres.

THE CIRCLE.



259. A ***Circle*** is a plane figure bounded by a curved line, called the *Circumference*, every point of which is equally distant from a point within called the *Centre*.

260. The ***Diameter*** of a circle is a straight line passing through the centre, and terminating at both ends in the circumference.

261. The ***Radius*** of a circle is a straight line drawn from the centre to any point in the circumference. The radius is one-half the diameter.

CASE I.

262. To find the circumference of a circle, when the diameter is given.

1. What is the circumference of a circle whose diameter is 25 ft.?

OPERATION.

$$25 \text{ ft.} \times 3.1416 = 78.54 \text{ ft.}$$

RULE.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

2. Find the circumference of a circle whose diameter is 63. *Ans.* 197.92.

3. A circular pond is 35 ft. in diameter; what is its circumference? *Ans.* 109.956 ft.

CASE II.

263. To find the diameter of a circle, when the circumference is given.

1. If the circumference of a circle is 25.1328 ft., what is its diameter?

OPERATION.

$$25.1328 \text{ ft.} \div 3.1416 = 8 \text{ ft.}, \textit{Ans.}$$

RULE.—*To find the diameter of a circle, divide the circumference by 3.1416.*

2. Find the diameter of a circle whose circumference is 50.2656. *Ans.* 16.

3. What is the diameter of a $\frac{3}{4}$-mile circular race course? *Ans.* 420 yd.

CASE III.

264. To find the area of a circle, when either its diameter or circumference is given, or when both are given.

1. Find the area of a circle whose diameter is 50 ft.

OPERATION.

$$50 \text{ ft.}^2 \times .7854 = 1963.5 \text{ sq. ft.}$$

2. Find the area of a circle whose diameter is 20 ft., and circumference 62.832 ft.

OPERATION.

$$\frac{62.832 \text{ ft.} \times 20 \text{ ft.}}{4} = 314.16 \text{ sq. ft.}$$

RULE.—*I. When only the diameter is given, multiply its square by .7854.*

II. When both the diameter and the circumference are given, find one-fourth of their product.

NOTE.—If only the circumference is given, multiply its square by .07958. Or, having found the diameter, follow Rule II.

3. What is the area of a circle whose circumference is 710 ft.? *Ans.* 40116.28 sq. ft.

4. If the radius of a circle is 75 ft., what is its area? *Ans.* 17671.5 sq. ft.

5. What is the value of a circular piece of ground whose circumference is 1500 ft., at $24 an acre? *Ans.* $98.65.

CASE IV.

265. To find the diameter or the circumference of a circle, when the area is given.

1. The area of a circle is 19.635 sq. ft.; what is the diameter?

OPERATION.

$$\sqrt{19.635 \div .7854} = \sqrt{25} = 5 \text{ ft., } \textit{Ans.}$$

2. What is the circumference of a circle whose area is 2578.23 sq. ft.?

OPERATION.

$$\sqrt{2578.392 \div .07958} = \sqrt{32400} = 180 \text{ ft., } \textit{Ans.}$$

RULE.—*I. To find the diameter, divide the area by .7854, and extract the square root of the quotient.*

II. To find the circumference, divide the area by .07958, and extract the square root of the quotient.

NOTE.—The circumference may also be found by first finding the diameter, and then applying the rule for Case I.

3. The area of a circle is 78.54 sq. yd.; what is the radius? *Ans.* 5 yd.

4. A horse is fastened to a post by a rope so that he may graze over 38.4846 sq. rods of grass; what is the length of the rope? *Ans.* 3½ rods.

5. What is the circumference of a circular fish-pond which has the same area as a square whose side is 40 rods? *Ans.* 141.78 rods.

SOLIDS.

266. A ***Solid,*** or *polyhedron,* has extension in length, breadth, and thickness or height.

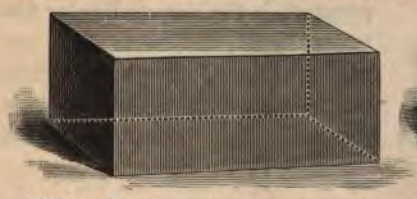
267. Solids include *Prisms, Pyramids, Cylinders, Cones, Spheres, etc.*

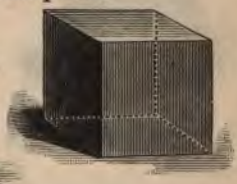
268. The planes bounding a solid are called *Faces,* and their intersections *Edges.*

269. A ***Prism*** is a solid whose bases are equal polygons, and whose sides are parallelograms.

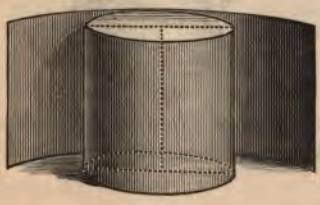
270. Prisms are called *triangular, quadrangular, pentangular,* etc., according to the form of their bases.

271. The parallelograms form the *Convex Surface,* and the perpendicular distance between the bases is called the *Altitude* of the prism.





272. A ***Parallelopipedon*** is a prism having parallelograms for its bases.



273. A ***Cube*** is a parallelopipedon having all its faces equal squares.

274. A ***Cylinder*** is a uniformly round body having equal and parallel circles for its bases.

275. The *Altitude* of a cylinder is a straight line joining the centres of its bases.

276. A ***Pyramid*** is a solid having for its base any polygon whatever, and for its lateral faces several triangles which meet in a common point called the vertex.

277. A ***Cone*** is a solid having a circular base, and its convex surface tapering uniformly to a point called the vertex.

278. The *Altitude* of a pyramid or cone, is the perpendicular distance from its vertex to the plane of its base.

279. The *Slant Height* of a pyramid is the perpendicular distance from the vertex to one of the sides of the base; while that of a cone is measured by a line drawn from the vertex to the circumference of the base.

280. When the top of a pyramid or cone is cut off by a plane parallel to the base, the part remaining is called the *Frustum*.

281. A ***Sphere*** is a solid bounded by a curved surface, every point of which is equally distant from a point within called the *Centre*.

282. A straight line drawn through the centre of a sphere and terminating at both ends in its surface, is called the *Diameter;* and a straight line drawn from the centre to any point in the surface, is termed the *Radius*.

CASE I.

283. To find the convex surface of any prism or cylinder.

1. What is the convex surface of a cube whose side is 8 ft.?

OPERATION.

8 ft. × 4 = 32 ft. = perimeter of base.
32 ft. × 8 ft. = 256 sq. ft., *Ans.*

NOTE.—Any two opposite faces of a cube may be considered as its bases.

2. What is the convex surface of a cylinder whose altitude is 10 ft., and the diameter of its base 10 ft.?

OPERATION.

3.1416 × 10 ft. = 31.416 ft. = perimeter of base.
31.416 ft. × 10 ft. = 314.16 sq. ft., *Ans.*

RULE.—*Multiply the perimeter of the base by the altitude.*

NOTE.—To find the entire surface, add the area of the bases to the convex surface.

3. Find the convex surface of a triangular prism whose altitude is 20 ft., and the sides of its base 8, 10, and 12 ft., respectively. *Ans.* 600 sq. ft.

4. How many sq. ft. of boards will it require to cover the entire outside of a cylindrical boiler, which is to be shipped to Boston, the diameter of the boiler being $3\frac{5}{8}$ ft. and the length 12 ft.? *Ans.* 213.39 sq. ft.

5. What is the entire surface of a cylindrical rolling-stone, 5 ft. in length, and 15 in. in diameter?

Ans. 22.09 sq. ft.

Case II.

284. To find the volume of any prism or cylinder.

1. What is the solidity of a triangular prism whose altitude is 15 ft., and the sides of its base, respectively, 3, 4, and 5 ft.?

OPERATION.

The area of the base is 6 sq. ft.
6 sq. ft. × 15 ft. = 90 cu. ft. = volume.

2. Find the volume of a cylinder whose altitude is 28 ft., and the diameter of its bases $2\frac{1}{2}$ ft.

OPERATION.

$2\frac{1}{2}$ ft.^{2} × .7854 = 4.90875 sq. ft. = area of base.
4.90875 sq. ft. × 28 ft. = 137.445 cu. ft., *Ans.*

Rule.—*Multiply the area of the base by the altitude.*

3. What is the cost of a block of Carrara marble 8 ft. long, 3 ft. 10 in. wide, and 6 ft. 6 in. high, at $2.25 a cu. ft.? *Ans.* $448.50.

4. How many bushels of wheat can a cylindrical vessel hold, that is 4 ft. 6 in. in diameter and 5 ft. 4 in. deep?
Ans. 68 bu. 5 qt.

CASE III.

285. To find the convex surface of a pyramid or cone.

1. What is the convex surface of a square pyramid, whose slant height is 60 in., and each side of its base 30 in.?

OPERATION.

30 in. $\times$ 4 = 120 in. = perimeter of the base.
120 in. $\times$ (60 in. $\div$ 2) = 3600 sq. in. = 25 sq. ft., *Ans.*

2. Find the convex surface of a cone whose diameter is 8 ft., and the slant height 20 ft.

OPERATION.

8 ft. $\times$ 3.1416 = 25.1328 ft. = circumference.
25.1328 ft. $\times$ (20 ft. $\div$ 2) = 251.328 sq. ft., *Ans.*

RULE.—*Multiply the perimeter of the base by one-half the slant height.*

NOTE.—Add the area of the base to the convex surface for the entire surface.

3. What is the entire surface of a pyramid whose base is a rectangle 12 ft. by 9 ft., and its slant height 8 ft. 6 in.? *Ans.* 286½ sq. ft.

4. The diameter of a cone is 20 in., and its slant height 20 ft.; what is its entire surface? *Ans.* 54.54 sq. ft.

CASE IV.

286. To find the solidity of a pyramid or cone.

1. What is the solidity of a square pyramid, the side of whose base is 9 ft., and the altitude 27 ft.?

OPERATION.

9 ft. $\times$ 9 ft. = 81 sq. ft. = area of base.
81 sq. ft. $\times$ ($\frac{1}{3} \times$ 27 ft.) = 729 cu. ft., Ans.

2. What are the solid contents of a cone, the diameter of whose base is 4 ft., and the altitude 15 ft.?

OPERATION.

4 ft.$^2 \times$ .7854 = 12.5664 sq. ft. = area of base.
12.5664 sq. ft. $\times$ ($\frac{1}{3} \times$ 15 ft.) = 62.832 cu. ft., *Ans.*

RULE.—*Multiply the area of the base by one-third of the altitude.*

3. Find the cost of a triangular pyramid of brown stone, whose altitude is 9 ft., each side of the base being 3 ft., at \$2$\frac{1}{2}$ per cu. ft. *Ans.* \$29.23.

4. What is the solidity of a cone generated by the revolution of a triangle about its perpendicular, whose hypothenuse is 13 ft. and base 5 ft.?

Ans. 314.16 cu. ft.

CASE V.

287. To find the convex surface of a frustum of a pyramid or cone.

1. Find the convex surface of a frustum of a square pyramid whose slant height is 19 ft., each side of the upper base 7 ft., and of the lower base 12 ft.

OPERATION.

48 ft. + 28 ft. = 76 ft. = sum of perimeters.
76 ft. $\times$ ($\frac{1}{2} \times$ 19 ft.) = 722 sq. ft., *Ans.*

RULE.—*Multiply the sum of the perimeters of the bases by one-half the slant height.*

NOTE.—The entire surface is equal to the convex surface plus the areas of the bases.

2. What is the whole surface of the frustum of a quadrangular pyramid, the slant height being 18 ft., each side of the upper end 5 ft., and each side of the lower end 14 ft.? *Ans.* 905 sq. ft.

3. Find the convex surface of the frustum of a cone whose slant height is 100 ft., and the diameters of whose bases are, respectively, 8 and 6 ft. *Ans.* 2199.12 sq. ft.

4. How much will it cost to line a cistern with cement, if it is 15 ft. square at the top, and 12 ft. square at the bottom, the slant being 8 ft., at $.25 a sq. ft.?

Ans. $144.

Case VI.

288. To find the solidity of a frustum of a pyramid or cone.

1. The altitude of a frustum of a cone is 12 ft. and the areas of the bases are, respectively, 4 and 9 sq. ft.; what is its solidity?

OPERATION.

$$\sqrt{4 \text{ sq. ft} \times 9 \text{ sq. ft.}} = 6 \text{ sq. ft.} = \text{mean base.}$$
$$4 \text{ sq. ft.} + 9 \text{ sq. ft.} + 6 \text{ sq. ft.} = 19 \text{ sq. ft.} = \text{sum of bases.}$$
$$19 \text{ sq. ft.} \times (\tfrac{1}{3} \times 12 \text{ ft.}) = 76 \text{ cu. ft.,} \quad \textit{Ans.}$$

Rule.—*To the sum of the areas of the bases add the mean proportional between them, and multiply the sum by one-third of the altitude.*

2. Find the volume of a frustum of a square pyramid, the sides of whose bases are 3 and 5 inches, respectively, and the altitude 18 in. *Ans.* 294 cu. in.

3. What is the volume of a frustum of a cone, whose height is 30 ft., and the radii of its bases, respectively, 8 and 5 ft.? *Ans.* 4052.664 cu. ft.

CASE VII.

289. To find the surface of a sphere.

1. Find the surface of a globe 50 in. in diameter.

OPERATION.

$$50 \text{ in.}^2 \times 3.1416 = 7854 \text{ sq. in.}$$

RULE.—*Multiply the square of the diameter by 3.1416.*

2. What is the surface of a sphere whose diameter is 4 ft. 8 in.? *Ans.* 9852 sq. in.

3. What would it cost to gild a ball for a flag-pole, at $1\frac{1}{4}$ cents a sq. in., the diameter of the ball being 4 in.? *Ans.* $0.63.

CASE VIII.

290. To find the solidity of a sphere.

1. Find the solidity of a globe whose diameter is 12 ft.

OPERATION.

$$6 \text{ ft.}^3 \times 4 \times 3.1416 \div 3 = 904.78 + \text{ cu. ft., } \textit{Ans.}$$

RULE.—*Multiply 4 times the cube of the radius by 3.1416, and divide the product by 3.*

NOTE.—The solidity of a sphere may also be found by multiplying the cube of the diameter by .5236.

2. What is the volume of a globe whose diameter is 18 ft.? *Ans.* 3053.64 cu. ft.

3. How many cu. ft. of gas in a balloon whose diameter is 40 ft.? *Ans.* 33510.4 cu. ft.

GAUGING.

291. ***Gauging*** is the process of finding the capacity of casks and other vessels.

NOTE.—A cask or barrel may be considered as a cylinder having the same length and a *mean* or *bulging* diameter.

1. How many gallons in a cask whose head diameter is 28 in., bung diameter 37 in., and length 40 in.?

OPERATION.

$$28 \text{ in.} + (37 \text{ in.} - 28 \text{ in.})\tfrac{2}{3} = 34 \text{ in.} = \text{mean diam.}$$
$$34 \text{ in.}^2 \times 40 \text{ in.} \times .0034 = 157.216 \text{ gal.}, \textit{ Ans.}$$

RULE.—*I. To the head diameter, add $\frac{2}{3}$ of the difference between the head and the bung diameter, and the result will be the mean diameter.*

II. Multiply the square of the mean diameter by the length, both expressed in inches, and their product by .0034.

2. Find the quantity of vinegar in a cask whose bung and head diameters are 32.5 and 26.5 in., respectively; the length being 42.5 inches. *Ans.* 134.42 gal.

3. How many gallons in a cask whose bung diameter is 38 in., head diameter 32 in., and length 40 in.? *Ans.* 176.256 gal.

4. What are the contents of a cask whose bung diameter is 36 in., head diameter 30 in., and length 48 in.? *Ans.* 188.66 gal.

5. Find the number of gallons in a cask whose length is 40 in., bung diameter 31 in., and head diameter 24 in. *Ans.* 111.76 gal.

SIMILAR POLYGONS AND SOLIDS.

292. ***Similar Polygons*** are such as have their angles equal, each to each, the same number of sides, and the sides about the equal angles proportional.

293. ***Similar Solids*** are those which have their solid angles equal and like placed, and are bounded by the same number of similar and like-placed polygons.

PRINCIPLES.—*I. The areas of similar polygons are proportional to the squares of their like dimensions.*

II. The volumes of similar solids are proportional to the cubes of their like dimensions.

1. If the diameter of a circular field containing 28.2744 sq. ft., is 6 ft., what is the diameter of a similar field containing 78.54 sq. ft.?

OPERATION.

$$28.2744 \text{ sq. ft.} : 78.54 \text{ sq. ft.} :: 6 \text{ ft.}^2 : x^2$$
$$x^2 = 100 \text{ sq. ft.}$$
$$x = 10 \text{ ft.}, \textit{ Ans.}$$

2. Two rectangles are to each other as 7 : 15; the length of the first is 80 ft.; what is the length of the second? *Ans.* 117.108 ft.

3. What is the area of a triangle whose base is 25 ft., if the base of a similar triangle containing 150 sq. ft., is $12\frac{1}{2}$ ft.? *Ans.* 600 sq. ft.

4. If the side of a cubical cistern containing 1728 cu. in., is 1 ft., what is the solidity of a similar cistern whose side is 4 ft.? *Ans.* 110592 cu. in.

5. If a cone 6 ft. in height contains 1539 cu. ft., what are the contents of a similar cone 4 ft. in height?

Ans. 456 cu. ft.

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